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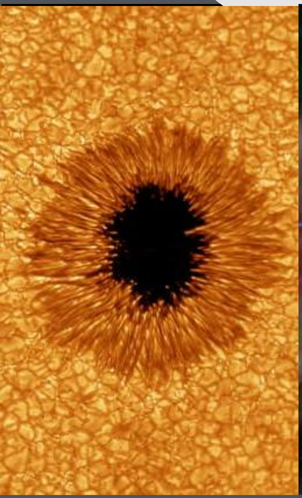
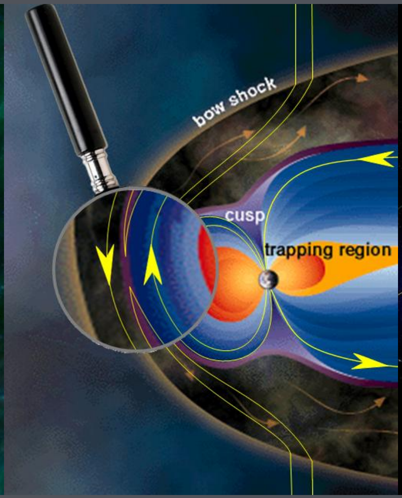
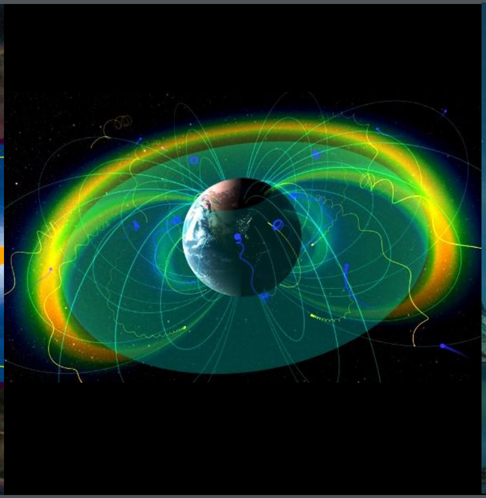







Optimum Coupling Functions for solar wind-magnetosphere interaction quantification

Solar Wind Magnetosphere Interaction Workshop
September 16, 2021

- My Definition

coupling function

/ˈkʌp(ə)lɪŋ ˈfʌŋkʃ(ə)n/

(*compound noun*)

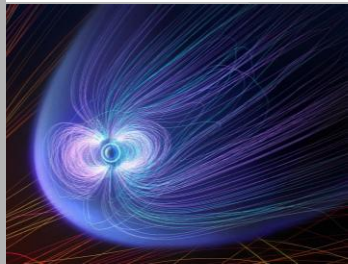
A combination of interplanetary parameters designed to best quantify and predict **a** terrestrial space weather disturbance indicator or index

Coupling functions

- Great many proposed since first correlations of terrestrial and interplanetary observations
- Theory-based

  (i) power input

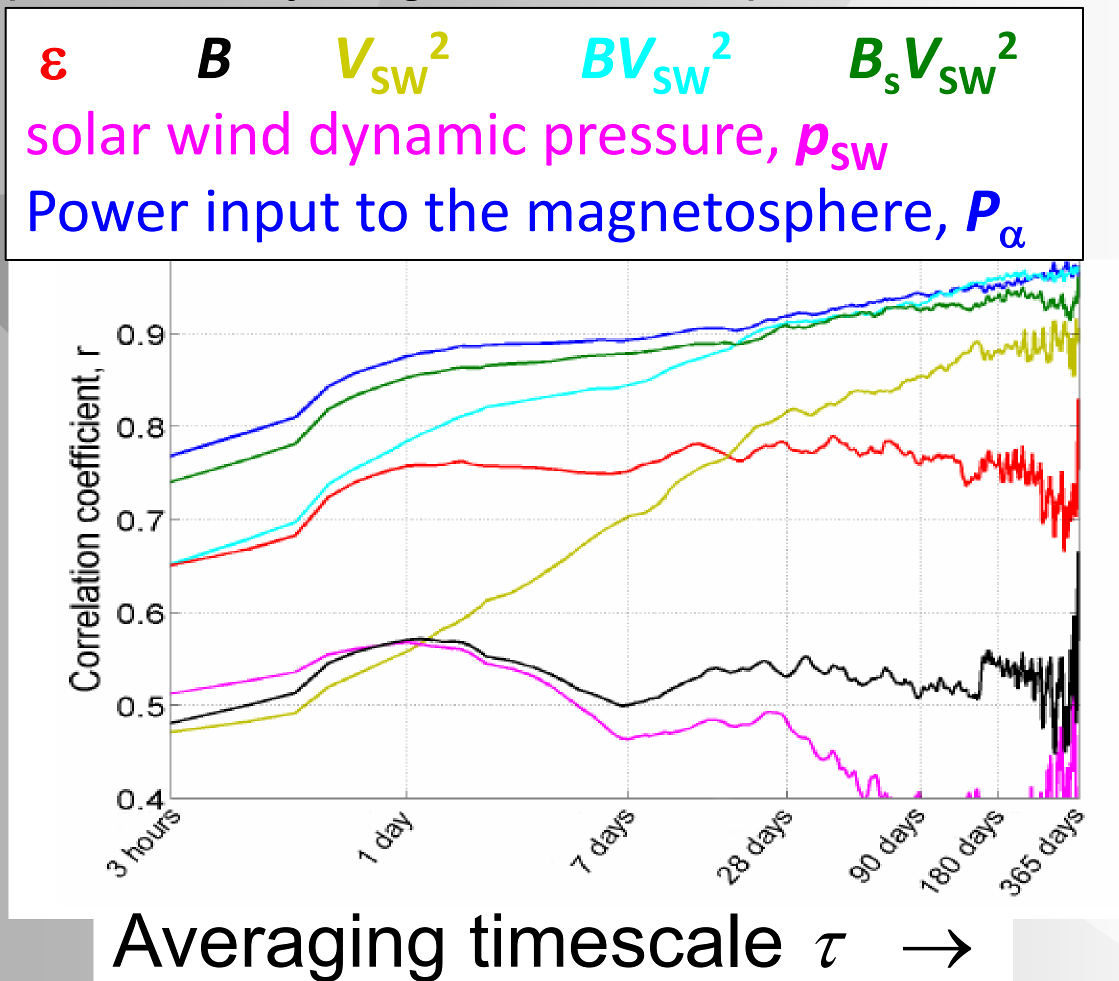
  (ii) voltage applied
- **Or** based on empirical correlations
- **Or** based on theory with empirically-derived factors
- **Or** from numerical global simulations
- **But** there are pitfalls with data in both derivation and testing



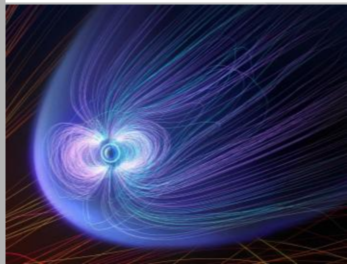
The epsilon factor, $\varepsilon = B^2 V_{sw} \sin^4(\theta/2)$

(comparisons by Finch and Lockwood, 2007)

- Incorrect theoretical basis: assumes relevant solar wind power is Poynting flux: result ε performs less well at all τ



- B = IMF, V_{sw} = solar wind speed, θ = IMF clock angle in GSM
- comparison allows for the effects of data gaps and of weightings given to datapoints by virtue of their frequency of occurrence
- ε (in red) performs less well at all timescales
- Do not use ε! (even to benchmark others as it is not a good test)



Simple theory-based coupling functions

1. Energy transfer into the magnetosphere

(Vasyluinas et al., 1982)

- Magnetospheric Power Input

$$P_\alpha = (\pi L_o^2) \times (m_{sw} N_{sw} V_{sw}^2/2) V_{sw} \times (t_r)$$

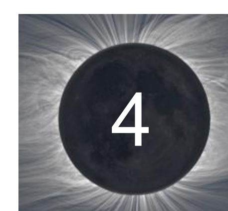
- from pressure balance & hemispheric dayside m'sphere:

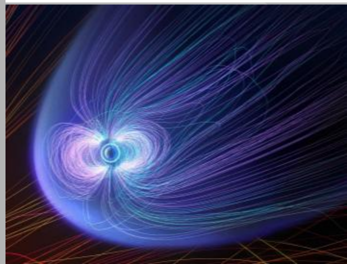
$$L_o = c L_s = c k_1 (M_E^2 / P_{sw} \mu_o)^{1/6}$$

- Vasyluinas et al. (1982) dimensionless transfer function:

$$t_r = k_2 M_A^{-2\alpha} \sin^d(\theta/2)$$

- Alfvén Mach #, $M_A = V_{sw} (\mu_o m_{sw} N_{sw})^{1/2} / B_{sw}$ • α is the "coupling exponent" (the one free fit parameter) • L_s is stand-off distance of nose and $c = L_o / L_s$ the magnetopause shape factor • M_E is the Earth's magnetic moment • N_{sw} is the solar wind concentration • V_{sw} is the solar wind velocity, • B_{sw} is the IMF magnitude • θ is the clock angle that the IMF makes with the north in the Earth's GSM frame of reference. • k_1 is the "blunt nose" sheath flow factor • k_2 is a constant • d is the IMF orientation factor exponent





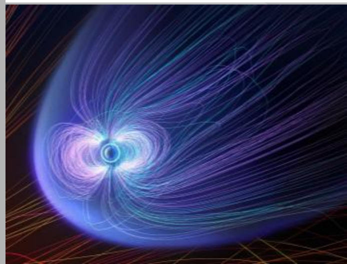
Simple theory-based coupling functions

(Vasyluinas et al., 1982)

- Gives magnetospheric Power Input

$$P_\alpha = \{M_E^{2/3} c^2 k_1 k_2 \pi / (2\mu_o^{(1/3-\alpha)})\}$$
$$\times\ B_\perp^{2\alpha}\ \rho_{sw}^{(2/3-\alpha)}\ V_{sw}^{(7/3-2\alpha)}\ sin^{d}(\theta/2)$$

- Use IGRF model to get M_E
- c taken to be a constant (OK for solar SW pressure changes but not so good for IMF B_z changes because of dayside erosion/tail flaring)
- $\{M_E^{2/3} c^2 k_1 k_2 \pi/(2\mu_o^{(1/3-\alpha)})\}$ can usually be taken to be a constant: I often use P_α/P_o where $P_o = <P_\alpha>_{\text{all data}}$, so this term cancels out
- Vasyluinas et al. point out that d is not and independent fit parameter but is set for a given α to ensure linearity with response
- Vasyluinas et al. find transverse component of $B_\perp = (B_z^2 + B_y^2)^{1/2}$ actually works better than B
- Derive the 1 **ONE** fit parameter, α, to maximise correlation of P_α with the terrestrial disturbance index to be predicted



Simple theory-based coupling functions

2. Applied voltage

(Reiff and Luhmann, 1986; Siscoe et al, 2002; Lockwood and McWilliams, 2021)

- Relevant solar wind electric field

$$E_{sw} = B_{\perp} V_{\mathrm{sw}} \sin^{\mathrm{d}}(\theta/2)$$

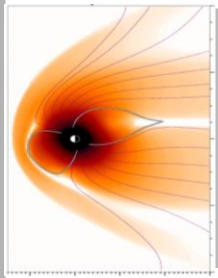
- Voltage across the diameter of the magnetosphere

$$\Phi_{\mathrm{M}} = 2L_{\mathrm{o}} B_{\perp} V_{\mathrm{sw}} \sin^{\mathrm{d}}(\theta/2) = c k_3 (M_{\mathrm{E}}^2 / P_{\mathrm{sw}} \mu_{\mathrm{o}})^{1/6} B_{\perp} V_{\mathrm{sw}} \sin^{\mathrm{d}}(\theta/2)$$

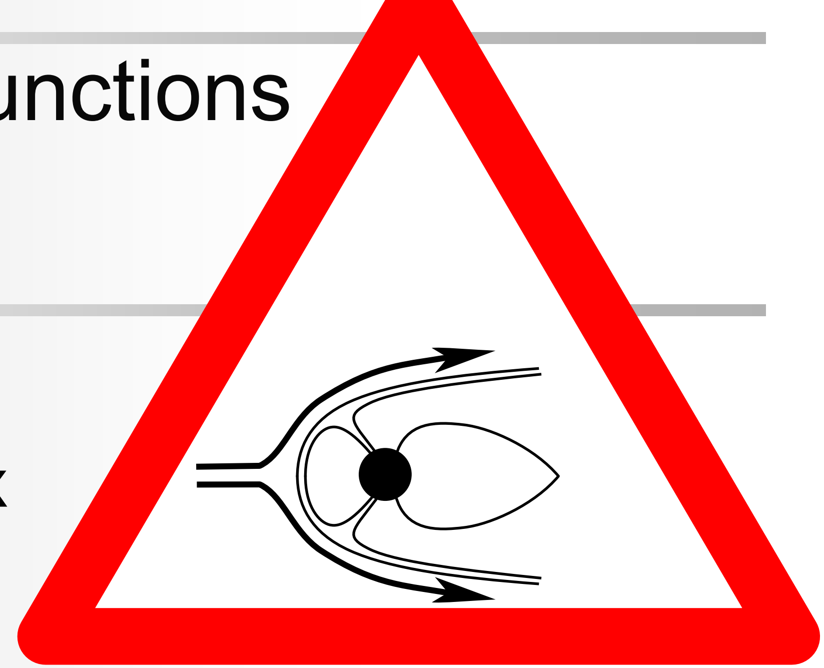
- Polar cap voltage for reconnection efficiency η :

$$\Phi'_{\mathrm{PC}} = \eta \Phi_{\mathrm{M}} = \eta k_4 B_{\perp} \rho_{\mathrm{sw}}^{-1/6} V_{\mathrm{sw}}^{2/3} \sin^{\mathrm{d}}(\theta/2)$$

- BUT η is not a constant *(e.g. Borovsky and Birn, 2014)*
- any dependence of η on $B_{\perp}$, ρ_{sw} or V_{sw} would change the exponents in the predicted Φ_{PC}



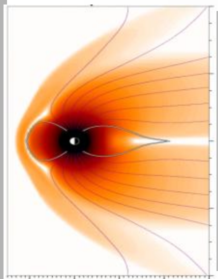
Empirical coupling functions



- There are many more complex theoretical functions
- A common empirical form also encompassing 1st-order theories is:

$$C_f = B_\perp^{a}\, \rho_{sw}^{b}\, V_{sw}^{c}\, \sin^{d}(\theta/2)$$

- Where the exponents a, b and c are fit parameters (c.f. for P_α with α = 1/3: a = 0.67, b = 0.33 and c = 1.67 for Φ'_{PC} with constant η: a = 1, b = –0.17 and c = 0.67)



A Pitfall

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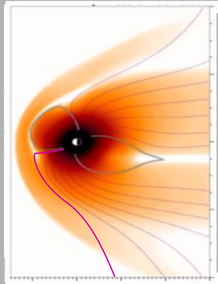
- with too many free fit parameters we can start to fit to the noise in the training data - resulting in the fit having reduced predictive power

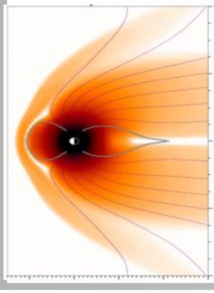
- recognised pitfall when signal-to-noise ratio is low in (e.g.) climate science and population growth but was often not recognised in space physics, especially before systems research and machine leaning were introduced into the field

- How to avoid it? In “k-fold” cross-validation, partition the data into k subsets (“folds”). Then, iteratively train the algorithm on all but one fold and use that as the test set (the “holdout fold”)



Sources of noise

- observation errors in interplanetary measurements
- observation errors in terrestrial disturbance index or indicator
- propagation errors: lag variations and spatial structure in interplanetary space
- data gaps
- effects of averaging & timescale
- non-linear response, pre-conditioning, time history
- dipole tilt effects (on ionospheric conductivities, magnetospheric structure and current sheets, and magnetopause coupling)



Effects of the persistence of interplanetary parameters

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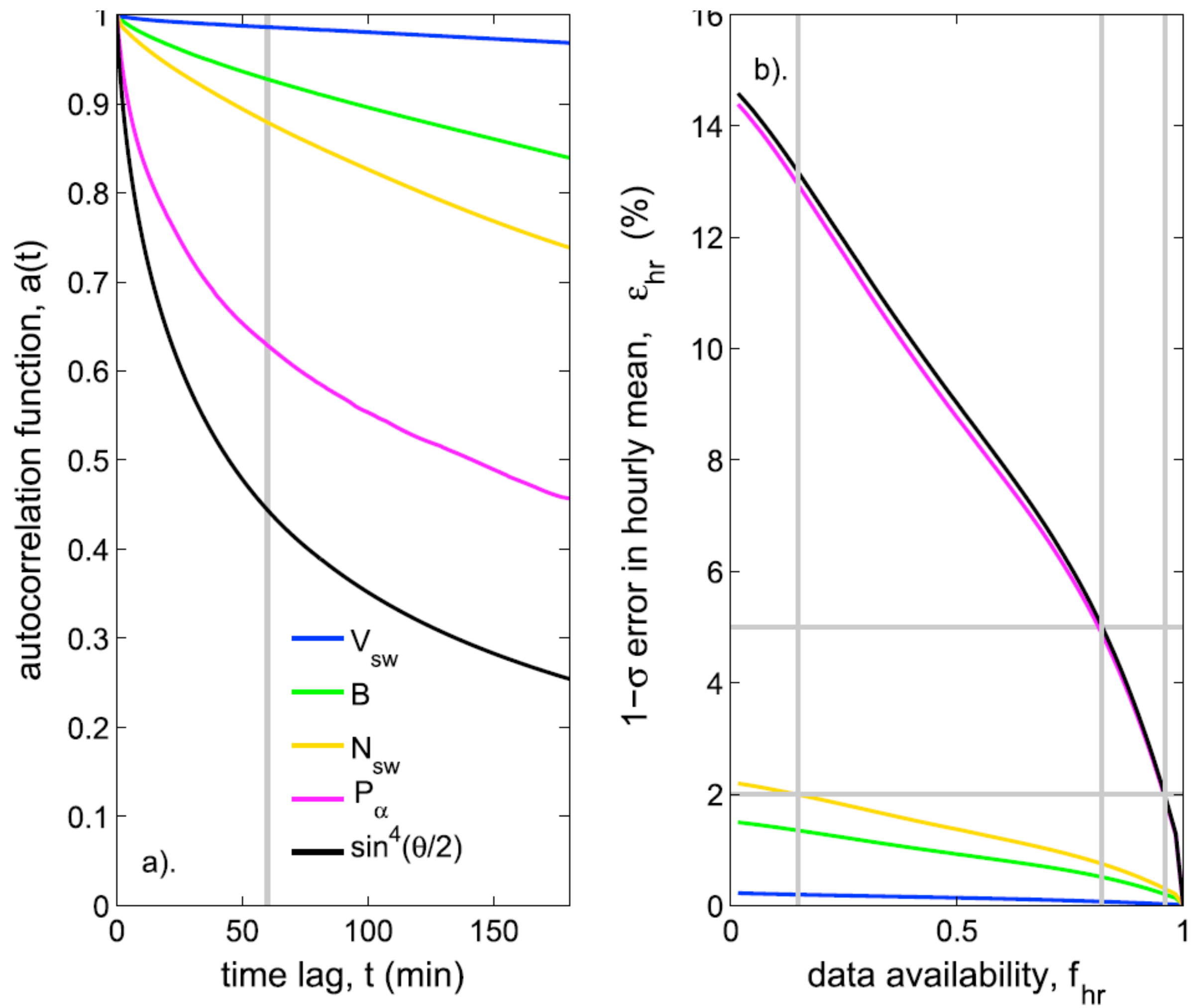
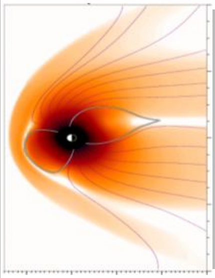
Lockwood et al. (2019)

18,063 hours with all 60 1-minute values 1996-2016 (1,083,797 1-min samples)

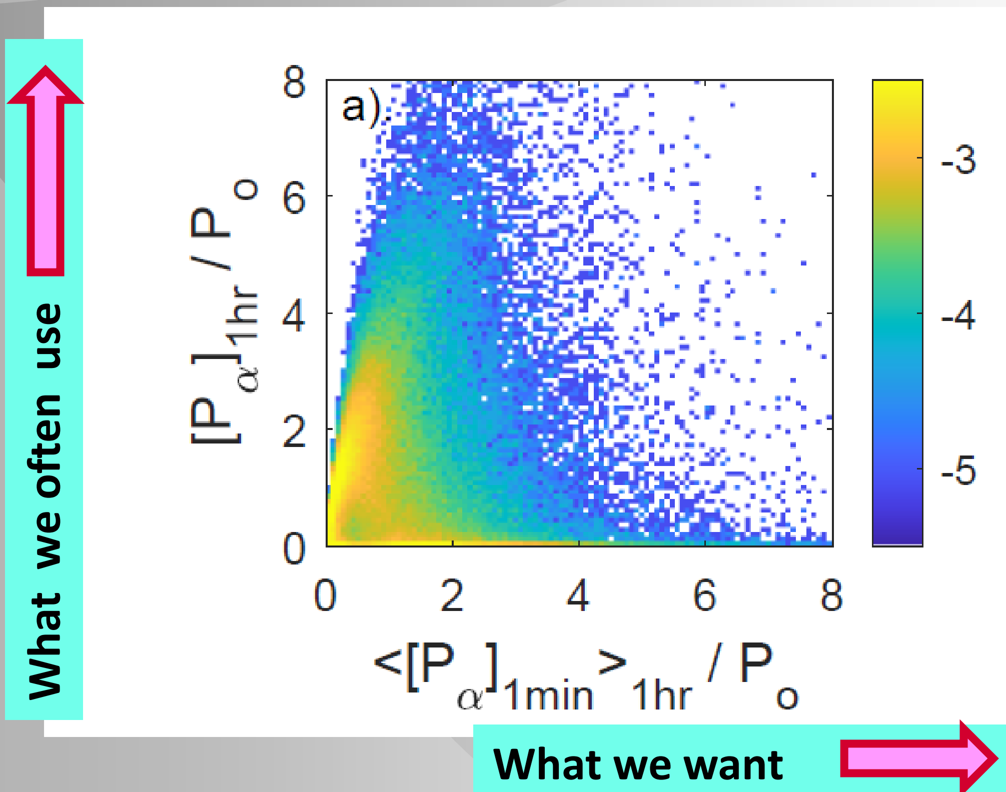


- Flat acf of V_{SW} (high persistence) means we only need one sample in an hour for mean value to be accurate to within 5%
- But for IMF orientation's low persistence means we need 82% of samples in an hour

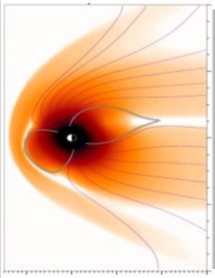


Timescale, averaging and weighting

- We want $\int_{\tau} C_f \, dt = \tau < C_f >_{\tau}$
- So we want $< C_f > = < B_{\perp}^{a} \rho_{sw}^{b} V_{sw}^{c} \sin^{d}(\theta/2) >$
- some studies use $[C_f] = <B_{\perp}>^{a} <\rho_{sw}>^{b} <V_{sw}>^{c} \sin^{d}(\theta_{1hr}/2)$



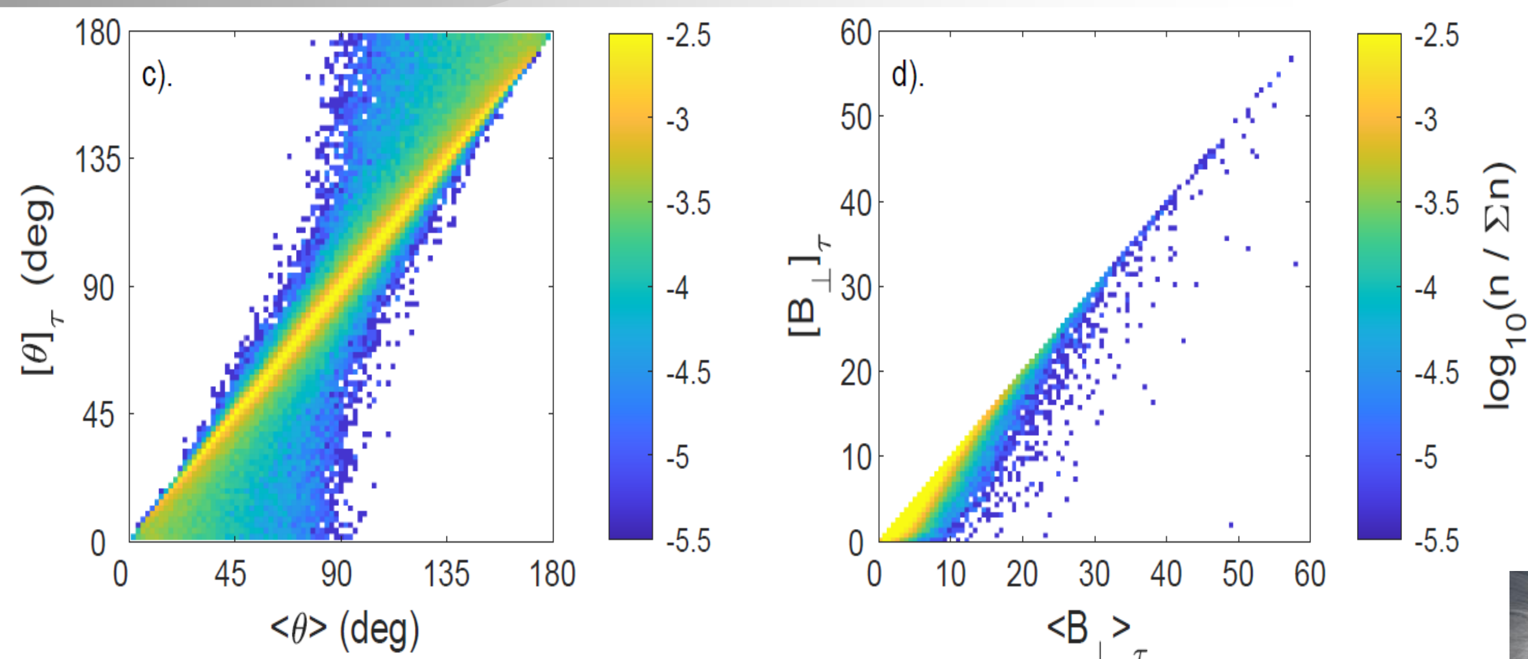
- where $\tan(\theta_{1hr})$ = $(|<B_y|>_{1hr}| / <B_z>_{1hr})$
- this is for example of P_{α} for $\alpha = 1/3$, giving $a = 2/3$, $b = 1/3$ and $c = 5/3$ with $d = 4$
- $r = 0.28$!



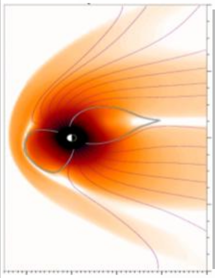
Timescale, averaging and weighting

- Big part of problem lies in using averaged B_x and B_y components to compute and $B_\perp$ and, in particular, θ
- Solved by computing $B_\perp$ and θ at high time resolution and then averaging *(e.g. McPherron et al., 2015)*

What we often use

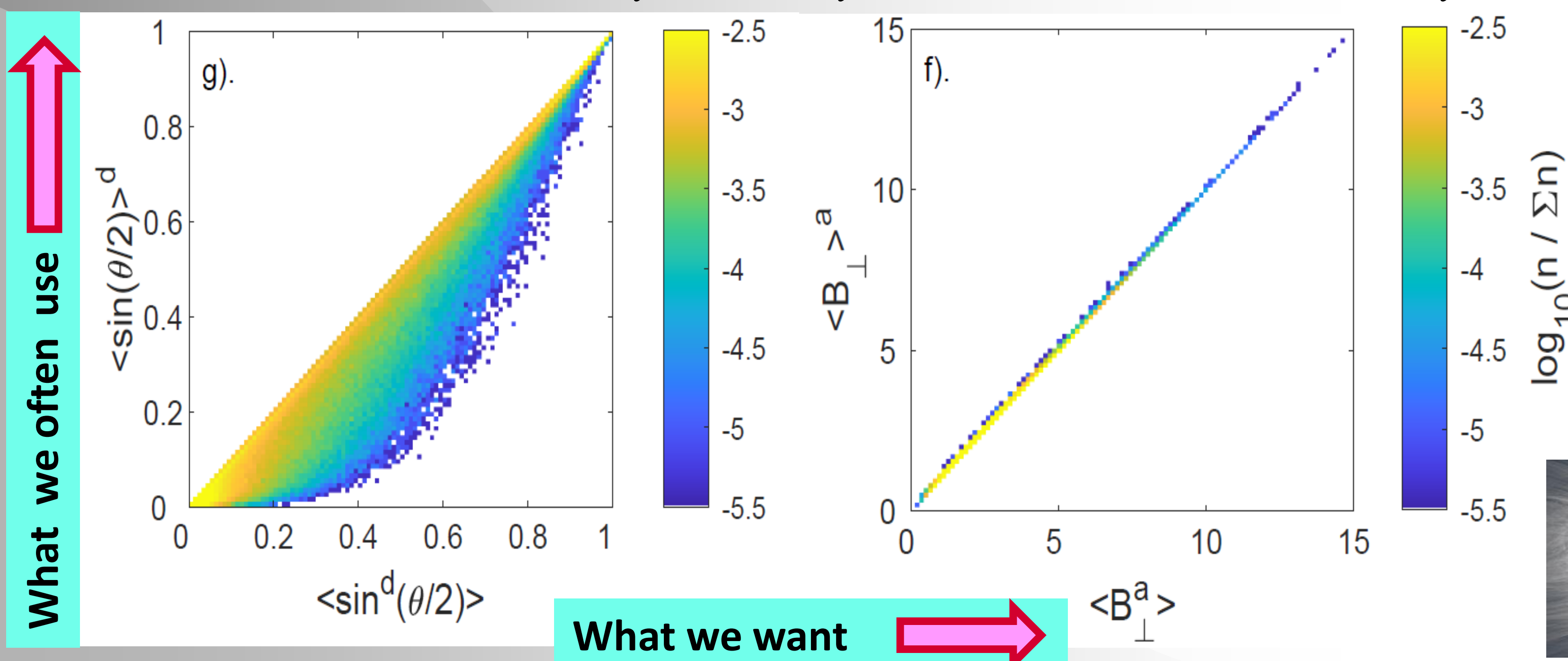


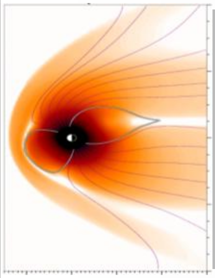
What we want



Timescale, averaging and weighting: $<x^p>_\tau \neq <x>_\tau^p$

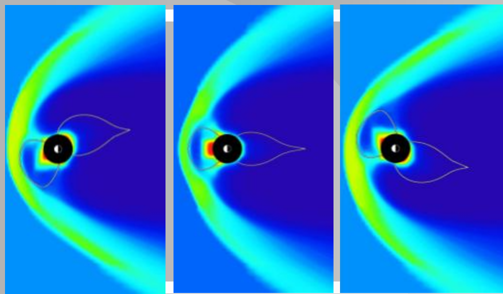
- But there is another problem which affects $\sin^d(\theta/2)$ most because of its much greater variability within the hour
- For $p > 1$ the Hölder ("power") mean $[<x^p>_\tau]^{1/p}$ exceeds arithmetic mean $<x>_\tau$ - and the difference increases with p and τ
- Hence for $p > 1$, $<x^p>_\tau > <x>_\tau^p$ (and for $p < 1$, $<x^p>_\tau < <x>_\tau^p$)



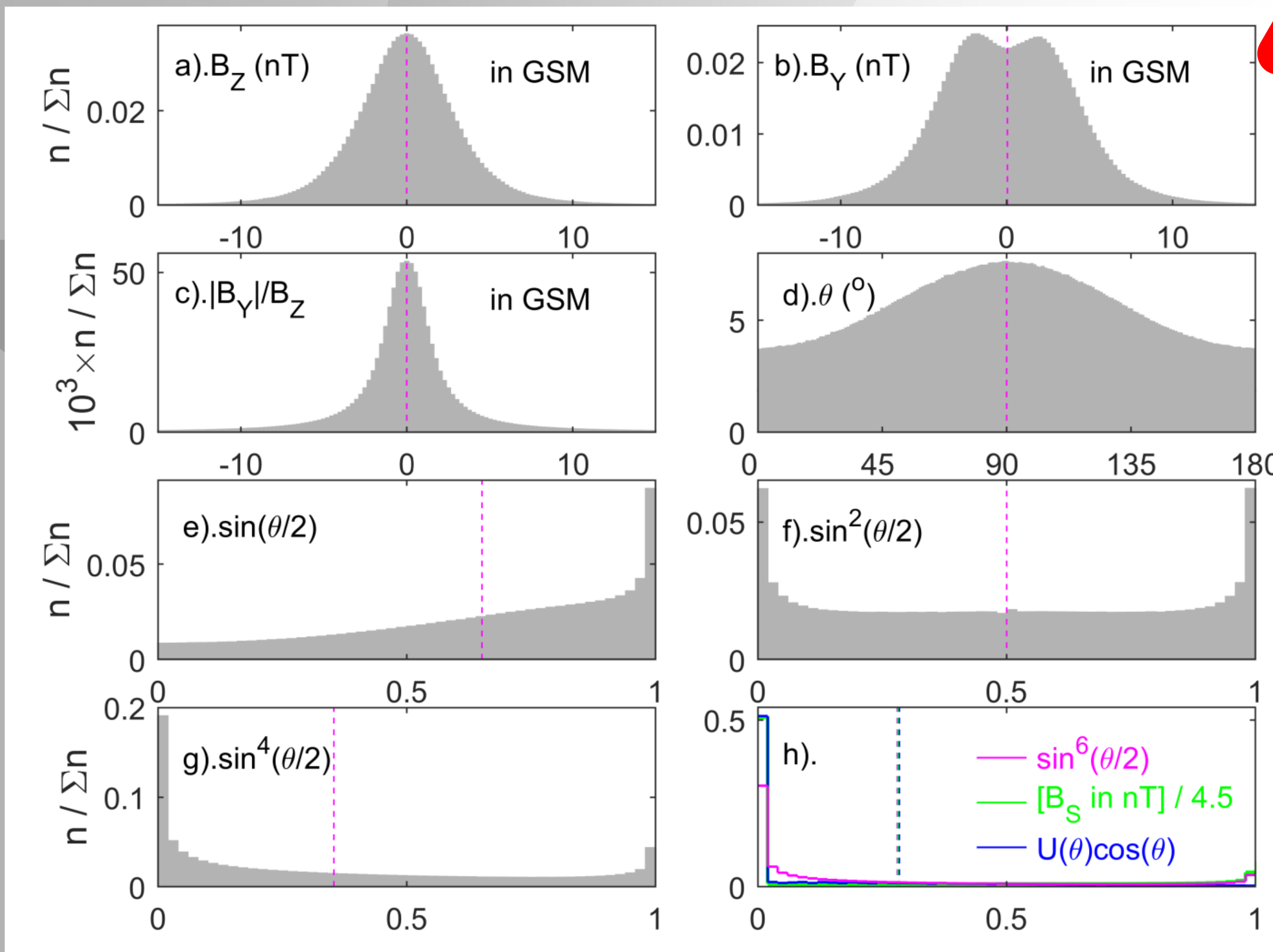


Want to "combine-then-average" Not "average-then-combine"

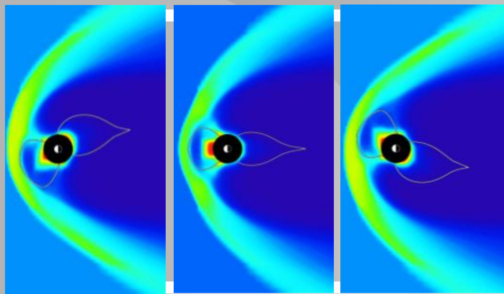
- Here use hourly data
  we want $< C_f >_{1hr} = < B_\perp^{a}\, \rho_{sw}^{b}\, V_{sw}^{c}\, sin^{d}(\theta/2) >_{1hr}$
- should not use $[C_f]_{1hr} = <B_\perp>^{a}\ <\rho_{sw}>^{b}\ <V_{sw}>^{c}\ < sin(\theta/2)>^{d}$
- In fact for hourly data $<B_\perp>^{a}$, $<\rho_{sw}>^{b}$ and $<V_{sw}>^{c}$ are actually to a high accuracy because of their high persistence
- Not so for $< sin(\theta/2)>^{d}$ for which high sub-hour variability introduces considerable noise
- Note that using $< sin(\theta/2)>^{d}$ introduces noise that increases with d – means that lower d fits are favoured and is one reason why derived d is too low in some studies.



Averaging and weighting



- The origin of the weird observed distribution of $sin^{d}(\theta/2)$
- 11,646,678 one-minute data samples 1995-2020



Averaging and weighting: changes with averaging timescale

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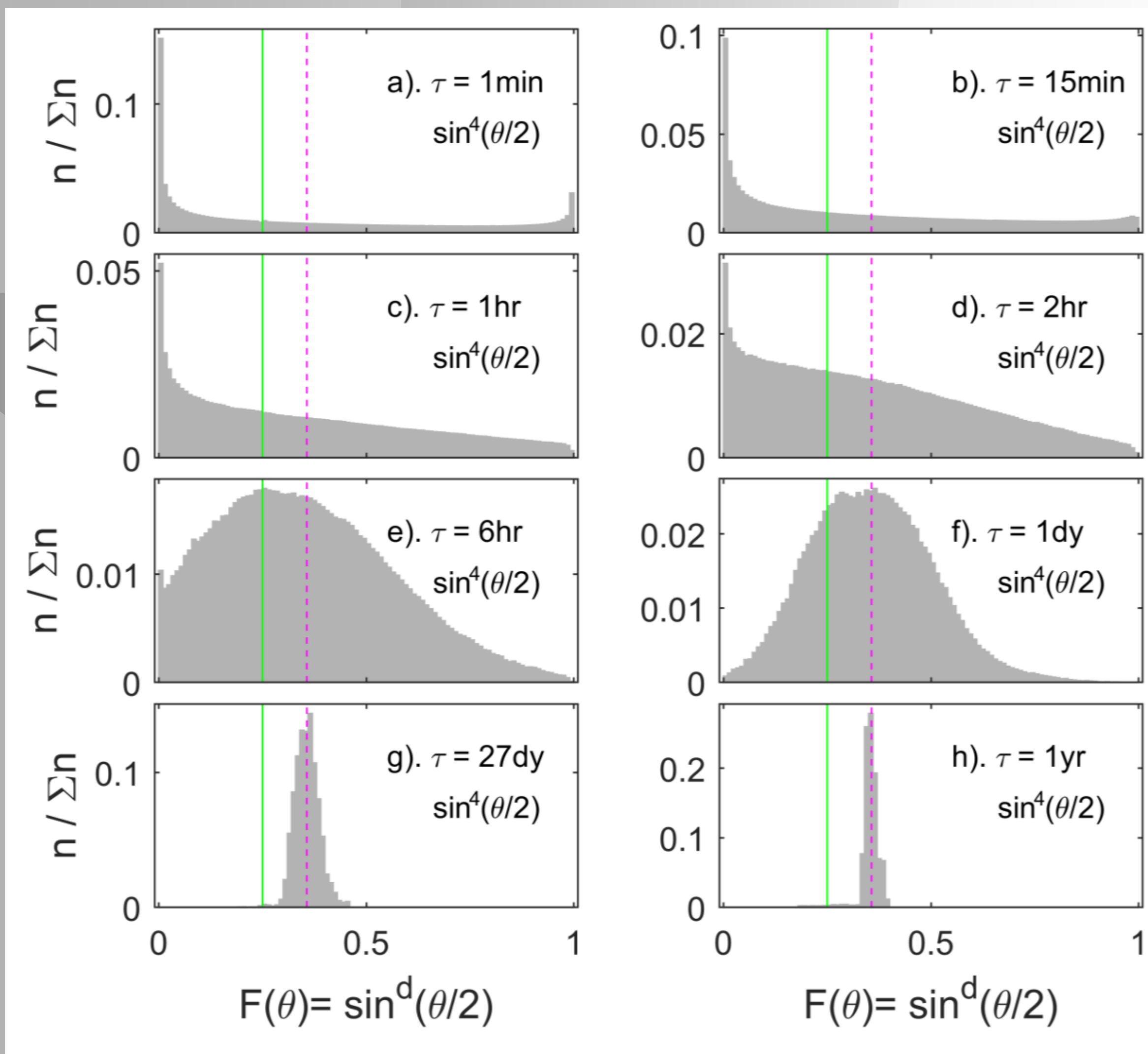
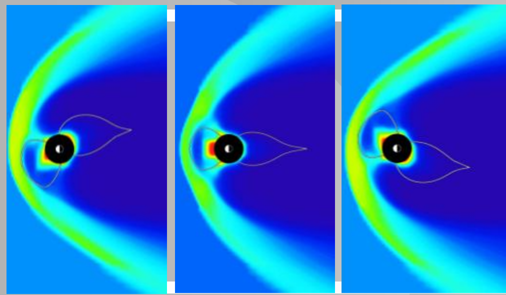
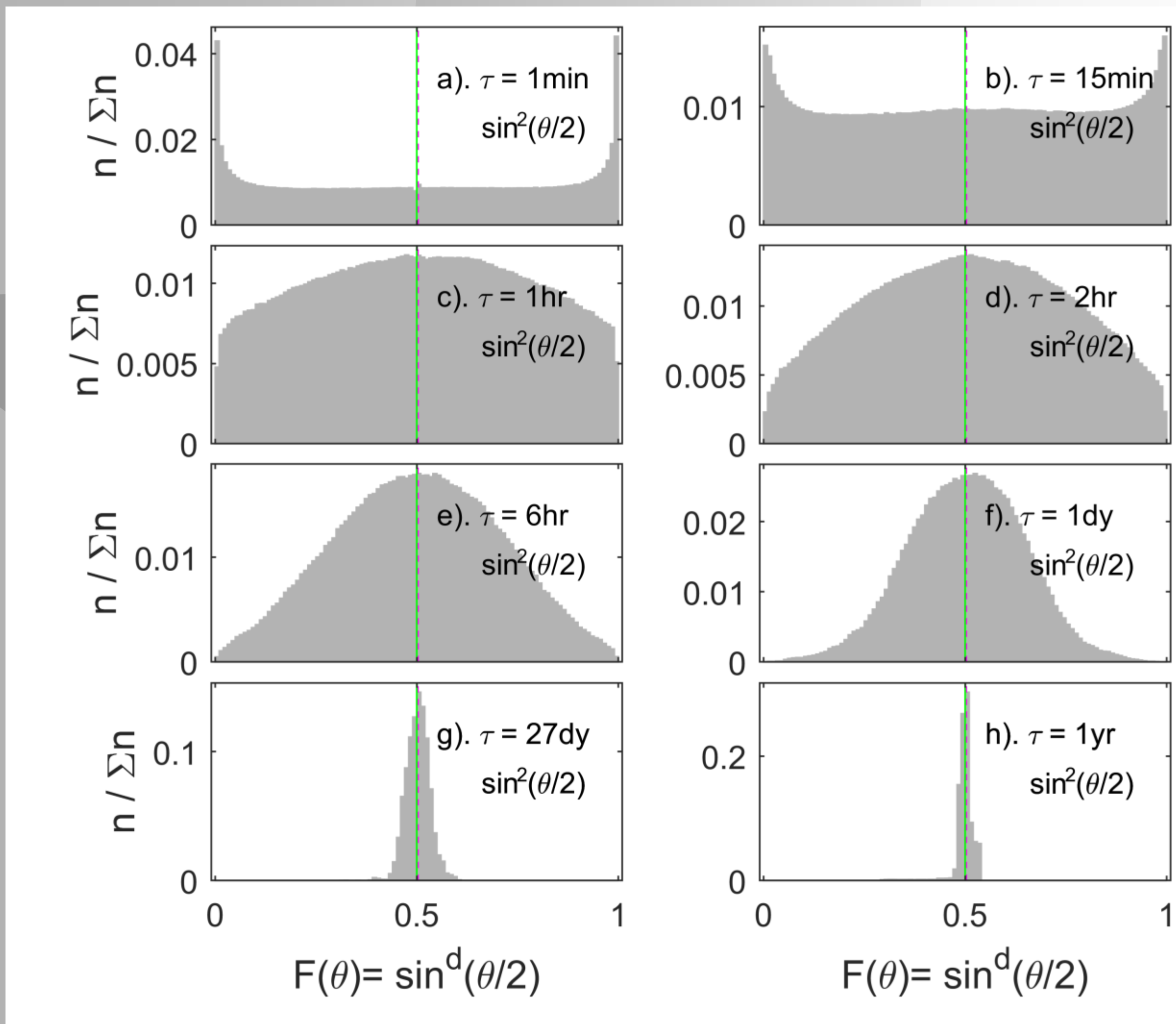


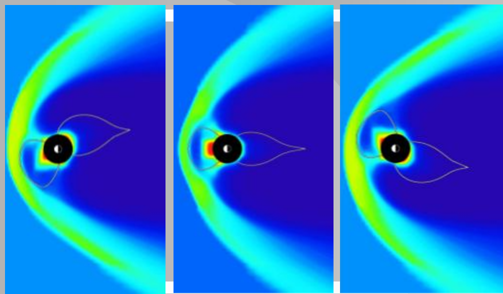
- The central limit theorem at work!
- This is for $sin^4(\theta/2)$
- Distribution evolves from weird, to less weird, to log-normal, to Gaussian, to delta function
- Explains why we do not need IMF orientation for annual means



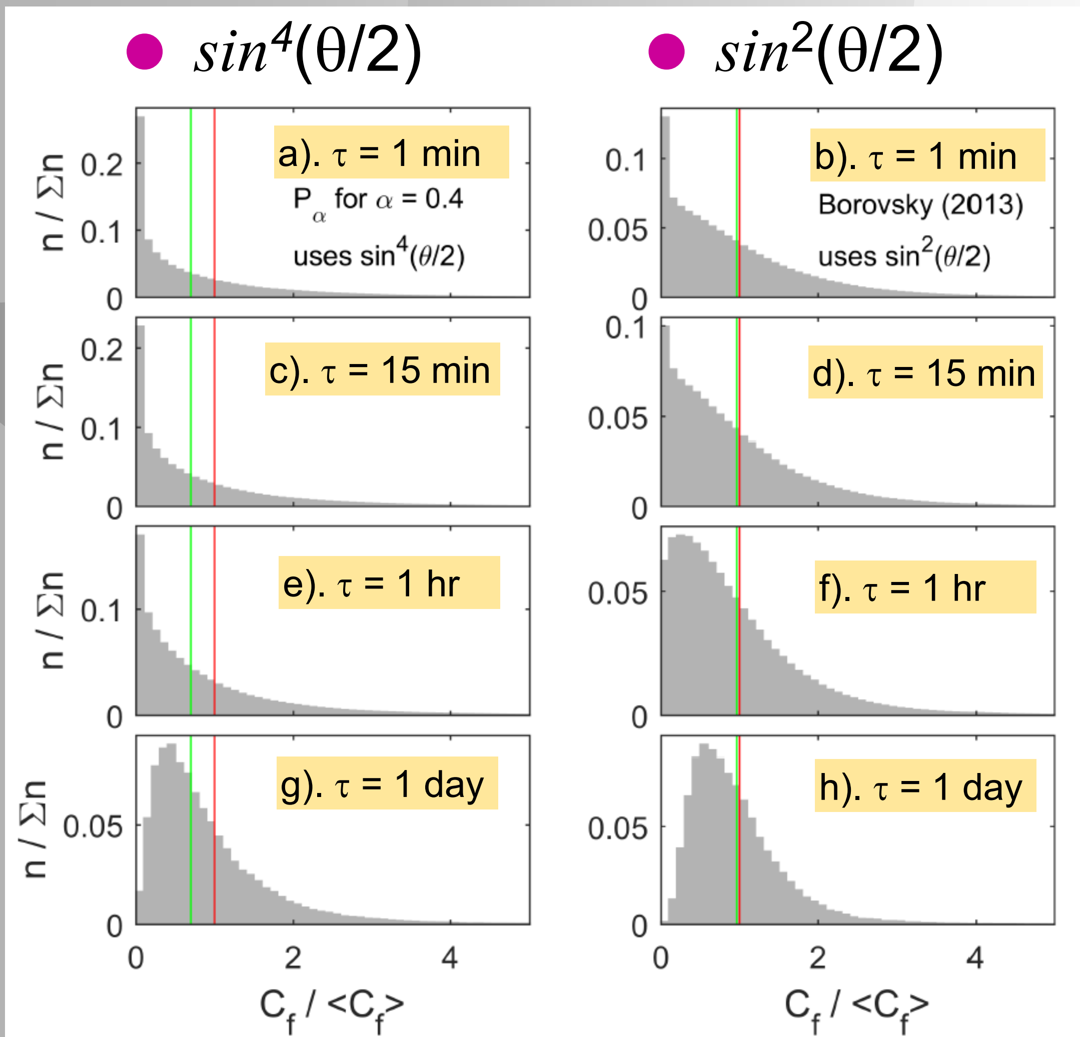
Averaging and weighting: changes with averaging timescale



- This is for $sin^2(\theta/2)$.
- Symmetric at all τ but for τ > 30 min it is dominated by $sin^2(\theta/2) = 0.5$ (i.e., $\theta = 90°$)
- Gives weighting to $\theta = 90°$ in correlations



What do these distributions of $sin^d(\theta/2)$ mean for coupling functions?



- Two examples – left for $sin^4(\theta/2)$ right for $sin^2(\theta/2)$.
- at all τ, $sin^4(\theta/2)$ gives much more weight to near zero values ($\theta \approx 0$) than $sin^2(\theta/2)$
- Gives low d an advantage in r from just the statistical weighting – not from the physics of θ control of the magnetosphere

Determination of optimum d for (e.g.) am data

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(Vasyluinas et al., 1982)

- Break out the IMF orientation factor $F(\theta)$ from the coupling function, C_f by defining G such that

$$G = C_f \,/\, F(\theta)$$

So for our general form $G = B_\perp^{\,a} \; \rho_{sw}^{\,b} \; V_{sw}^{\,c}$ & $F(\theta) = sin^{d}(\theta/2)$

- we need C_f to be a linear predictor with am

$$am = s_{am}\, C_f + i_{am} = s_{am} G\, F(\theta) + i_{am}$$

- Which yields

$$F(\theta) = (1/s_{am}) \times (am - i_{am}) \,/\, G = \text{“}\, R_{am} \,\text{”}$$

So if we bin the data by $F(\theta)$ and plot $<F(\theta)>$ against $<R_{am}>$ we get proportionality if our $F(\theta)$ is correct

R_{am} and G are set by a, b and c and so therefore are $F(\theta)$ and d – i.e., d is not a free fit parameter

Determination of optimum d for am data

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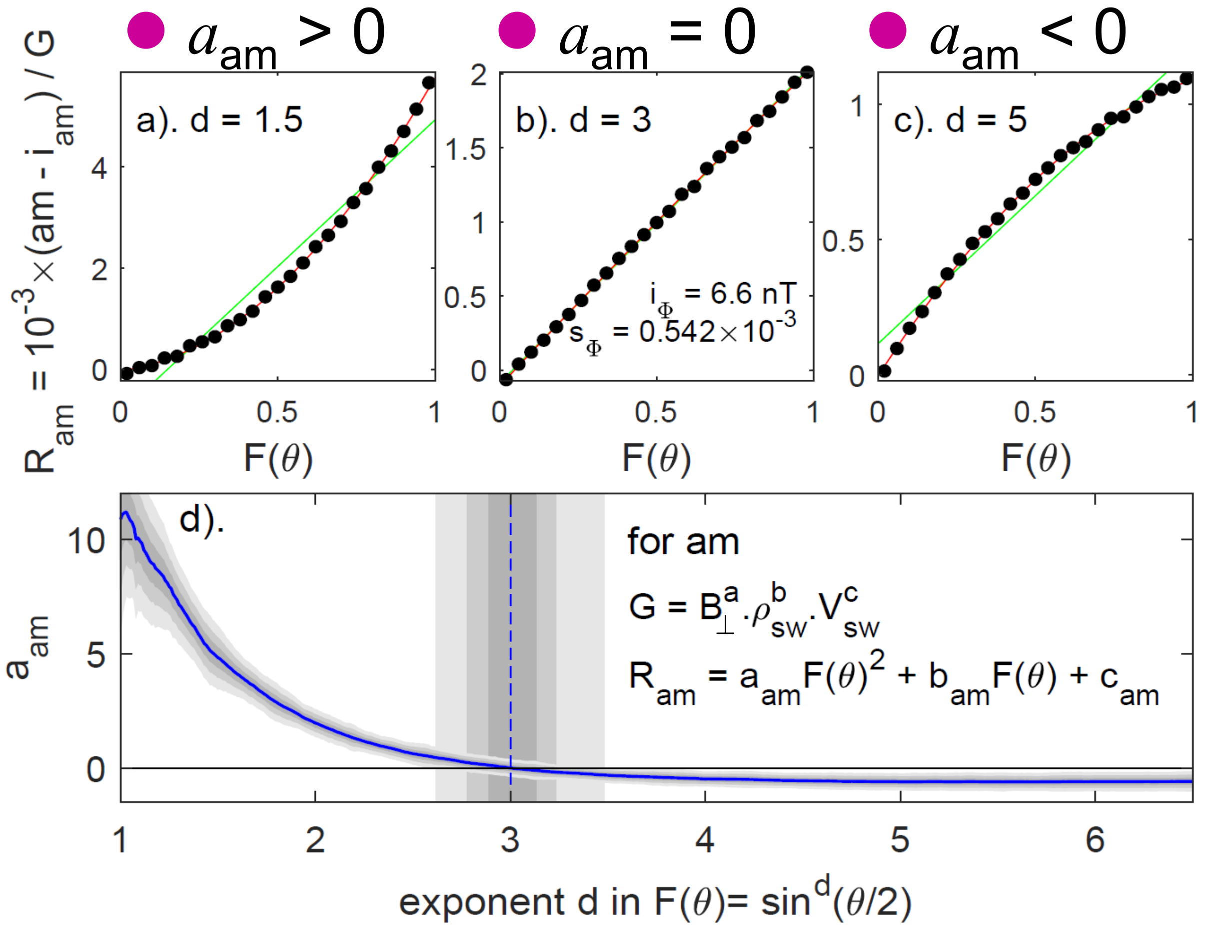


- To make C_f proportional to am, R_{am} needs to vary linearly with $F(\theta)$
- Sets d needed as where the quadratic term of fit polynomial, a_{am} falls to zero with 1-σ, 2-σ and 3-σ uncertainties

- a, b & c for each d found by a Nelder-Mead simplex search to minimize r.m.s. fit deviation of C_f to $am \rightarrow G$

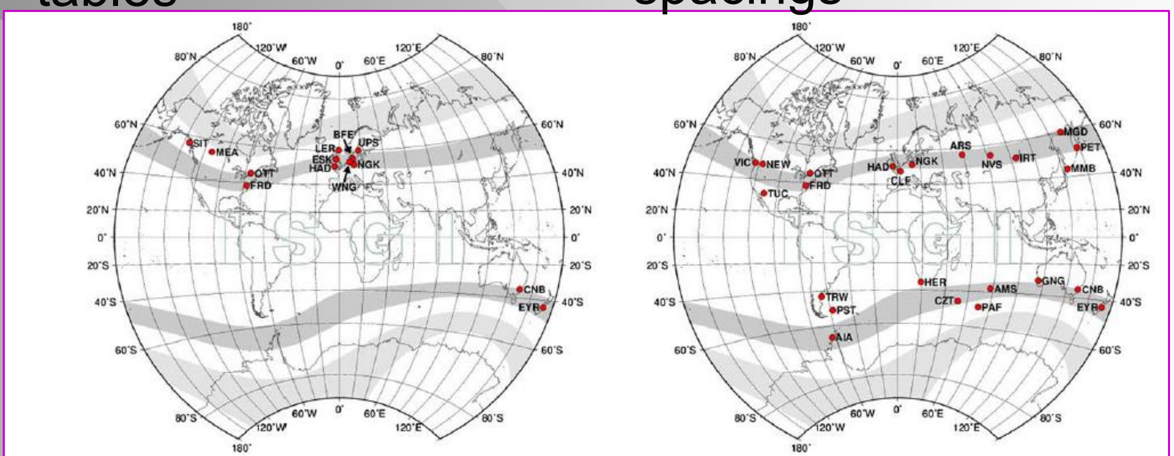
Response of Indices

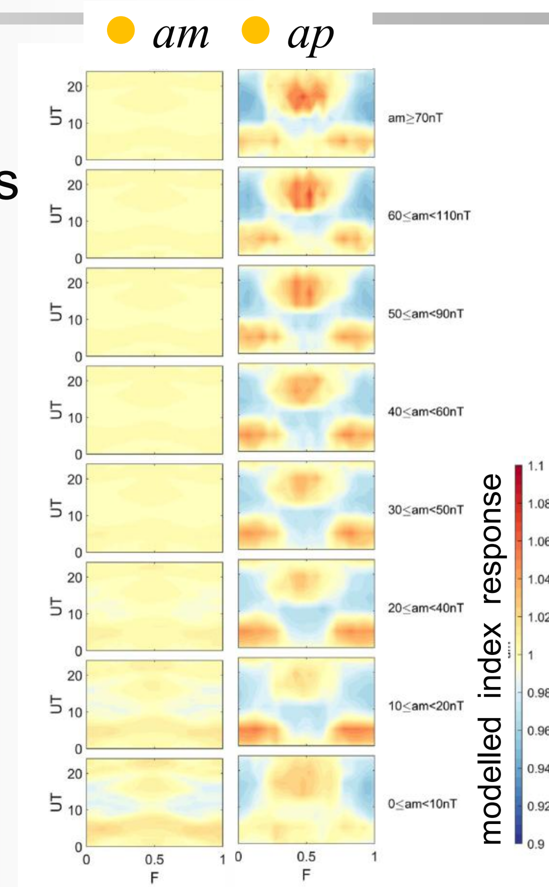
Lockwood et al. (2018)

Time-of-day / time-of-year response functions of planetary geomagnetic indices, doi: 10.1051/swsc/2019017

- current *ap* (*kp*) stations
- Longitudinal distribution is uneven in both hemispheres
- Stations mapped to Potsdam using look-up tables

- current *am* stations
- Longitudinal distributions as even as oceans allow
- Use weightings to allow for longitude spacings





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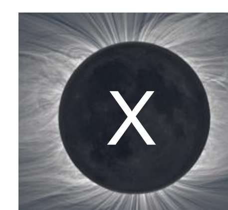


Response of *am* Indices

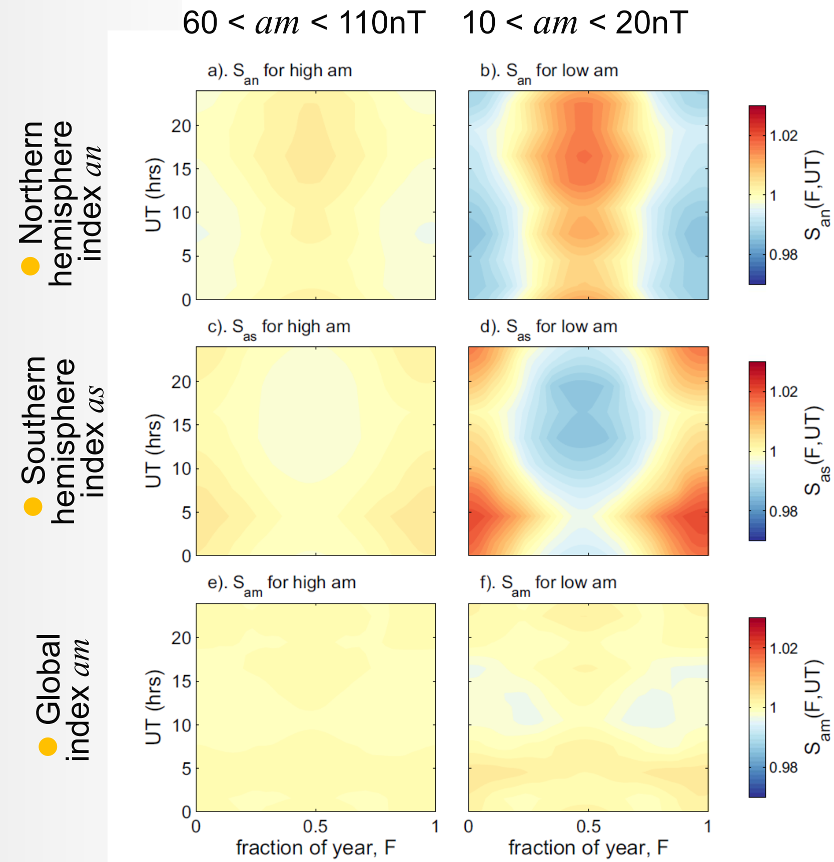
Lockwood et al. (2018)

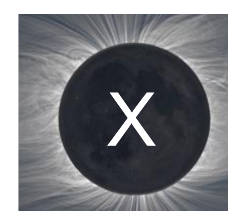
Time-of-day / time-of-year response functions of planetary geomagnetic indices, doi: 10.1051/swsc/2019017

- *ap* (*kp*) index is not suitable as has large spurious responses in both *F* and *UT* (±10%)

- Hemisphere indices *an* and *as* have a constant response in *F* and *UT* to within ±2% for low *am* and to within ±0.5% for high *am*

- Global index *am* has a constant response in *F* and *UT* to within ±1% for low *am* and to within ±0.1% for high *am*





Coupling Functions

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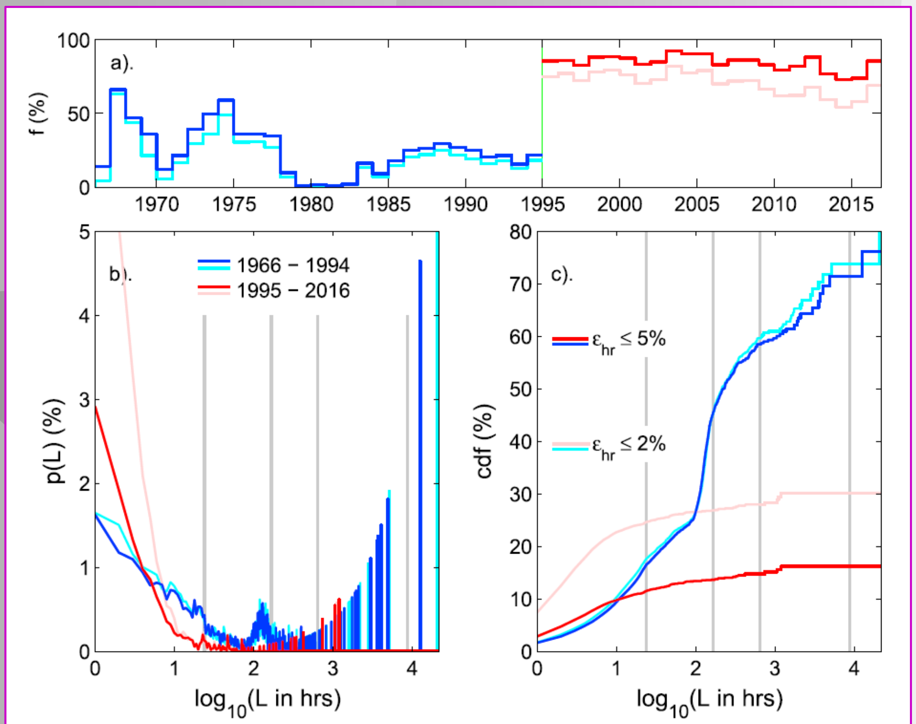
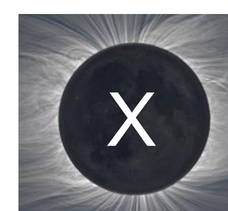
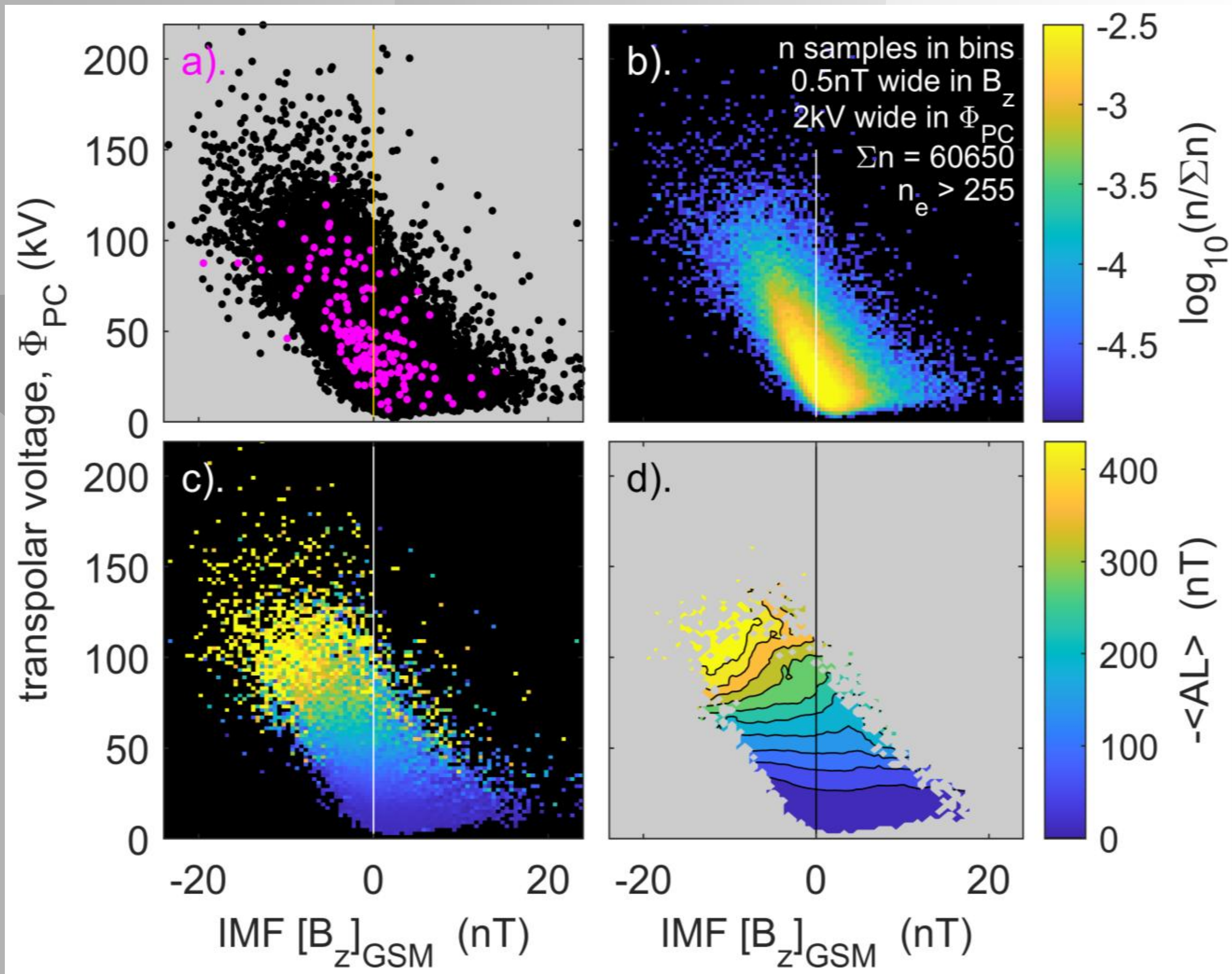


- For <5% and <2% errors in hourly coupling function using B, V_{SW}, m_{SW}, N_{SW}, and θ_{GSM}
- Data Gaps much more common and longer before 1995 (WIND and the ACE became available)

Lockwood et al (2019b), The development of a space climatology: 1. Solar-wind magnetosphere coupling as a function of timescale and the effect of data gaps, doi: 10.1029/2018SW001856



SuperDARN radar observations of transpolar voltage Φ_{PC} 1995-2020



- 214410 hourly values of Φ_{PC}
- to minimise effect of data assimilation model we require mean number of radar echoes > 255
- yields 65133 samples
- We generate simultaneous *am* index data set by linear interp-olation

Lockwood and McWilliams (2021)

SuperDARN radar observations of transpolar voltage Φ_{PC} 1995-2020

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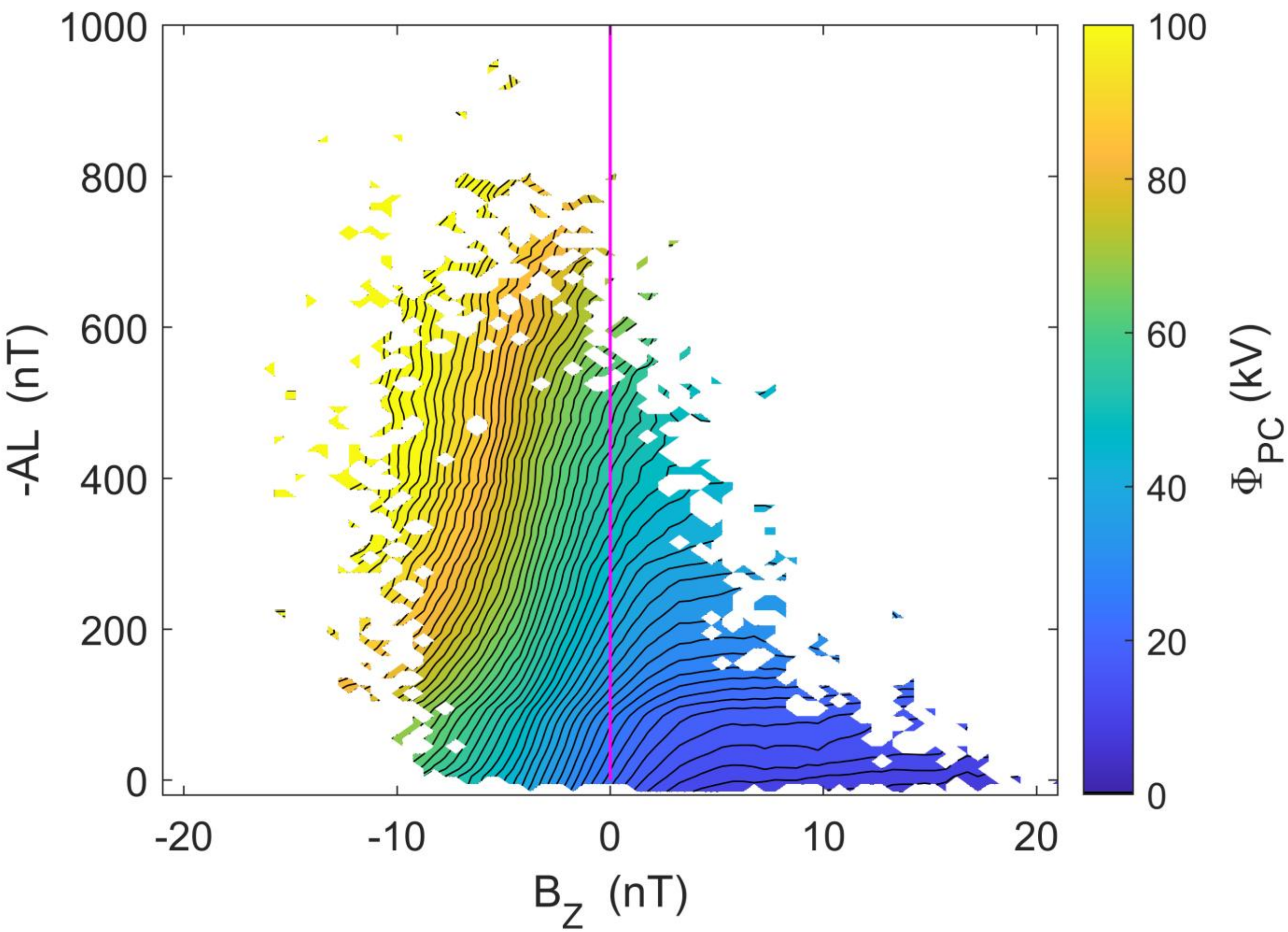
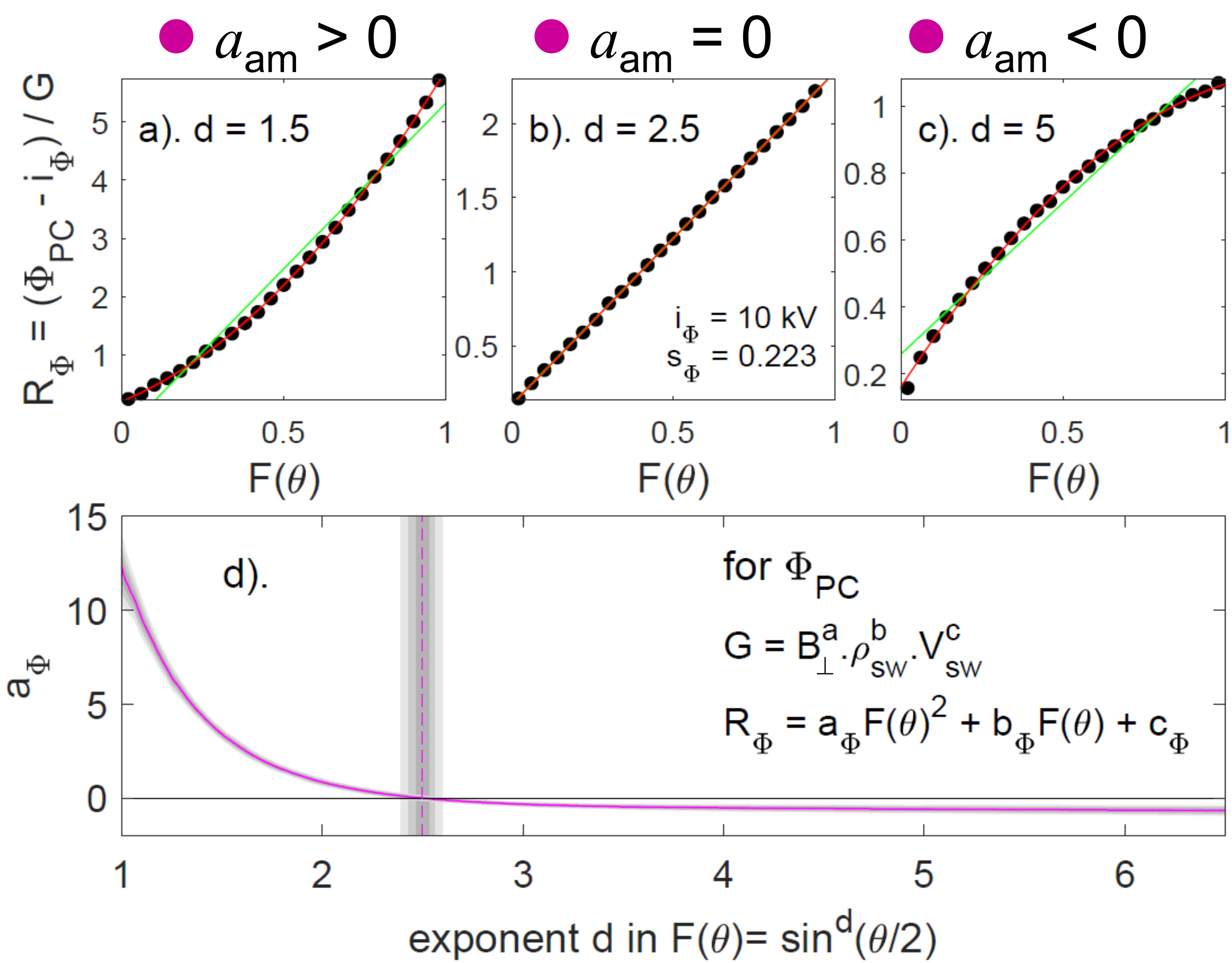


- Same data: here Φ_{PC} is contoured as a function of IMF B_z (along x axis) and $-AL$ (along y axis)
- Contours of Φ_{PC} are 2 kV apart

Lockwood and McWilliams (2021)

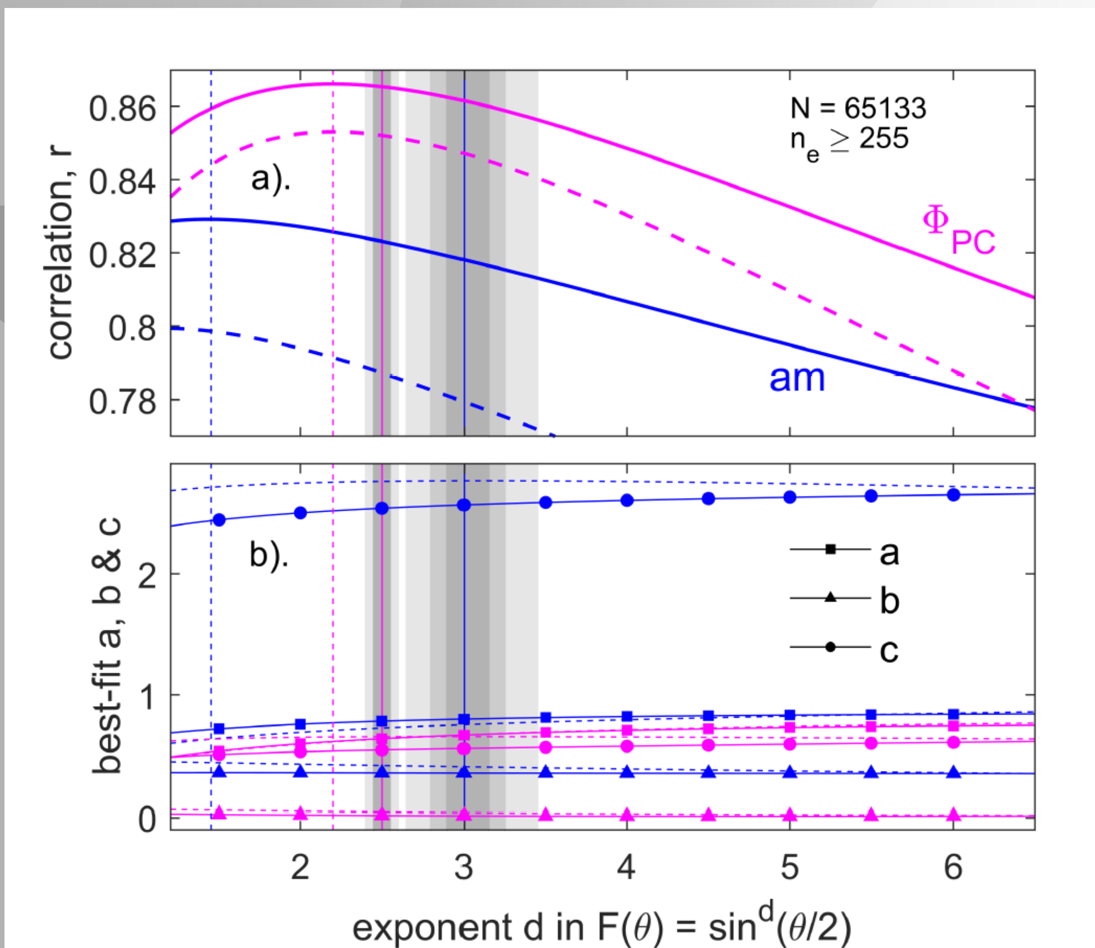
Determination of optimum d for Φ_{PC} data



- More tightly defined for Φ_{PC}
- Note significantly lower d for Φ_{PC} (2.50±0.07) than for *am* (3.00±0.22)

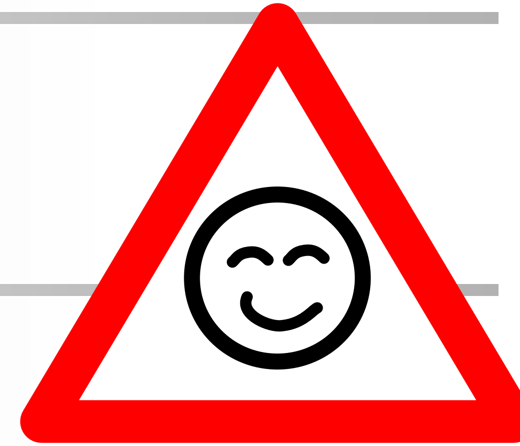
Correlations as a function of IMF orientation factor exponent, *d*

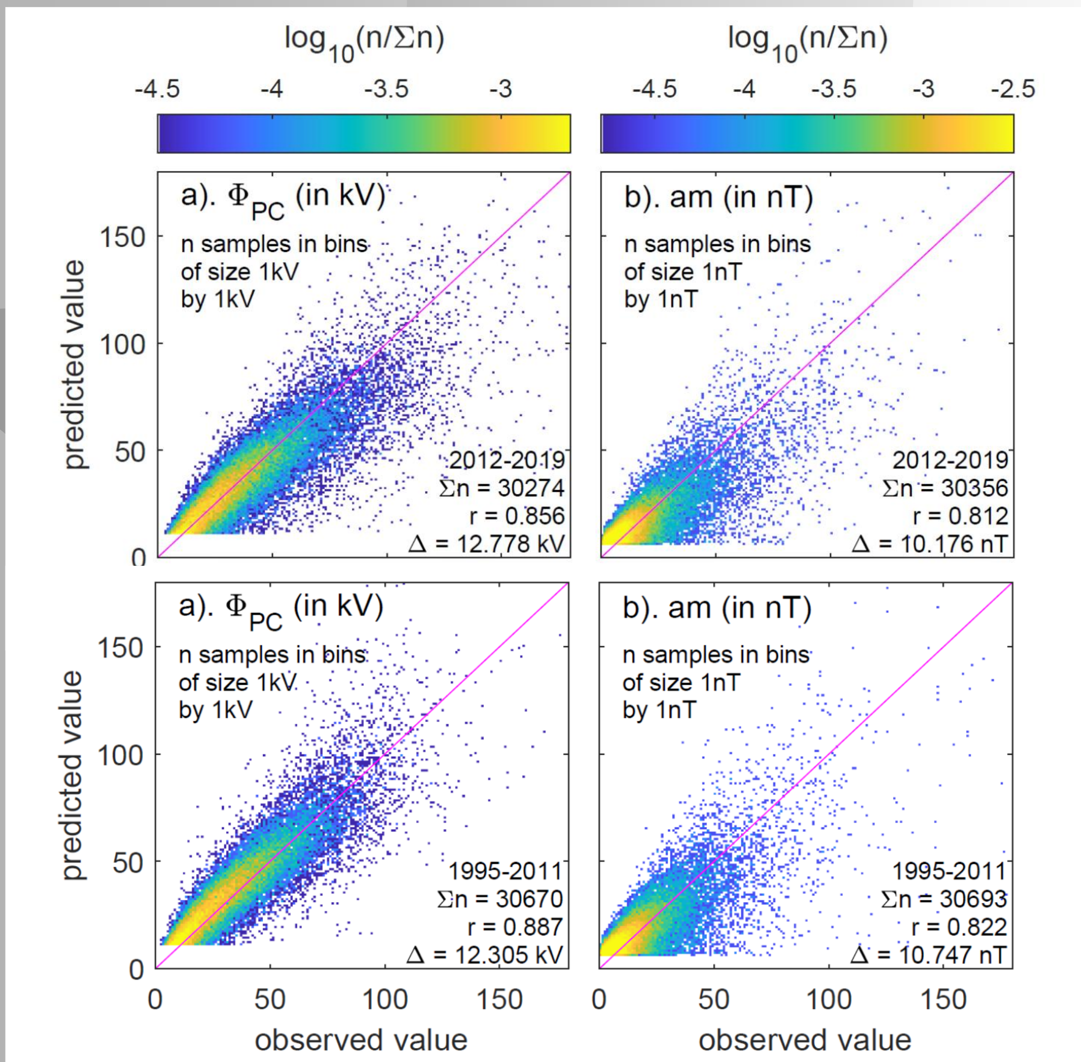
- $C_f = B_\perp^{\,a}\ \rho_{sw}^{\,b}\ V_{sw}^{\,c}\ \sin^d(\theta/2)$



- Use lag correlograms (for d = 3 and best fit a, b and c) with 1-hr running means of C_f to find optimum lag of 18.5±1.3 min for Φ_{PC} and 30.5±4.0 min for *am*.
- Vary d over proposed range of 1-6
- For each d use Nelder-Mead simplex search to find a, b and c that give maximum correlation

2-fold overfitting test

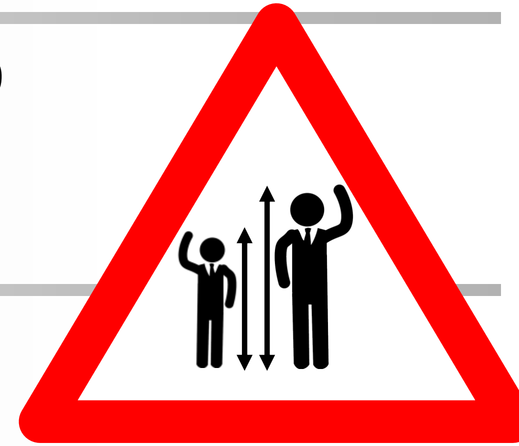




- fit data
  2012-2020
  30356 samples

- test data
  1995-2011
  30693 samples

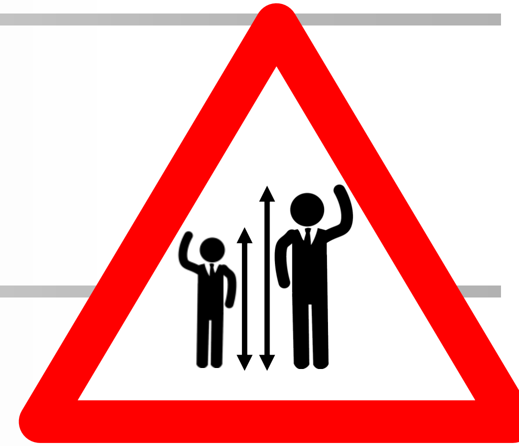
Best-fits compared to simple theories



- all C_f made using best-practice "combine-then-average"
- compare with simple theories for coupling of energy (for am) and voltage (for Φ_{PC})

T	lag, δt (min)	C_f	optimum values					
			d	r_p	r_p^2	a	b	c
Φ_{PC}	18.5 ±1.3	best fit	2.50±0.07	0.865	0.748	0.642±0.019	0.018±0.008	0.550±0.047
		Φ'_{PC} for constant η	4	0.823	0.677	1	−0.167	0.667
am	47.0 ±4.0	best fit	3.00±0.22	0.858	0.736	0.802±0.022	0.360±0.012	2.560±0.072
		P_α for α = 0.4	3	0.852	0.725	0.800	0.267	1.533

Variance of transpolar voltage Φ_{PC} explained by various C_f



- Note this is an independent test to an entirely new dataset (no chance of overfitting) of all C_f except the new one proposed here

Coupling function, C_f	trained on	δt_p (min)	a	b	c	d	r^2 (%)		
							$\sin^d([\theta]_\tau)$	$\langle\sin(\theta)\rangle^d$	$\langle\sin^d(\theta)\rangle$
This work	Φ_{PC}	18	0.64	0.02	0.55	2.5	62.8	73.1	74.6
Newell et al (2007)	Φ_D	17	0.67	0	1.33	2.67	58.7	69.2	69.2
Boyle et al. (1977) term2	Φ_{PC}	19	1	0	0	3	57.5	67.5	68.9
Boyle et al. (1977)	Φ_{PC}	18	-	-	-	-	4.30	68.6	68.8
Wygant et al. (1983)	Φ_{PC}	18	1	0	1	4	48.2	64.2	66.4
Borovsky (2013)	Φ_D	18	0.93	0.04	1.07	2	64.1	66.9	66.6
Milan et al. (2021)	Φ_D	18	1	0	1.33	4.5	34.0	60.6	63.1
Lockwood et al. (2019b)	am	18	0.84	0.24	1.49	4	45.0	61.5	62.7
McPherron et al. (2015)	AL	18	0.79	0.10	1.92	3.67	46.3	61.0	61.5
Scurry and Russell (1991)	am	26	1	0.5	2	4	32.9	44.7	44.9
Perreault & Akasofu (1978)	AL	26	2	0	1	4	23.0	28.0	29.0
(Boyle et al. (1977) term1	Φ_{PC}	0	0	0	2	0	4.3	4.3	4.3)

- Note Boyle formula is $10^{-4}V_{SW}^2 + 11.7B\ sin^3(\theta/2)$

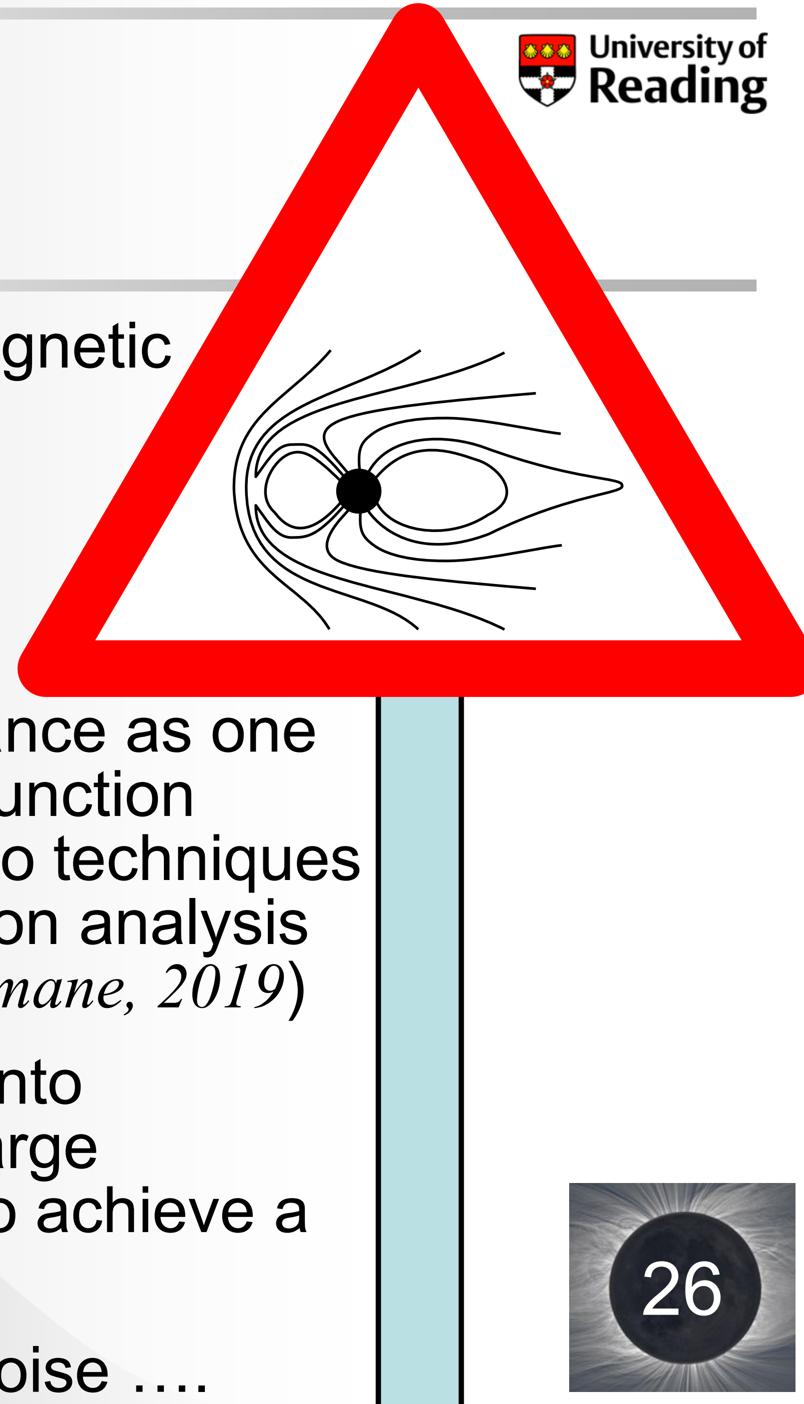
Variance of full 3-hourly *am* index dataset explained by various C_f

- Note this is an independent test for all C_f in this case
- Data for 1995-2020

Coupling function, C_f	trained on	δt_p (min)	a	b	c	d	r^2 (%)		
							$\sin^d([\theta]_\tau)$	$<\sin(\theta)>^d$	$<\sin^d(\theta)>$
This work	am	48	0.8	0.36	2.56	3.00	57.8	67.4	73.6
McPherron et al. (2015)	AL	47	0.79	0.10	1.92	3.67	49.8	60.7	72.1
Lockwood et al. (2019b)	am	47	0.84	0.24	1.49	4	47.0	58.1	71.0
Borovsky (2013)	Φ_D	47	0.93	0.04	1.07	2	61.0	64.0	68.3
Milan et al. (2021)	Φ_D	47	1	0	1.33	4.5	43.0	52.5	67.0
Scurry and Russell (1991)	am	47	1	0.5	2	4	46.8	58.2	67.0
Wygant et al. (1983)	Φ_{PC}	47	1	0	1	4	44.4	52.1	64.8
Boyle et al. (1977)	Φ_{PC}	47	-	-	-	-	23.1	56.9	63.4
Newell et al (2007)	Φ_D	47	0.67	0	1.33	2.67	49.9	57.5	57.5
Boyle et al. (1977) term2	Φ_{PC}	56	1	0	0	3	39.9	42.0	49.2
Perreault & Akasofu (1978)	AL	47	2	0	1	4	31.6	35.9	41.9
(Boyle et al. (1977) term1	Φ_{PC}	0	0	0	2	0	23.1	23.1	23.1)

Conclusions

- best coupling functions for geomagnetic Index such as *am* and Φ_{PC} are significantly different
- coupling functions need to be tailored to what they aim to predict
- So rather than thinking of disturbance as one response, predicted by a universal function coupling function we need to move to techniques such as coupling canonical correlation analysis using state vectors (*Borovsky and Osmane, 2019*)
- analysis inserting synthetic gaps into continuous data sets at random in large ensembles sets availability criteria to achieve a required accuracy
- But there are always sources of noise ….



Conclusions: Pitfalls

- Coupling function reflects errors in IMF orientation caused by data gaps: reject means of insufficient samples
- Distributions and averaging give statistical biases to certain clock angles, so IMF orientation factor must be determined by the linearity condition, not correlation
- need to always "compute-then-average" especially for $F(\theta)$ at large d and τ, and worry about effect of datagaps
- Note that for larger τ the difference between $<x^a>$ and $<x>^a$ will become significant for ρ_{sw} and then $B_\perp$: the higher persistence of V_{sw} means it will be the least affected

Coupling Functions

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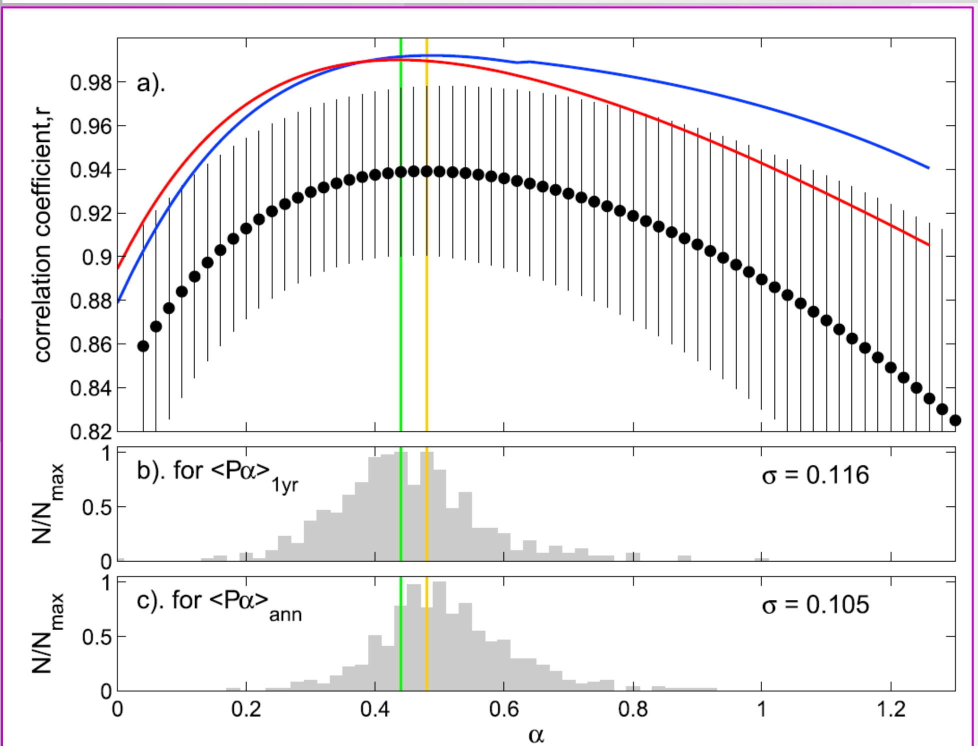
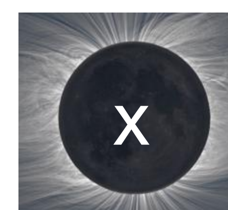
correlating P_α with am (post 1995 data)

- Even for one fit parameter, α there are pitfalls



- Red and middle panel is $r(\alpha)$ for combine-then-average construction of $< P_\alpha >_{1yr}$ whereas the blue (and bottom panel) is for average then combine (here called $< P_\alpha >_{ann}$)
- Black dots and error bars are from 100,000 runs with synthetic datagaps introduced at random with the distribution for 5% error prior to 1995 & here neglecting their effect
- Green and orange lines mark optimum α for the two cases and the grey is the probability distribution around it from the 100,000 runs

Lockwood et al (2019b), The development of a space climatology: 1. Solar-wind magnetosphere coupling as a function of timescale and the effect of data gaps, doi: 10.1029/2018SW001856



Coupling Functions

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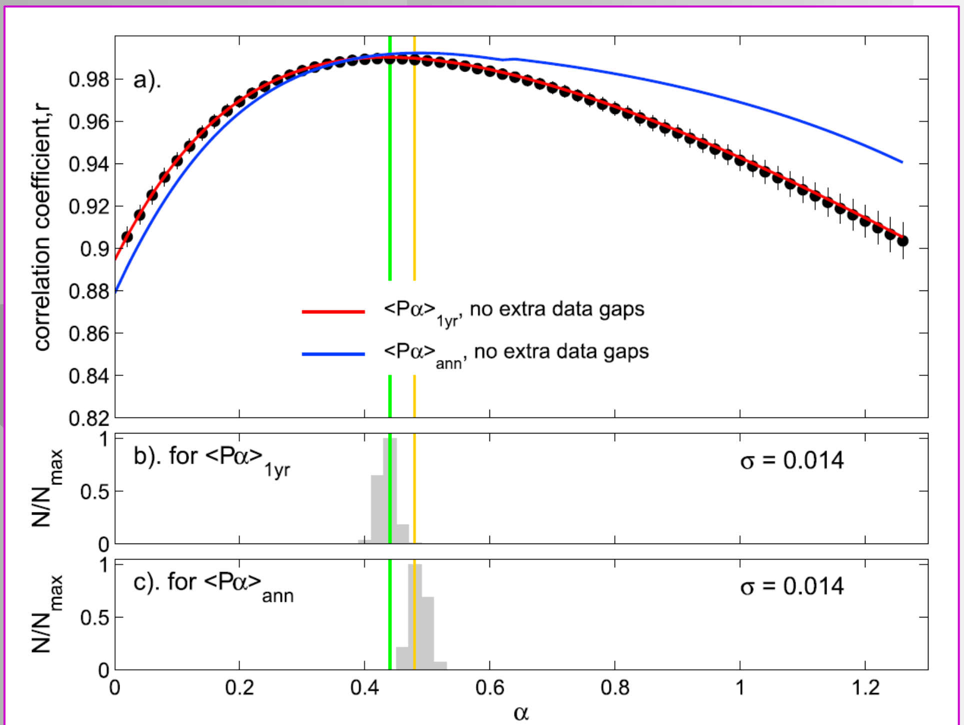
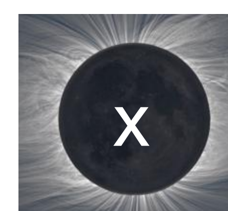
- Even for one fit parameter, α there are pitfalls



- Same as last plot but instead of neglecting effect of data gaps we use only “simultaneous” data (allowing for propagation lag) - what I call piecewise removal of non-matching data
- Much lower uncertainty
- Combine-the-average works best (as one would expect)
- Interpolation to fill gaps turns out to be worse than just neglecting gaps (because of very low correlation time if IMF orientation)

Lockwood et al (2019b), The development of a space climatology: 1. Solar-wind magnetosphere coupling as a function of timescale and the effect of data gaps, doi: 10.1029/2018SW001856



Department of Meteorology

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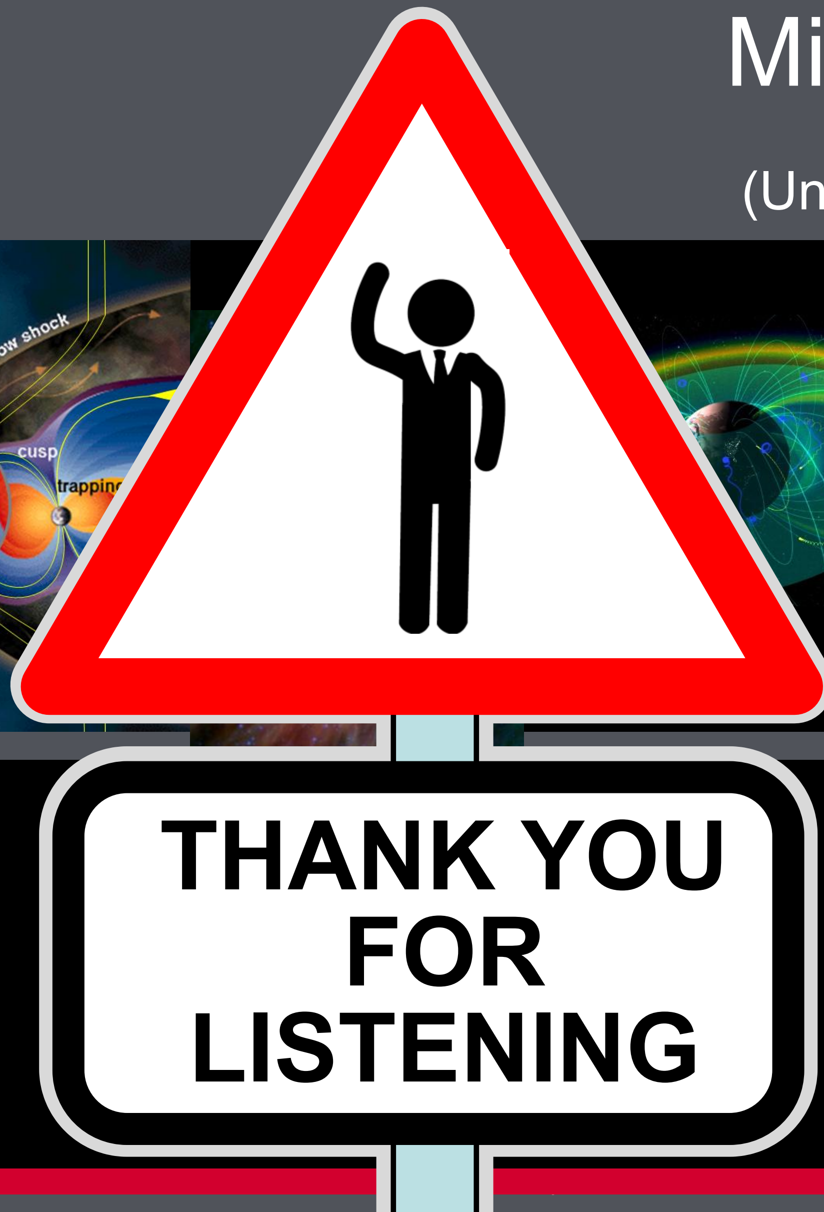
Mike Lockwood

(University of Reading, UK)



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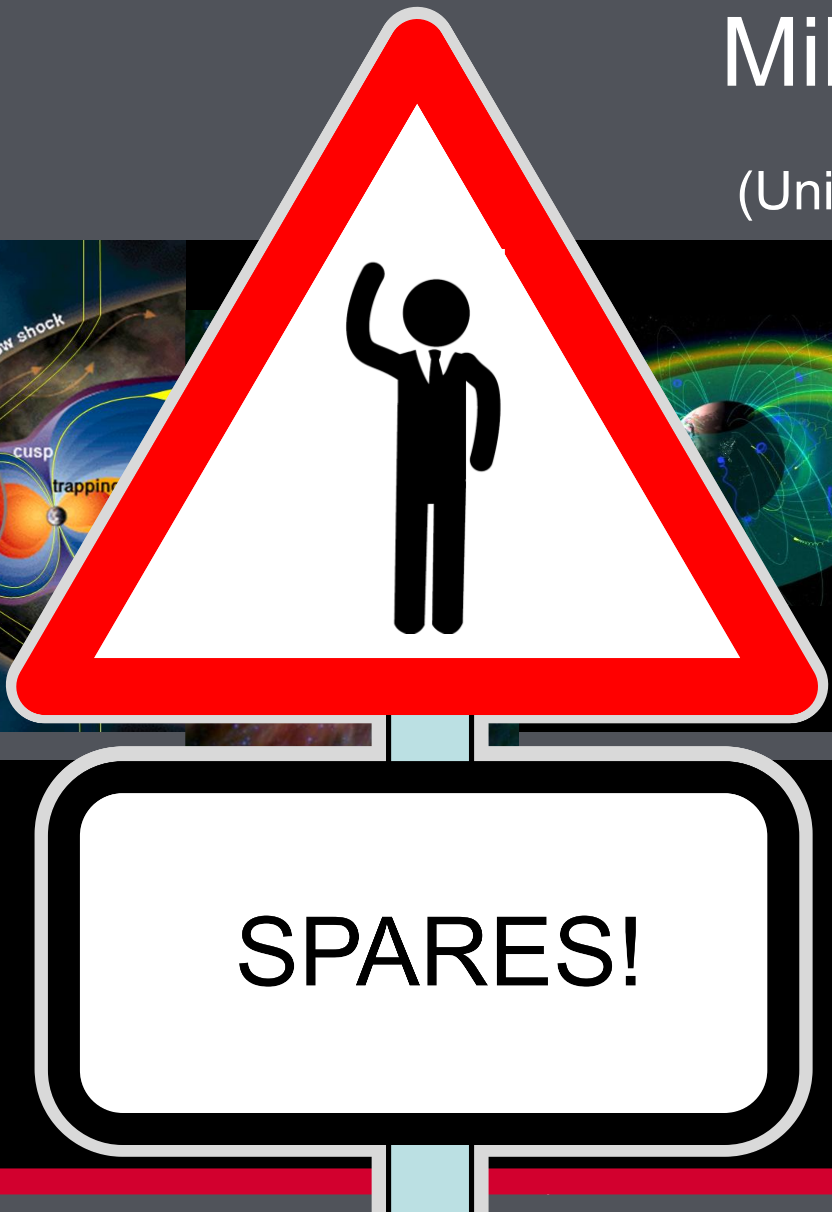

Energy flow in the magnetosphere

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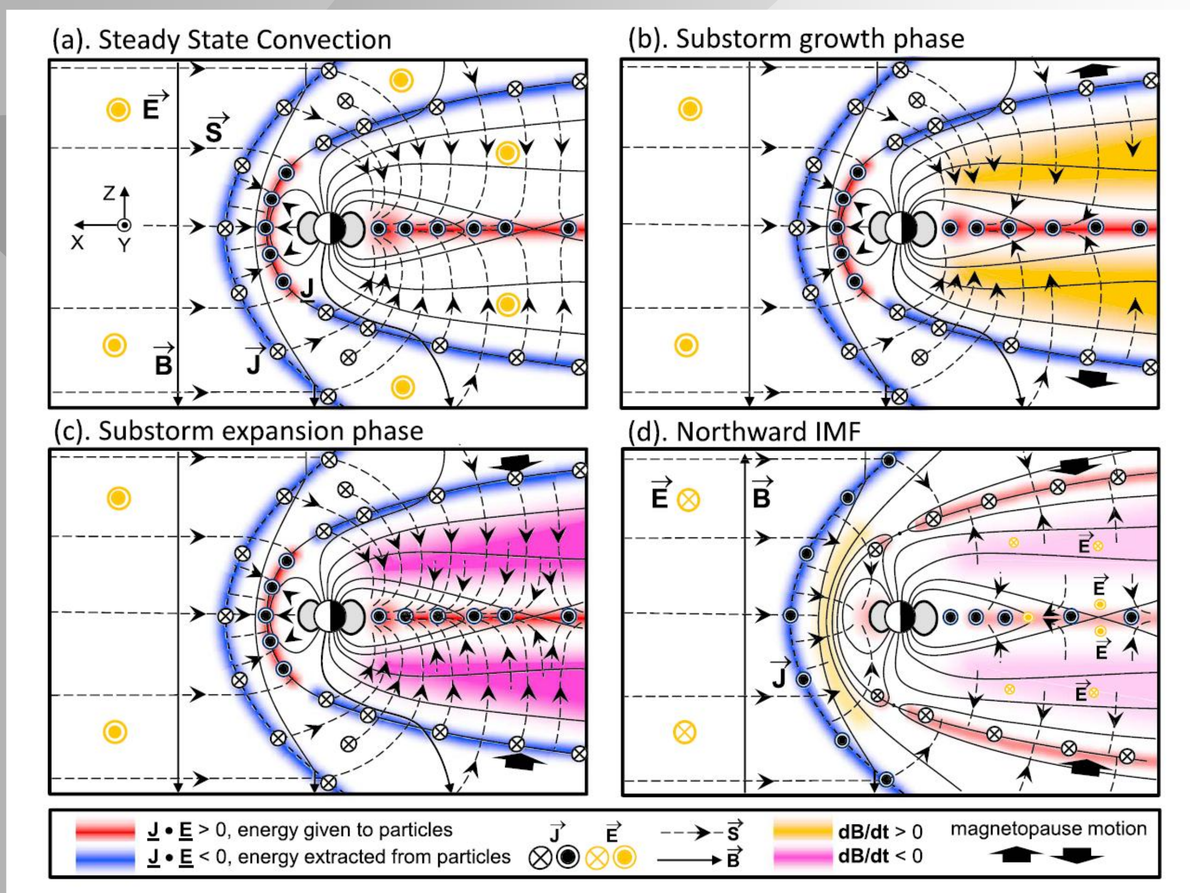
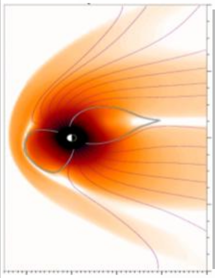
(Cowley, 1991, Lockwood, 2004, 2019; Ebihara et al.,2019)

- dominant power in the solar wind is the kinetic energy flux of particles
- Solar wind Poynting flux is smaller by a factor of about 50



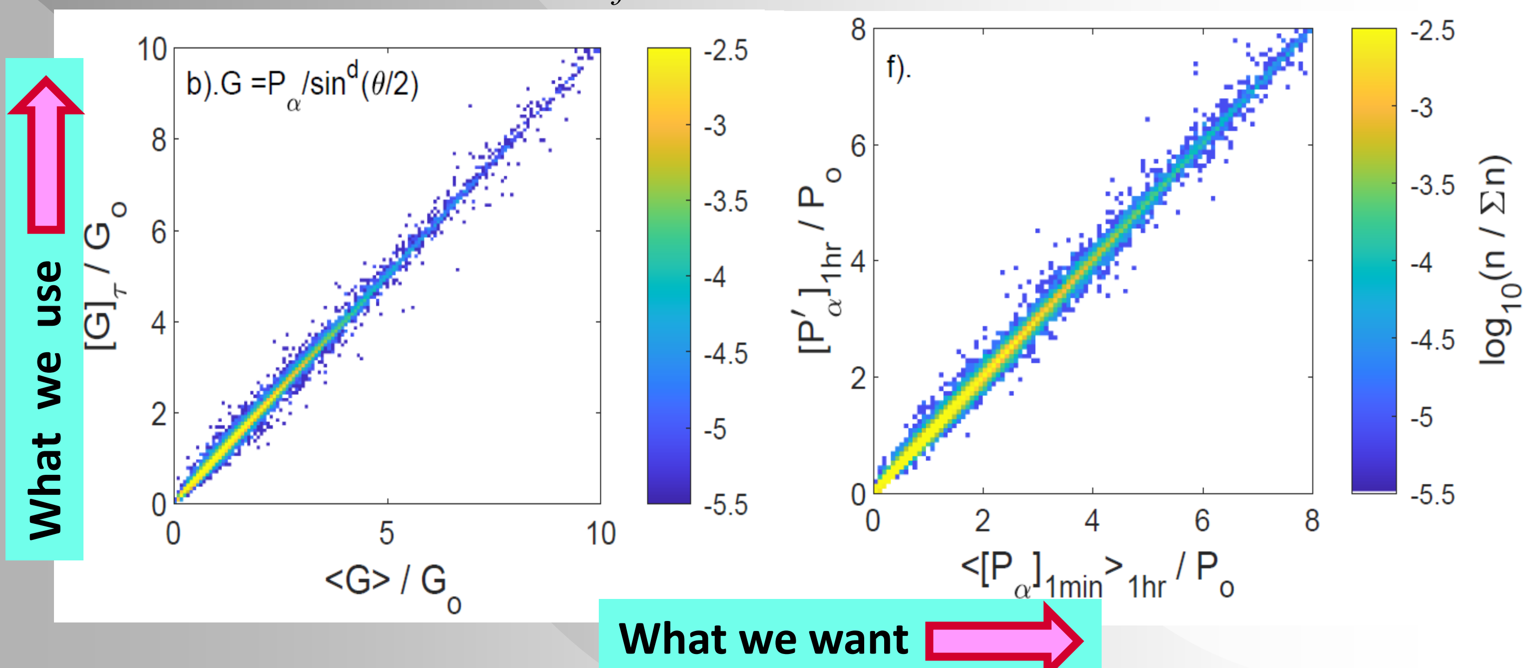
- Poynting flux is generated from kinetic energy of SW particles by regions of $\vec{J}.\vec{E} < 0$ in the shock, sheath and tail magnetopause
- Only about 10% of the energy entering the magnetosphere comes from Poynting flux in the undisturbed, upstream solar wind



Want to "combine-then-average" Not "average-then-combine"

- want $< C_f >_{1hr} = < B_\perp^{a} \rho_{sw}^{b} V_{sw}^{c} \sin^{d}(\theta/2) >_{1hr}$
- but this gives problems for iteration of a, b, & c
- should not use $[C_f]_{1hr} = <B_\perp>^{a} <\rho_{sw}>^{b} <V_{sw}>^{c} < \sin(\theta/2)>^{d}$
- We find a compromise that works well for τ = 1hour is

$$[C'_f]_{1hr} = <B_\perp>^{a} <\rho_{sw}>^{b} <V_{sw}>^{c} <\sin^{d}(\theta/2)>$$



- allows us to iterate a, b & c for a given d

Proposed Coupling Functions (1)

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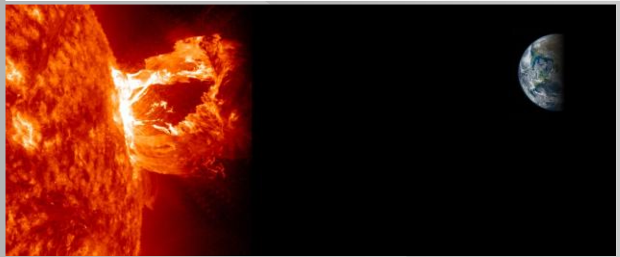
Basis	coupling function $B^a \rho_{sw}^b V_{sw}^c F(\theta_{GSM})^d$	a	b	c	d	$F(\theta)$	τ	Reference
IMF (empirical fit to inter-diurnal geomagnetic data)	B	1	0	0	0	-	1 yr	*Svalgaard & Cliver* (2005)
solar wind speed	V_{sw}	0	0	1	0	-	1 yr	*Feynmann & Crooker* (1978)
(benchmark test)	V_{sw}^2	0	0	2	0	-	1day-1yr	*Finch & Lockwood* (2007)
empirical fit to inter-diurnal geomagnetic data	$BV_{sw}^{-0.1}$	1	0	–0.1	0	-	1 yr	*Lockwood et al.* (2014)
empirical fit to range geomagnetic data	$BV_{sw}^{1.7}$	1	0	1.7	0	-	1 yr	*Lockwood et al.* (2014)
southward IMF in GSM (benchmark test)	$[B_S]_{GSM}$	1	0	0	1	h.w.r.	1day-1yr	*Finch & Lockwood* (2007)
h.w.r. interplanetary electric field applied to *Dst*	$E_{sw} = [B_S]_{GSM} V_{sw}$	1	0	0	1	h.w.r.	2.5 min	*Burton et al.* (1975)
h.w.r. interplanetary electric field applied to Φ_{PC}	$E_{sw} = [B_S]_{GSM} V_{sw}$	1	0	1	1	h.w.r.	~ 10 min	*Cowley* (1984)
dawn-dusk interplanetary electric field applied to Φ_{PC}	$BV_{sw} sin^4(\theta_{GSM}/2)$	1	0	1	4	$sin^d(\theta/2)$	1 hr	*Wygant et al.* (1983)
(benchmark test)	$[B_S]_{GSM} V_{sw}^2$	1	0	2	1	h.w.r.	1day-1yr	*Finch & Lockwood* (2007)
solar wind Poynting flux (basis of ε)	$B_\perp^2 V_{sw}$	2	0	1	0	-	-	-

Proposed Coupling Functions (2)

solar wind kinetic energy flux (basis of P_α)	$\rho_{sw} V_{sw}^3$	0	1	3	0	-	-	-
solar wind Poynting flux with θ_{GSM} control	$B_\perp^2 V_{sw} sin^4(\theta_{GSM}/2)$	2	0	1	4	$sin^d(\theta/2)$	-	-
epsilon factor	$\varepsilon = B^2 V_{sw} sin^4(\theta_{GSM}/2)$	2	0	1	4	$sin^d(\theta/2)$	-	*Perreault & Akasofu* (1978)
solar wind dynamic pressure (benchmark test)	$p_{sw} = \rho_{sw} V_{sw}^2$	0	1	2	0	-	1day-1yr	*Finch & Lockwood* (2007)
empirical fit to *am*	$B_\perp \rho_{sw}^{1/2} V_{sw}^2 sin^4(\theta_{GSM}/2)$	1	0.5	2	4	$sin^d(\theta/2)$	3 hr	*Scurry and Russell* (1991)
empirical fit to Φ_D	$B_\perp V_{sw}^{4/3} sin^{9/2}(\theta_{GSM}/2)$	1	0	1.33	4.5	$sin^d(\theta/2)$	5 min	*Milan et al* (2012)
empirical fit to *Dst*	$B V_{sw}^2 N_{sw}^{1/2} sin^6(\theta_{GSM}/2)$	1	0.5	2	6	$sin^d(\theta/2)$	1hr	*Temerin & Lee* (2006)
near-universal coupling function 1: based on Φ_D	$B^{2/3} V_{SW}^{4/3} sin^{8/3}(\theta_{GSM}/2)$	0.67	0	1.33	2.67	$sin^d(\theta/2)$	1 hr	*Newell et al.* (2007)
near-universal coupling function 2: fit to *Dst*	$B^{2/3} \rho_{sw}^{1/2} V_s^{7/3} sin^{8/3}(\theta_{GSM}/2)$	0.67	0.5	2.33	2.67	$sin^d(\theta/2)$	1hr	*Newell et al.* (2007)
theory of Φ_{PC}, Φ_{SW}	$B \rho_{sw}^{-1/6} V_{sw}^{2/3} sin^4(\theta_{GSM}/2)$	1	–0.17	0.67	4	$sin^d(\theta/2)$	-	*this paper*
empirical fit to *Dst*	$B_\perp \rho_{sw}^{1/3} V_{sw}^{5/3} sin^4(\theta_{GSM}/2)$	1	0.33	1.67	4	$sin^d(\theta/2)$	1 hr	*Murayama* (1986)
empirical fit to *Dst*	$B_\perp \rho_{sw}^{1/2} V_s^{7/3} sin^6(\theta_{GSM}/2)$	1	0.5	2.33	6	$sin^d(\theta/2)$	1 hr	*Balikhin et al.* (2010)
theoretical estimate of Φ_D	$B V_{sw} sin^2(\theta_{GSM}/2)$	1	0	1	2	$sin^d(\theta/2)$	-	*Kan and Lee* (1979)
power input to the magnetosphere	$P_\alpha = B^{2\alpha} V_{sw}^{(7/3-2\alpha)} \rho_{sw}^{(2/3-\alpha)} sin^4(\theta_{GSM}/2)$	2α	$2/3-\alpha$	$7/3-2\alpha$	4	$sin^d(\theta/2)$	All	*Vasyluinas et al* (1982)
P_α fitted to *AL*	P_α for $\alpha = 0.50$	1	0.27	1.33	4	$sin^d(\theta/2)$	1 min	*Bargatze et al* (1986)



Proposed Coupling Functions (3)

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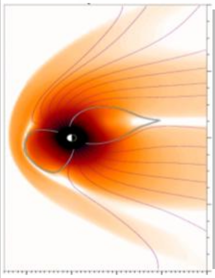
P_α fitted to *AL* data, allow for data gaps	P_α for $\alpha = 0.42$	0.84	0.25	1.49	4	$sin^d(\theta/2)$	1 hr	*Lockwood et al* (2019a)
P_α fitted to *AL* data allow for data gaps	P_α for $\alpha = 0.44$	0.88	0.23	1.45	4	$sin^d(\theta/2)$	1 yr	*Lockwood et al* (2019a)
P_α fitted to range geomagnetic data	P_α for $\alpha = 0.36$	0.72	0.31	1.61	4	$sin^d(\theta/2)$	1 day	*Lockwood* (2019)
Theory and fits to various geomagnetic data	$\approx B^{0.93} N_{sw}^{0.04} V_{sw}^{1.07} sin^2(\theta_{GSM}/2)$	0.93	0.04	1.07	2	$sin^d(\theta/2)$	1 hr	*Borovsky (2013)*
Theory and fits to various geomagnetic data	$\approx B^{1.26} N_{sw}^{-0.13} V_{sw}^{0.74} sin^2(\theta_{GSM}/2)$	1.26	–0.13	0.74	2	$sin^d(\theta/2)$	1 hr	*Borovsky (2013)*
empirical fit to *AL*	$B_\perp^{0.7} V_{sw}^{1.92} N_{sw}^{0.1} \sin^{3.67}(\theta_{GSM}/2)$	0.9	0.05	2.14	4.85	$sin^d(\theta/2)$	1 min	*Luo et al.* (2013)
numerical simulation	$B_\perp^{0.86} V_{sw}^{1.47} N_{sw}^{0.24} \{\sin^{2.70}(\theta_{GSM}/2)+0.25\}$	0.86	0.24	1.47	2.70	$sin^d(\theta/2)$	-	*Wang et al.* (2014)
empirical fit to *AL*	$B_\perp^{0.7} V_{sw}^{1.92} N_{sw}^{0.1} \sin^{3.67}(\theta_{GSM}/2)$	0.70 ±0.01	0.096 ±0.009	1.92 ±0.04	3.67 ±0.04	$sin^d(\theta/2)$	1 min	*McPherron et al.* (2015)
empirical fit to *am*	$B^{0.955} \rho_{sw}^{0.355} V_{sw}^{2.434} sin^{3.87}(\theta_{GSM}/2)$	0.95 ±0.02	0.35 ±0.01	2.40 ±0.05	3.40 (3.05 - 4.00)	$sin^d(\theta/2)$	1 hr	*this paper*
empirical fit to Φ_{PC}	$B^{0.792} \rho_{sw}^{0.031} V_{sw}^{0.457} sin^{3.84}(\theta_{GSM}/2)$	0.73 ±0.02	0.03 ±0.01	0.43 ±0.03	2.60 ±0.10	$sin^d(\theta/2)$	1 hr	*this paper*

- and there are many more !
- (many of them more complex in form)



Want to "combine-then-average" Not "average-then-combine"

- Here use hourly data

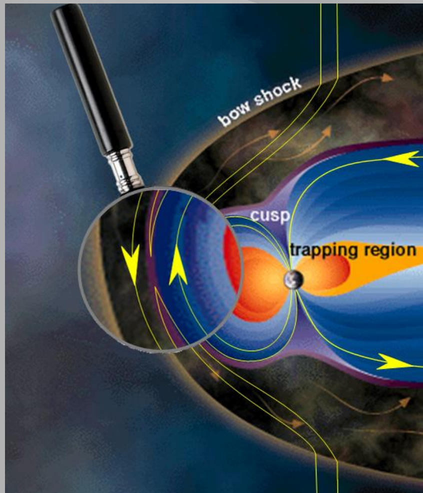
  we want $< C_f >_{1hr} = < B_\perp^{a} \rho_{sw}^{b} V_{sw}^{c} \sin^{d}(\theta/2) >_{1hr}$

- should not use $[C_f]_{1hr} = <B_\perp>^{a} \ <\rho_{sw}>^{b} \ <V_{sw}>^{c} \ < \sin(\theta/2)>^{d}$

- as d is not independent of a, b, & c

  we here take a fixed d – use Nelder-Mead simplex search to find exponents a, b, & c that give best correlation between $< C_f >_{1hr}$ and $< T >_{1hr}$ (where the terrestrial index T is at the optimum lag)

- then use the *Vasyluinas et al (1982)* test (and not correlation because of a weighting issue) to which d gives linearity between $< C_f >_{1hr}$ and $< T >_{1hr}$

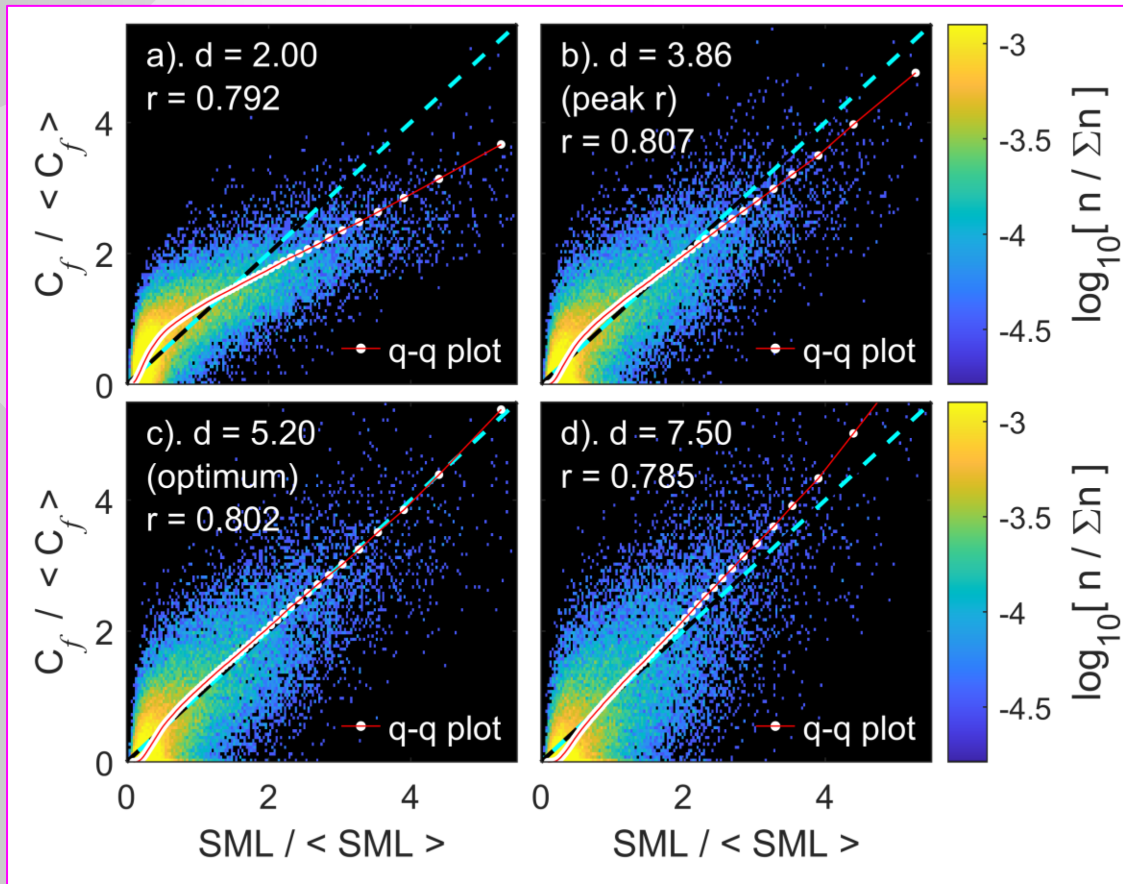


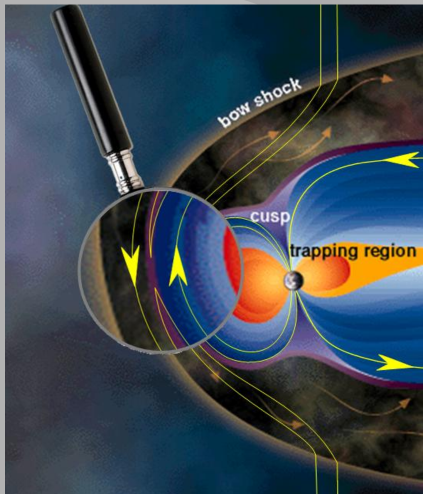
Solar Wind-Magnetosphere Coupling Functions

- Commonly used form

$$C_f = B_\perp^{a} \rho_{sw}^{b} V_{sw}^{c} \sin^{d}(\theta/2)$$

- *SML* is like *AL* but from many more (north hemisphere) stations
- q-q plots test shows if distributions are the same
- optimum (d = 5.2) gives linearity between C_f and *SML* and a more linear q-q plot
- But peak correlation (r = 0.81) is for d = 3.86





Solar Wind-Magnetosphere Coupling Functions

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- Commonly used form

$$C_f = B_\perp^{\,a}\,\rho_{sw}^{\,b}\,V_{sw}^{\,c}\,\sin^d(\theta/2)$$

- None of the C_f distributions match that for SML at low values (largely when IMF $B_Z > 0$)

- But for larger d they do fit the distribution for larger SML (disturbed conditions, dominated by IMF B_Z < 0)