



RESEARCH ARTICLE

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Key Points:

- Correlation of sunspot and geomagnetic indices masks solar cycle dependence
- Ratio of international and group sunspot numbers is variable
- A composite proposed can be used to quantify open flux emergence

Supporting Information:

- Readme
- Text S1

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Centennial variations in sunspot number, open solar flux, and streamer belt width: 2. Comparison with the geomagnetic data

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Abstract We investigate the relationship between interdiurnal variation geomagnetic activity indices, *IDV* and *IDV*(1d), corrected sunspot number, R_C , and the group sunspot number R_G . R_C uses corrections for both the “Waldmeier discontinuity,” as derived in Paper 1, and the “Wolf discontinuity” revealed by Leussu *et al.* (2013). We show that the simple correlation of the geomagnetic indices with R_C or R_G masks a considerable solar cycle variation. Using *IDV*(1d) or *IDV* to predict or evaluate the sunspot numbers, the errors are almost halved by allowing for the fact that the relationship varies over the solar cycle. The results indicate that differences between R_C and R_G have a variety of causes and are highly unlikely to be attributable to errors in either R_G alone, as has recently been assumed. Because it is not known if R_C or R_G is a better predictor of open flux emergence before 1874, a simple sunspot number composite is suggested which, like R_G , enables modeling of the open solar flux for 1610 onward in Paper 3 but maintains the characteristics of R_C .

1. Introduction

In this second paper of a series of three, we study the relationship between geomagnetic interdiurnal range indices and sunspot numbers in detail. We investigate the degree to which sunspot data are consistent with the geomagnetic data and aim to define sunspot data sequences to use as inputs to the open solar flux model prediction in Paper 3 [Lockwood and Owens, 2014].

As discussed in Paper 1 [Lockwood *et al.*, 2014c], there are two main long-term indices in common use to quantify sunspot activity; the sunspot number and the group sunspot number data sequences which extend back to 1749 and 1610, respectively, in monthly means. We here start from the sunspot number R sequence published by the World Data Centre for the Sunspot Index and Long-term Solar Observations (*WDC-SILSO*) of the Royal Observatory of Belgium, Brussels, which is a composite of the international, Zürich, and Wolf sunspot numbers. The international number has been compiled from data generated by a number of observers (currently numbering 86 in 29 different countries) from 1 January 1981 onward. This uses the same algorithm as, and is a continuation of, the daily Zürich sunspot numbers, R_z , which were based upon observations made at Zürich and its two branch stations in Arosa and Locarno. Only one observer was used to make each daily R_z estimate (who was the highest ranked observer available on that day in a hierarchy ordered by perceived reliability). The “Waldmeier discontinuity” discussed in Paper 1 relates to a putative change in observing and processing practices for compiling R_z around 1945.

As discussed by Clette *et al.* [2007], the basic sunspot number formulation was first used in 1849 by Rudolf Wolf, Director of the Zürich Observatory. He was succeeded by his former assistant Alfred Wolfer, who introduced an important change in 1882. In generating his sunspot number, R_W , Wolf did not count the smallest spots when quantifying the number of individual spots, N_S , in order to try to maintain compatibility with earlier observers. However, this requires a subjective choice of which spots to include. Wolfer abandoned this practice and his new procedure was calibrated against Wolf’s sunspot numbers over a 16 year interval (1877–1892). This yielded a seemingly constant scaling factor of 0.6 that is still used today to scale the sunspot observations to the pre-1882 Wolf sunspot number, R_W . In addition, Wolf had become so convinced of a linear correlation between his geomagnetic and sunspot data that he recalibrated his sunspot numbers for before 1849 upward by about 20%. An additional problem is that the data before 1849 are sparse, leading to greater uncertainties [Usoskin, 2013]. Another inhomogeneity in the data series arises in 1818, before when only monthly values have been compiled, whereas after then the basic data available are daily. Annual means of R_W are available from 1700 [Chernosky and Hagan, 1958], but they are not generally regarded as reliable because the data are so sparse and highly interpolated and consequently these are rarely used [Usoskin, 2013].

Leussu et al. [2013] have recently used a homogeneous series of recently digitized sunspot drawings by Schwabe [*Arlt et al.*, 2013] to study the calibration of the Wolf sunspot numbers, R_W . They find a systematic error in the scaling of R_W before 1849, calling for a 20% reduction in values of R for all years before 1849. This almost certainly is reversing Wolf's 20% upward recalibration of sunspot numbers to agree with geomagnetic activity data, a practice that is now generally thought to be both unreliable and undesirable [*Mursula et al.*, 2009, *Lockwood et al.*, 2014c]. Hence, we will refer to the correction called for by the work of Leussu et al. as the "Wolf discontinuity."

The group sunspot number series was introduced by *Hoyt and Schatten* [1994, 1998] and is extremely valuable because it extends back to before the Maunder minimum in sunspot activity. However, it has to be compiled from observations with sparse availability for the early years (N in equation (2) of Paper 1 is small and some observers may not have been active throughout any 1 year). *Usoskin et al.* [2003a] studied the effect of this and made some minor corrections, and other corrections have been made, based on additional historic observations that have come to light [*Vaquero et al.*, 2011; *Vaquero and Trigo*, 2014]. The Royal Greenwich Observatory (RGO)/Solar Observing Optical Network (SOON) sunspot group data discussed in Paper 1 cover 1874 to the present and form the backbone of the R_G data sequence, and the correlation demonstrated in Paper 1 is used to ensure that it is very similar to the international sunspot number since 1874. However, it is well known that international/Zürich/Wolf composite and group sunspot numbers diverge as one goes back in time before this date, as indeed was noted by *Hoyt and Schatten* [1994, 1998] when they first derived the R_G sequence [e.g., *Hathaway et al.*, 2002].

Paper 1 quantified the magnitude of the Waldmeier discontinuity to be 11.6%. In addition, *Leussu et al.* [2013] have quantified the Wolf discontinuity to be 20%. We here implement both these corrections before establishing the relationship between the geomagnetic variation and sunspot number and how it varies over the solar cycle. To ensure that there is no confusion, we here call the corrected international/Zürich/Wolf sunspot number composite R_C where $R_C = R$ for 1946 and after, $R_C = 1.116R$ for 1849–1945, and $R_C = (1.116 \times 0.8)R$ for 1848 and before, where R is the sunspot number composite, as published by WDC-SILSO. In the present paper we also suggest a simple extension of the resulting R_C back to 1610 using $1.3R_G$. (Note that this is not, in any sense, a recalibration of R_G ; rather, it is a means of extending the R_C data series back to the Maunder minimum that is greatly preferable to using the sparse, highly interpolated early annual means of the Wolf number).

Svalgaard and Cliver [2005] introduced the IDV index based on the interdiurnal variability u index of *Bartels* [1932]. The major difference between u and IDV is that the latter is based on near-midnight values only, whereas the u index used whole-day means. *Svalgaard and Cliver* [2007] did not see this as a significant difference because they used *Bartels*' u index data to extend the IDV sequence back to 1835, which is just 3 years after *Gauss*' establishment of the first magnetometer station in Göttingen. The data used by *Svalgaard and Cliver* [2007] to extend IDV to before 1880 were compiled by *Bartels*, but it is not a homogeneously constructed index, being compiled in a different way after 1872 to before then. *Bartels* notes that before 1872, no proper data to generate an interdiurnal index were available to him and so he used other correlated measures of the diurnal variation as proxies. *Bartels* himself stresses that his u values before 1872 are "more for illustration than for actual use" and describes data for 1835–1847 as "least reliable," 1847–1872 as "better," and 1872–1930 as "satisfactory." Given that *Bartels* does not include his data before 1872 in his satisfactory classification, it is not just a semantic point that he regarded the data before 1872 as "unsatisfactory." *Svalgaard and Cliver* [2007] carried out some tests to justify employing the u proxy data for before 1872 which enabled them to make a reconstruction of geomagnetic activity back to 1835. However, Figure 5 of Paper 1 [*Lockwood et al.*, 2014c] shows that the early IDV data have different characteristics to the later data. Specifically, the early IDV index was much more similar to the sunspot number data than at later times strongly suggesting that the independence of the two data sets had not been maintained and that at some point calibration of one using the other had taken place. This may partially have come about through Wolf's use of geomagnetic data to recalibrate R_W , but as shown in Paper 1, other geomagnetic data do not show the same error as IDV . This is a pitfall we remain conscious to avoid in this paper.

Lockwood et al. [2013a] introduced the $IDV(1d)$ index because of the following: (1) they were concerned about the overstrong correlation between u (and hence early IDV) and sunspot numbers in the early years and (2) because they found that the dependence on interdiurnal variation data on interplanetary parameters depended on station latitude (at all latitudes, not just near the auroral oval) and because IDV was compiled from an evolving mix of stations, its dependence on interplanetary parameters in the past would be different from that in the space age. These authors also returned to *Bartels*' original concept of using whole-day means which has advantages

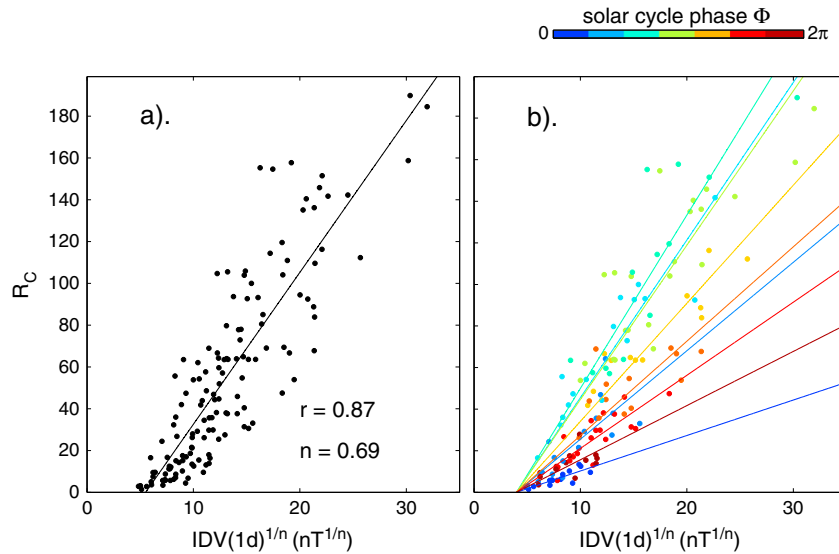


Figure 1. Scatterplots of annual means for 1874 to 2012 (inclusive) of the corrected international sunspot number, R_C , as a function of $IDV(1d)^{1/n}$, where n is the best fit exponent ($n = 0.69$), derived in Paper 1 and R_C employs the optimum corrections f_{R1} and f_{R2} discussed in the text. (a) Takes no account of the solar cycle phase, whereas in (b) the data are sorted into seven equal-sized bins of solar cycle phase, Φ , derived from the RGO/SOON data using the mean heliographic latitude of sunspot groups and the method of *Owens et al.* [2011]. Points for each bin are color coded and fitted with a linear regression constrained to pass through the point $R_C = 0, IDV(1d) = 2.6$ ($IDV(1d)^{1/n} = 4$); the linear regression fits have slope F and are shown by the lines using the same color coding.

in suppressing both instrumental and geophysical noise: indeed, the range of correlations for stations at different latitudes is found to be as great as that for one station at different UTs, and hence, 24 stations of IDV data are required to achieve the same noise suppression as one station giving $IDV(1d)$. This has the advantage of allowing $IDV(1d)$ to be compiled from just one magnetic observatory in the same geographical region, avoiding artifacts caused by the response to interplanetary variations varying with the location of the observatory. *Lockwood et al.* [2013a] were able to extend the $IDV(1d)$ data sequence back to 1845 using data from the Helsinki observatory. *Svalgaard* [2014] pointed out a calibration error in the Helsinki data that applied to 7 years during cycle 11, and *Lockwood et al.* [2014a] have studied the optimum correction method and implemented it to allow for this. It is important to note that although *Svalgaard* [2014] used R_C to identify the calibration change in the Helsinki data, the correction implemented by *Lockwood et al.* [2014a] did not employ any form of sunspot number and so the full independence of the geomagnetic and sunspot number data sequences was maintained.

From modeling using R to quantify emergence rate, *Wang et al.* [2005] found an approximate relationship between open solar flux and R^n , with an exponent of n near 0.5. However, these authors also noted that the correlation was high for photospheric flux but poor for open solar flux. *Svalgaard and Cliver* [2005] found that their IDV index correlated with R^n . They also noted that IDV correlates with the near-Earth interplanetary magnetic field (IMF) B with very little influence of the solar wind speed (see *Lockwood* [2013]) so *Svalgaard and Cliver* [2005] concluded that B also varies with R^n . As noted by *Lockwood et al.* [2014a], the peak correlation coefficient between the sunspot number to the power n, R^n , and the observed IMF is $r = 0.84$ for $n = 0.4$, with significance (computed against the AR-1 autoregressive “red-noise” model) of $S = 96.3\%$. Hence, the simultaneous R^n “explains” about 70% of the overall observed IMF variation. Paper 1 shows the peak correlations for IDV , and $IDV(1d)$ are 0.85 and 0.83 for $n = 0.54$ and $n = 0.69$, respectively. Both are more statistically significant than the correlation with the IMF data ($S > 99.9\%$) because of the larger number of data samples. We here investigate these correlations of IDV and $IDV(1d)$ with sunspot numbers in more detail.

2. The Nature of the Relationship Between Interdiurnal Variation Indices and Sunspot Number

Figure 1a shows a scatterplot of annual means of R_C , the corrected WDC-SILSO sunspot composite (corrected using the optimum correction factors $f_{R1} = 1.116$ for the Waldmeier discontinuity, as derived in Paper 1, and the

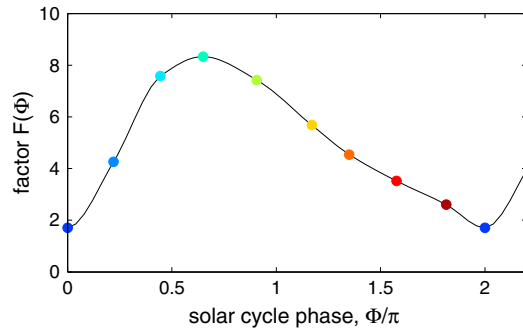


Figure 2. Variation of the slope of the regressions lines in Figure 1b, $F(\Phi)$ as a function of Φ . The points are the best fit values for the seven bins (shown using the same color coding as Figure 1b) and the line is an interpolation using splines fitting. Points are shown for the mean Φ of the data in each bin.

the method of *Owens et al.* [2011] which exploits the regular solar cycle (butterfly wing) variation of the mean latitude of sunspot groups from the RGO/SOON data: Φ is defined as zero (2π) at the start of each new cycle when the mean latitude of sunspots increases suddenly. Points are color-coded by their Φ bin and for each a linear regression is fitted, each constrained to pass through a set common origin (and shown in Figure 1b using the same color scheme). The fits all pass through the point $IDV(1d) = 2.6$ (which gives $IDV(1d)^{1/n} = 4$), $R_C = 0$. This point was derived by varying the intercept and evaluating the probability density functions (pdfs) from the p values for each fit: these were very similar in form and taking the peak of the product of the pdfs gives the optimum intercept value for all seven fits. It can be seen that the relationship between $IDV(1d)^{1/n}$ and R_C depends strongly on Φ with a coherent solar cycle variation giving a much larger slope, F , at sunspot maximum (the light blue and green lines) than at sunspot minimum (the red/brown and blue lines). Hence, using an overall fit shown in Figure 1a would introduce systematic differences at both sunspot minimum and sunspot maximum.

The best fit slope $F (= dR_C/d(IDV(1d)^{1/n}))$, of each linear regression fit is shown in Figure 2, using the same color coding as Figure 1b, as a function of the sunspot cycle phase Φ . The black line is an interpolation using a Piecewise Cubic Hermite Interpolating Polynomial fit that is here used to compute F for any value of Φ . Figure 3a

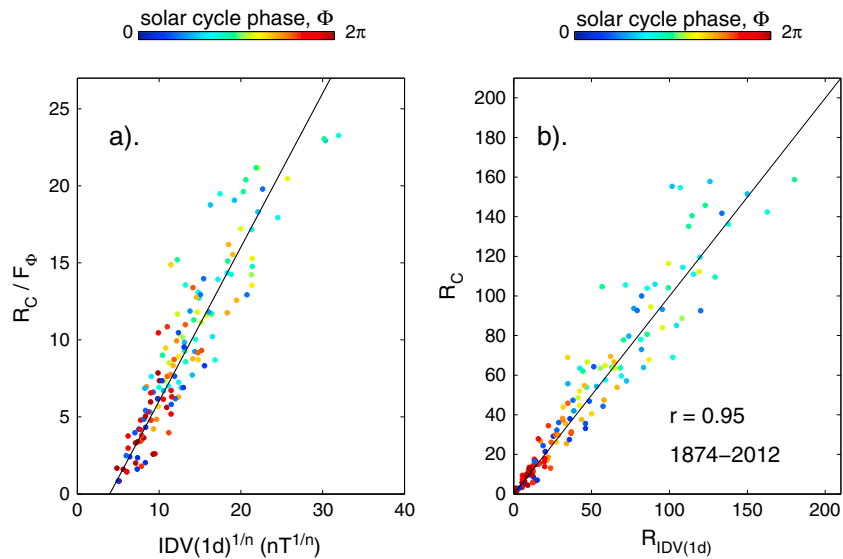


Figure 3. (a) Scatterplot of $R_C/F(\Phi)$, determined using the R_C and Φ value for each annual mean with the $F(\Phi)$ value interpolated using Figure 2, as a function of $IDV(1d)^{1/n}$ for the optimum n of 0.69. The black line is the best linear regression fit $R_C/F(\Phi) = s IDV(1d)^{1/n} + c$ for the best fit coefficients of $s = 1.000$ and $c = -3.966$. (b) The corrected sunspot number R_C as a function of its estimate from estimate from $IDV(1d)$, $R_{IDV(1d)} = F(\Phi) \times [s IDV(1d)^{1/n} + c]$. In both panels the annual data points are color coded according to their solar cycle phase, Φ .

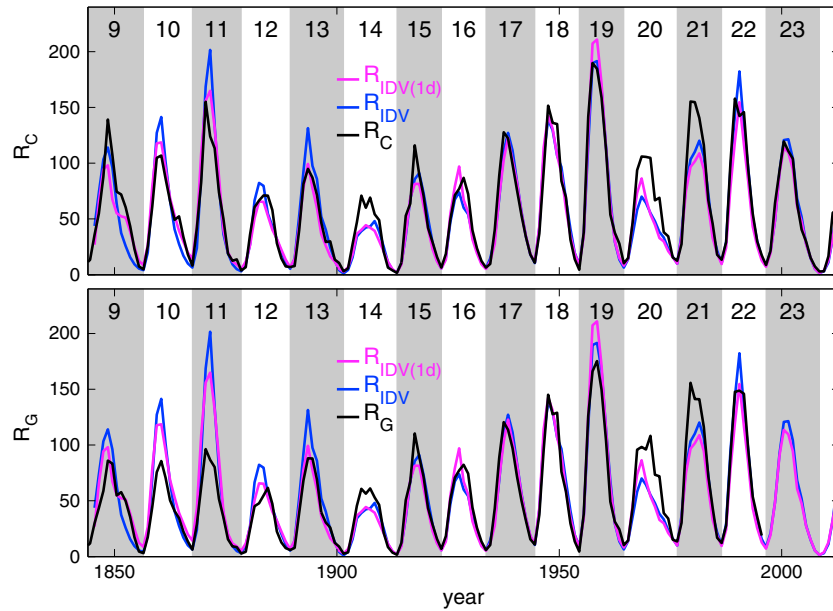


Figure 4. (top and bottom) Variations of annual means for 1845–2012. The black line in Figure 4 (top) is the corrected international sunspot number, R_C , and that in Figure 4 (bottom) is the group sunspot number, R_G . In both panels the blue line is R_{IDV} , constructed from the IDV geomagnetic index and the mauve line is $R_{IDV(1d)}$, constructed from the $IDV(1d)$ geomagnetic index. Odd-numbered solar cycles are shaded grey, and the cycle number is given along the top of each panel.

shows a scatterplot of the annual values of $R_C/F(\Phi)$ against $IDV(1d)^{1/n}$ obtained this way. It can be seen that the scatter has been reduced by the use of $F(\Phi)$ compared to Figure 1a and that the fit now applies at all phases of the solar cycle with no systematic biases of the residual with solar cycle phase. The black line is the best linear regression of slope s and intercept c , using which the value of $IDV(1d)$ can be used to compute the best fit to R_C , which we term $R_{IDV(1d)} = F(\Phi) \times [s IDV(1d)^{1/n} + c]$, where the best fit coefficients are $s = 1.000$ and $c = -3.966 \text{ nT}^{1/n}$. Note that s is unity, as it should be from the definition of $F(\Phi)$. A scatterplot of $R_{IDV(1d)}$ against R_C is shown in Figure 3b. By allowing for the phase of the solar cycle, Φ , the correlation coefficient has been raised from $r = 0.87$ ($r^2 = 0.75$) in Figure 1a to $r = 0.95$ ($r^2 = 0.90$) in Figure 3b and so this enables us to “explain” a further 15% of the variation in R_C . The RMS deviation between observed and fitted R_C values has been reduced from 22.4 to 13.8.

This is not just a property of the $IDV(1d)$ index: the IDV index behaves in exactly the same way. Without allowance for Φ , the correlation of R_C with against $IDV^{1/n}$ peaks at $r = 0.87$ for $n = 0.54$ ($r^2 = 0.75$), whereas R_{IDV} and R_C give $r = 0.94$ ($r^2 = 0.88$) and the RMS deviation between observed and fitted R_C values is reduced from 22.7 to 14.5 by including an allowance for Φ .

Hence, if using $IDV(1d)$ or IDV to predict or evaluate the sunspot number R_C , the prediction errors are almost halved by allowing for the fact the relationship varies over the solar cycle. Because R_C and R_G are very similar over much of the interval of the geomagnetic observations, very similar results were obtained by fitting to R_G rather than R_C .

3. Comparison of Sunspot Number Estimates

Figure 4 compares the time series of the corrected sunspot numbers (R_C , in black in top) and the group sunspot numbers (R_G , in black in bottom) with values estimated with allowance for Φ from $IDV(1d)$ and IDV , $R_{IDV(1d)}$ and R_{IDV} , which are shown in both panels by the mauve and the blue lines, respectively. Agreement is generally good using both geomagnetic indices—the slightly higher correlation (and lower RMS fit residual) obtained using $IDV(1d)$ is mainly because IDV tends to give some solar cycle maxima that are too large, compared to the maxima in both R_C and R_G . The white and gray stripes show even- and odd-numbered solar cycles, respectively; the cycle numbers being given along the top of both panels.

Table 1. Analysis of Minimum-to-Minimum Sunspot Cycle Means for Solar Cycles 1–16^a

Cycle Number	Start Year	End Year	$\langle R_C \rangle$	$\langle R_G \rangle$	$\langle R_C \rangle / \langle R_G \rangle$
1	1754	1765	37.11	28.57	1.30
2	1765	1775	50.03	43.22	1.16
3	1775	1783	65.95	39.71	1.66
4	1783	1798	61.73	34.63	1.49
5	1798	1810	21.27	12.14	1.75
6	1810	1823	16.14	10.82	1.49
7	1823	1833	34.84	33.52	1.04
8	1833	1843	58.28	50.73	1.15
9	1843	1856	56.07	41.65	1.35
10	1856	1867	55.34	41.72	1.33
11	1867	1878	63.12	43.65	1.45
12	1878	1889	38.55	30.12	1.28
13	1889	1901	43.27	40.30	1.07
14	1901	1913	34.76	32.28	1.08
15	1913	1923	49.28	49.07	1.00
16	1923	1933	45.81	46.46	0.99
17	1933	1944	61.38	59.01	1.04
18	1944	1954	76.16	72.26	1.05
19	1954	1964	94.89	86.86	1.09
20	1964	1976	58.80	58.39	1.01
21	1976	1986	82.98	83.32	1.00

^aIn addition to the cycle start and end dates, the cycle means of the corrected international sunspot number, $\langle R_C \rangle$, and the sunspot group number, $\langle R_G \rangle$, are given along with the ratio of the two $\langle R_C \rangle / \langle R_G \rangle$.

Paper 1 discusses the sunspot number variation after the start of the RGO data in 1874 (solar cycles 12–13), and we here concentrate in the behavior before then, namely, cycles 9–11. Figure 4 shows that the sunspot number variation predicted by $IDV(1d)$ is consistent with the R_C over solar cycle 11, whereas the group number is considerably smaller. The value for IDV is larger than all other values for this cycle. $R_{IDV(1d)}$ and R_{IDV} both increase with time over cycles 9–11 such that although both are much closer to R_C for cycle 11, for cycle 9 they both lie midway between R_G and R_C . In this comparison, it should be remembered that the higher values of $R_{IDV(1d)}$ in cycle 11 arise from recalibration of the Helsinki data proposed by Svalgaard [2014]. Svalgaard's correction was based on sunspot numbers but the $IDV(1d)$ index implemented here is that of Lockwood *et al.* [2014a] who used only geomagnetic data (from Helsinki and St. Petersburg) to make this correction, thereby maintaining the independence of the sunspot and geomagnetic data.

Table 1 gives the means over the sunspot cycles of both R_C and R_G and their ratio: it shows that cycle 11 gives $\langle R_C \rangle / \langle R_G \rangle$ of 1.45 which is a maximum for after the Dalton minimum but this ratio declines as we go back in time and is essentially unity for cycle 7 (immediately after the Dalton minimum). Going to yet earlier cycles, $\langle R_C \rangle / \langle R_G \rangle$ rises again, reaching a peak of 1.75 in cycle 5 before falling again to 1.30 for cycle 1.

4. Discussion

Figure 5a shows the annual means of R_G than R_C (in red and blue), and Figure 5b gives the 22 year running means $\langle R_G \rangle_{22}$ and $\langle R_C \rangle_{22}$ (in the same colors) and Table 1 gives the means over each solar cycle (minimum to minimum) and their ratio. Cliver *et al.* [2013] study the ratio of annual values R_C/R values (where R is the WDC-SILSO sunspot number composite), omitting years with R below 5 to avoid generating large values. They define a discontinuity around 1882 which they attribute to an error in the calibration of R_G . They then assume that the R calibration is constant (so all differences between R and R_G are assumed to be erroneous and all errors are assumed to be in R_G), thereby generating a new sunspot number sequence. In addition, they use an overly large estimate for f_{R1} of 1.2 (remember that Paper 1 finds an optimum value of 1.116 and that the probability of $f_{R1} = 1.2$ is miniscule), and they neglect the Wolf discontinuity (i.e., they use f_{R2} of 1). Allied together these three decisions have the effect of making pre-1885 values of sunspot number much larger and so make the long-term drift in sunspot numbers appear much smaller [see Svalgaard, 2011].

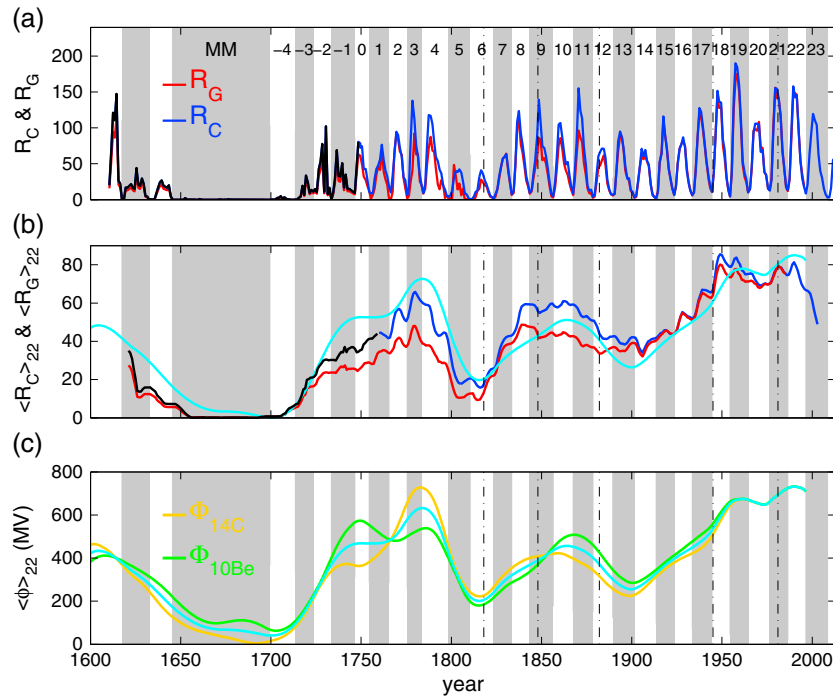


Figure 5. (a) Annual means of (in red) the group sunspot number, R_G , and (in blue) the corrected international sunspot number, R_C . The black line is $1.3R_G$. (b) Twenty-two year running means of the variations in Figure 5a. The cyan line is the best linear regression of the average heliospheric modulation potential ϕ_{av} . (c) Twenty-two year means of heliospheric modulation parameters ϕ , as given by *Usoskin* [2013] from the ^{14}C ($\phi_{14\text{C}}$, orange) and ^{10}Be ($\phi_{10\text{Be}}$, green) cosmogenic isotopes. The cyan line is the mean of the two $\phi_{av} = (\phi_{14\text{C}} + \phi_{10\text{Be}})/2$. In all panels, odd-numbered solar cycles (minimum to minimum) are shaded grey and even-numbered ones in white and the cycle number is given along the top of (a). The Maunder minimum (MM) is also shaded grey. The vertical dash-dotted lines give significant dates in the development of the R_C composite: 1818 (start of daily data); 1848 (the correction for the Wolf discontinuity); 1882 (change from Wolf to Zürich sunspot numbers with the start of counting all sunspots); 1945 (the correction for the Waldmeier discontinuity); 1981 (change from Zürich to international sunspot numbers with the start of use of a network of observers).

However, Figures 4 and 5 and Table 1 cast major doubt that R_C values before 1882 are essentially correct and the variations in the ratio $\langle R_C \rangle / \langle R_G \rangle$ are due to errors in $\langle R_G \rangle$ alone. The vertical dash-dotted lines in Figure 5 show the times of significant changes in the compilation of the R (and hence R_C) composite. The last of these is the change from Zürich to international sunspot number. The annual means of R_C and R_G in Figure 5a show no relative change across this date and although $\langle R_C \rangle_{22}$ values in Figure 5b only extend for a few years after this date, there is no sign of any discontinuity. Before this is the Waldmeier discontinuity in 1945 which sits close to the minimum between cycles 17 and 18, and Table 1 shows that $\langle R_C \rangle / \langle R_G \rangle$ is essentially the same for these two cycles. Hence, any variation between cycles 15 and 21 is not attributable to the Waldmeier discontinuity (which has now been corrected for) and is associated with measurement uncertainties in R_C or R_G , or both, or with a real variation in the ratio of the numbers of spots to groups (N_S/N_G) which would influence R_C and R_G differently.

The dash-dotted line at 1882 (during cycle 12) marks the change from Wolf to Wolf's sunspot counting procedure. Table 1 shows that $\langle R_C \rangle / \langle R_G \rangle$ increases strongly as we go back in time through this date, which can also be seen in Figures 5a and 5b. *Hoyt and Schatten* [1994, 1998] were aware of this when they compiled R_G and, quite reasonably, attributed it to inadequate calibration of the known change from Wolf to Wolf's method and hence an error in R . On the other hand, *Cliver et al.* [2013] accept the validity of the 0.6 Wolf calibration factor as a constant and claim the error is in R_G . The geomagnetic data do give some support to Cliver et al.'s view in that in Figure 4 the sunspot reconstructions for cycle 11 from geomagnetic activity, $R_{IDV(1d)}$ and R_{IDV} both agree somewhat better with R_C than R_G . However, Wolf's derivation of the 0.6 factor was based on data from 1877 to 1892, and Table 1 shows that as we go back in time to before 1877, into cycles 10 and then 9, the ratio $\langle R_C \rangle / \langle R_G \rangle$ falls, and Figure 4 shows that the geomagnetic reconstructions lie

midway between R_C and R_G in cycle 9 and indeed the more reliable test is $IDV(1d)$ (because, as shown in Paper 1, it is uncontaminated by sunspot number) and this gives $R_{IDV(1d)}$ and that is more similar to R_G than R_C in cycle 9. Cycles 7 and 8 give $\langle R_C \rangle / \langle R_G \rangle$ close to unity because of the correction by *Leussu et al.* [2013] for the Wolf discontinuity (the dash-dotted line at 1848 during cycle 9). Hence, the relative behavior of R_C and R_G and the work of *Leussu et al.* [2013] shows that using Wolf's constant factor of 0.6 before 1877, as assumed by *Cliver et al.* [2013] and *Svalgaard* [2011], is too simplistic and invalid.

Table 1 and Figure 5b show that the corrections for the Wolf and Waldmeier discontinuities make R_C and R_G more similar in their long-term variation than R and R_G . Nevertheless, there are still differences which are considerably greater before 1882 than after that date, and those differences do fluctuate with time. Given the subjectivity of the decisions on which individual spots to include in Wolf's sunspot numbers before 1882, we agree with the many authors who regard R_G as more robust and homogeneous than R (and hence R_C) (see reviews by *Usoskin* [2013], *Hathaway* [2010], *Hathaway and Wilson* [2004], and references therein). However, it is possible that calibration errors in both sequences contribute to these fluctuations at different times and to varying degrees, and we can never be sure that there has not been a long-term variation in sunspot behavior which means that at least a component of the divergences or R_C and R_G are real phenomena rather than measurement artifacts.

In the context of this last point, we here note that recent cycles do not lead us to expect the ratio R_C/R_G to be exactly constant, as shown by Figure 5 and Table 1 (cycles 16 to 21 show a 10% peak-to-peak fluctuation in $\langle R_C \rangle / \langle R_G \rangle$). *Penn and Livingston* [2006] have created much interest in the variability of magnetic fields within sunspots. They deduced a decline over the interval 1998–2005, whereas other authors report only a solar cycle variation with little change from one cycle to the next [*Pevtsov et al.*, 2011]. *Nagovitsyn et al.* [2012] note that these data can be reconciled by the observation that there has been a change over 1998–2005 in the fraction of small and large sunspots which has the potential to cause different changes in R and R_G and hence to vary the ratio R/R_G . In addition, the results of *Kilcik et al.* [2011, Figure 3] show changes in the ratios R/N_{GS} and R/N_{GL} where N_{GS} is the number of small sunspot groups and R/N_{GL} is the number of large sunspot groups. Specifically, solar cycle means of $R/(N_{GS} + N_{GL})$ are approximately 1.1, 1.2, 1.0, and 0.8 for solar cycles 20–23 so there is a $\pm 20\%$ variation about the mean. Furthermore, there have been changes in the distribution of group sizes with $(N_{GS}/N_{GL}) \approx 2.1, 1.6, 2.0,$ and 1.4 for those cycles which, for a given definition of what constitutes a group, will have different effects on R and R_G . Hence, some variations in R/R_G (and hence R_C/R_G) are real and should not be attributed to calibration errors.

Separating any calibration drifts in both R_C and R_G and so identifying any effects of real changes in the behavior of the individual spots will require a great deal of careful checking of individual historic sunspot observations that is beyond the scope of this paper. The geomagnetic test reported on here may well be improved and/or extended back toward the ultimate start date of 1832 (the date of the establishment of the first magnetometer station by Gauss) as more digital data become available, but the calibration of that early data must be kept fully independent of sunspot records if it to be of any value. Other tests may be possible, for example, using auroral sighting catalogues.

However, there is one comparison that we can make at this stage, and that is with the heliospheric modulation potential ϕ derived from cosmogenic isotopes. Figure 5c shows two examples of ϕ reconstructions (22 year means) from ^{10}Be (in green) and ^{14}C (in orange) from the data sets by, respectively, *Usoskin et al.* [2003b] and *Muscheler et al.* [2007], as reviewed by *Usoskin* [2013]. It can be seen that they are not identical, and it must be noted that there are other reconstructions which are different again. Both show considerable similarities to both the smoothed sunspot number records shown in Figure 5b. We here take the arithmetic mean of these two ($\phi_{av} = 0.5(\phi_{10\text{Be}} + \phi_{14\text{C}})$, shown in cyan) and Figure 5b shows (again in cyan) a linear regression of ϕ_{av} against smoothed sunspot number for 1880–2000. Several features of this variation from cosmogenic isotopes are also seen in both the $\langle R_C \rangle_{22}$ and $\langle R_G \rangle_{22}$ variations before 1873. Interestingly, the cosmogenic isotope data, like the geomagnetic data, suggest that R_C is more accurate than R_G during solar cycle 11, whereas the opposite appears to be true in cycle 9.

5. Definition of a Usable and Consistent Composite for Modeling Purposes

This paper has discussed the considerable similarities and relatively small differences between various proxies of the solar magnetic activity level. It has been shown that implementing corrections to the sunspot

number record for the Wolf and Waldmeier discontinuities improves agreement, but we caution that, because they are different indices, one should not expect perfect agreement between sunspot numbers and group numbers. However, thus far, we have not achieved the stated aim of defining a sunspot number (as opposed to group number) variation that can be used as an input into the modeling in Paper 3, which requires extending it back to into or before the Maunder minimum because of the effect of the prior history of sunspot numbers and associated open flux emergence. From the above, we cannot conclude if R_G or R_C before 1874 contains calibration errors, or both or even neither (in which case all the differences would be associated with real changes in sunspot characteristics). Most importantly, for the purposes of Paper 3, we do not know if R_C or R_G better quantifies open flux emergence before 1874. This being the case, the best option is to investigate the implications of both. This is straightforward in the case of R_G because it extends back to before the Maunder minimum, but for R_C we require an extension to earlier years.

We here suggest the simplest algorithm possible. To extrapolate R_C to dates before 1749, we suggest using $1.3R_G$ which applies to solar cycle 1, thereby eliminating any discontinuity at 1749. This extension is shown in Figures 5a and 5b in black. This simple combination is reasonably similar to the average cosmogenic isotope variation. We note there are several refinements to this simple algorithm that could be made (for example, using uncorrected R_G in solar cycle 9 where it appears to agree better with both geomagnetic data and cosmogenic isotopes); however, given the large uncertainties in the calibration of both R_G and R_C that remain, we do not believe that they can be justified. We do not make any claims that this variation in sunspot number is in any way definitive; indeed, we expect more precise variations to emerge as a result of the several ongoing studies of historic sunspot observations to recalibrate both the international and group sunspot number and from improved ice cores yielding better cosmogenic isotope records. However, we do note that this variation is reasonably consistent with both the geomagnetic data and cosmogenic isotope abundances which provide the best independent tests available to us. It is important to stress that we do not mean to suggest, in any way, that this extended R_C is more accurate than R_G ; indeed, we subscribe to the view that the relative simplicity of R_G means that it is much more robust and homogeneous, even after the corrections used in R_C have been implemented. However, because it remains possible that the differences between R_C and R_G may be real and that R_C may be a better quantifier of open flux emergence, we need to be able to use both as an input to open flux continuity models. Hence, the last paper of this series of three [Lockwood and Owens, 2014] uses both the R_G and extended R_C sequences of sunspot number estimates as an input into a model of open solar flux variation. The output from this model is compared with the reconstructed open solar flux variation which has been generated from the geomagnetic data [Lockwood, 2013; Lockwood et al., 2013a, 2013b, 2014a, 2014b]. The R_G and R_C data sequences are available in the supporting information. The finalized IDV(1d) index data are given in the paper by Lockwood et al. [2014c] and in the supporting information by Lockwood and Owens [2014].

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