THE EVOLUTION OF THE SUN'S OPEN MAGNETIC FLUX

I. *A Single Bipole*

D. H. MACKAY[1], E. R. PRIEST[1] and M. LOCKWOOD[2]
[1] *School of Mathematics and Statistics, University of St. Andrews, St.Andrews, Fife, Scotland, KY16 9SS*
[2] *World Data Centre C1, Rutherford Appleton Laboratory, Chilton, Oxfordshire, U.K.*

(Received 1 March 2002; accepted 9 March 2002)

Abstract. In this paper the origin and evolution of the Sun's open magnetic flux are considered for single magnetic bipoles as they are transported across the Sun. The effects of magnetic flux transport on the radial field at the surface of the Sun are modeled numerically by developing earlier work by Wang, Sheeley, and Lean (2000). The paper considers how the initial tilt of the bipole axis (α) and its latitude of emergence affect the variation and magnitude of the surface and open magnetic flux. The amount of open magnetic flux is estimated by constructing potential coronal fields. It is found that the open flux may evolve independently from the surface field for certain ranges of the tilt angle. For a given tilt angle, the lower the latitude of emergence, the higher the magnitude of the surface and open flux at the end of the simulation. In addition, three types of behavior are found for the open flux depending on the initial tilt angle of the bipole axis. When the tilt is such that $\alpha \geq 2°$ the open flux is independent of the surface flux and initially increases before decaying away. In contrast, for tilt angles in the range $-16° < \alpha < 2°$ the open flux follows the surface flux and continually decays. Finally, for $\alpha \leq -16°$ the open flux first decays and then increases in magnitude towards a second maximum before decaying away. This behavior of the open flux can be explained in terms of two competing effects produced by differential rotation. Firstly, differential rotation may increase or decrease the open flux by rotating the centers of each polarity of the bipole at different rates when the axis has tilt. Secondly, it decreases the open flux by increasing the length of the polarity inversion line where flux cancellation occurs. The results suggest that, in order to reproduce a realistic model of the Sun's open magnetic flux over a solar cycle, it is important to have accurate input data on the latitude of emergence of bipoles along with the variation of their tilt angles as the cycle progresses.

1. Introduction

In recent years there has been much interest in studying through observations and theory the origin and evolution of the Sun's open magnetic flux. The open magnetic flux which originates in coronal holes extends outwards from the Sun into interplanetary space and becomes the interplanetary magnetic field (IMF). This IMF surrounds the Earth and interacts with the Earth's magnetosphere in many subtle ways. Recent results reveal that cosmogenic isotopes show highly consistent variations with paleo-climatic indicators of the Earth's global average temperature during the Holocene (e.g., Bond *et al.*, 2001). Lockwood (2001) has estimated that 80% of the variation in the cosmic-ray fluxes that produce these isotopes

Solar Physics **207:** 291–308, 2002.

is explained by the variation in the open solar magnetic flux. Two explanations linking the open solar flux and climate have been proposed. Firstly, the cosmic-ray fluxes may influence climate directly, and Svensmark and Friis-Christensen (1997) and Svensmark (1998) have proposed such a mechanism via their effect on global cloud cover. An alternative explanation would be that a correlation between open solar flux and total solar irradiance, found during the last three solar cycles (Lockwood, 2002), has applied throughout the Holocene. To understand fully how the open magnetic flux and IMF may affect us we need to understand its origin and variation as the surface magnetic flux varies through its 11-year activity cycle (Priest, 1982).

Several papers in recent years have aimed to determine how the open flux varies over both the short term (few years) and also the long term (many cycles). Papers by Lockwood, Stamper, and Wild (1999) and Wang, Lean, and Sheeley (2000) show that the open magnetic flux (and therefore the IMF) varies throughout the solar cycle. They find that with each solar cycle the modulation of the open flux lags the total surface flux (and sunspot number) by 1–2 years (see also Wang and Sheeley, 2002). Lockwood, Stamper, and Wild (1999) also found that the maximum magnetic flux leaving the Sun varies strongly from one cycle to the next with an increase in average values of 2.3 since 1901. Wang, Lean, and Sheeley (2000) determine the open flux variation through the reconstruction of potential magnetic fields from observed synoptic magnetograms, while Lockwood, Stamper, and Wild (1999) use the indirect method of the aa index (Mayaud, 1971). To determine why the open flux lags behind the surface flux Wang, Sheeley, and Lean (2000) consider a magnetic flux transport model (Wang, Nash, and Sheeley, 1989; van Ballegooijen, Cartledge, and Priest, 1998) where the radial magnetic field at the solar surface is evolved under the effects of differential rotation, meridional flow and supergranular diffusion for a number of years. With this they show for one or two large bipoles how the open flux may evolve independently from the surface flux and give a suggestion about why the observed lag should occur. In contrast, Solanki, Schüssler, and Fligge (2000) took a much simpler approach and constructed a semi-empirical model which relates open flux emergence rate to the observed sunspot number and assume a linear loss rate of open flux with a best-fit loss time constant. With this approach a good representation of how the open flux has varied over the last 300 years could be found.

In this paper we continue the work of Wang, Sheeley, and Lean (2000) to consider in more detail how the open magnetic flux varies for a single bipole depending on its initial tilt angle and latitude of emergence. In the future the simulations shall be extended to include the interaction of multiple bipoles over many solar cycles. The paper is outlined as follows. In Section 2 the model is discussed along with its assumptions. In Section 3 the case of a single bipole is reconsidered where the evolution of the surface and open flux is determined as a function of the bipole tilt angle and latitude of emergence. Finally, in Section 4 the results and consequences for full solar cycle simulations are discussed.

2. The Model

To consider the evolution of the Sun's open magnetic flux we shall use a magnetic flux transport model (DeVore, Sheeley, and Boris, 1984; Sheeley, DeVore, and Boris, 1985: Sheeley, Nash, and Wang, 1987; Wang, Nash, and Sheeley, 1989; van Ballegooijen, Cartledge, and Priest, 1998). The model evolves the radial component of the Sun's magnetic field at the solar surface under the combined effects of flux emergence, differential rotation, meridional flow and supergranular diffusion. Let $B_r(R_\odot, \theta, \phi, t)$ be the radial magnetic field at ($r = R_\odot$) where r is the radial distance from the Sun's center, θ the polar angle, ϕ the azimuthal angle and t time. Here B_r represents the large-scale field of the Sun which is averaged over spatial scales larger than a supergranule (30 Mm). The evolution of the field at the solar surface $r = R_\odot$ is then described by the equation

$$\frac{\partial B_r}{\partial t} = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\left(-u(\theta)B_r + D\frac{\partial B_r}{\partial\theta}\right)\right] + \frac{D}{\sin^2\theta}\frac{\partial^2 B_r}{\partial\phi^2} - \Omega(\theta)\frac{\partial B_r}{\partial\phi}, \quad (1)$$

where $u(\theta)$ is the meridional flow, $\Omega(\theta)$ the differential rotation profile and $D = 600$ km s^{-1} is the photospheric diffusion constant (Leighton, 1964).

The meridional flow which is directed poleward is given as a function of latitude ($\lambda = \pi/2 - \theta$) as

$$u(\lambda) = \begin{cases} -u_0 \sin(\pi\lambda/\lambda_0) & |\lambda| < \lambda_0 \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

so above λ_0 the flow velocity vanishes. The particular values for these constants are $\lambda_0 = 75°$ and $u_0 = 11$ m s^{-1} (Hathaway, 1996; Snodgrass and Dailey, 1996). The differential rotation profile is given by (Snodgrass, 1983)

$$\Omega(\theta) = 13.38 - 2.30 \cos^2\theta - 1.62 \cos^4\theta - 13.20 \quad \text{deg day}^{-1}. \quad (3)$$

For simplicity and to provide information on the evolution of the dipole components of the field (Wang, Sheeley, and Lean, 2000), the radial magnetic field is expressed in terms of spherical harmonic functions as

$$B_r(r, \theta, \phi, t) = \sum_{l=1}^{N} b_l(r, \theta, \phi, t) = \sum_{l=1}^{N}\sum_{m=0}^{l} b_{lm}(r, \theta, \phi, t), \quad (4)$$

where $b_l(r, \theta, \phi, t)$ represents a multipole, l and $b_{lm}(r, \theta, \phi, t)$ represents each spherical-harmonic component at the radius r, where l is the harmonic degree and m is the azimuthal mode number. For the simulation $N = 63$, which is sufficient to resolve structures on the size of a supergranule (30 Mm).

As the surface field is evolved, a coronal magnetic field is extrapolated from it at regular intervals to determine the amount of open flux. The coronal field obtained from $B_r(R_\odot, \theta, \phi)$ is assumed to be potential ($\nabla \times \mathbf{B} = 0$) with a source surface

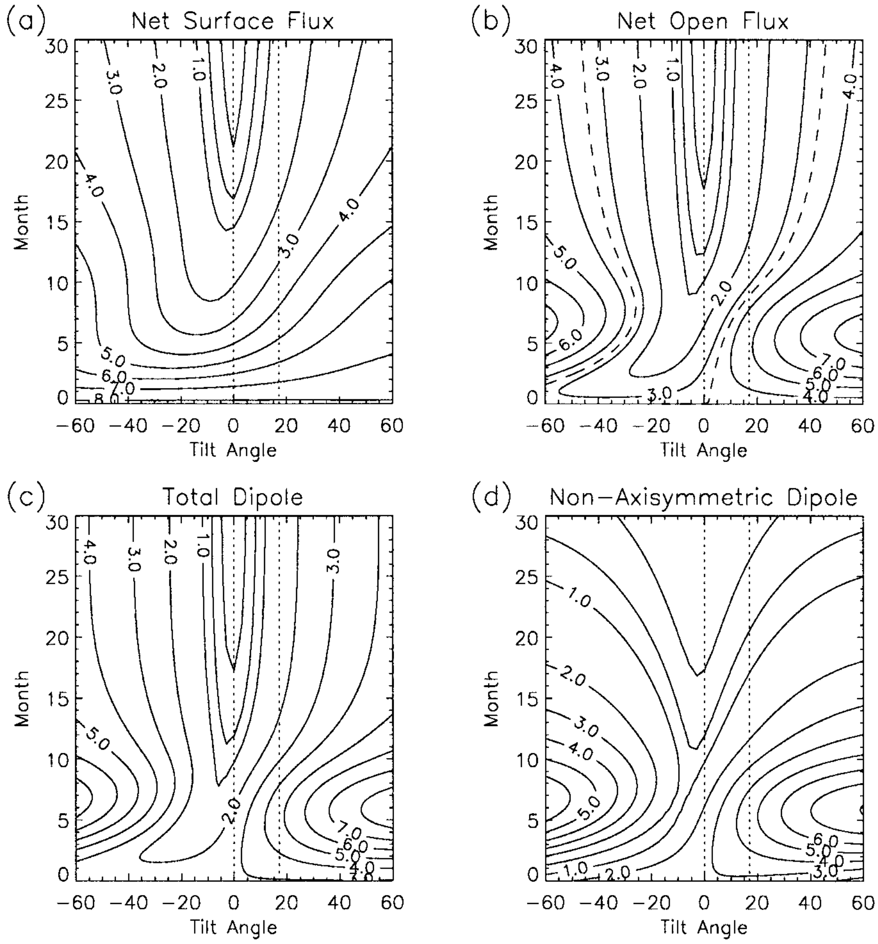

Figure 1. Contour plots of (a) total (net) surface flux ($\Phi_{tot}/4\pi R_{\odot}^2$), (b) total open flux ($\Phi_{\rm open}/4\pi R_e^2$), (c) the total dipole ($<b_1>(R_{ss}/R_e)^2$), and (d) the non-axisymmetric dipole ($<b_{11}>(R_{ss}/R_e)^2$) as a function of the tilt angle of the bipole axis for a single bipole. In each case a 30-rotation period is considered.

at $r = R_{ss} = 2.5\ R_{\odot}$, where $B_\theta = B_\phi = 0$. At the source surface (Schatten, Wilcox, and Ness, 1969) the field is assumed to become purely radial and field lines extending out to it are classified as open. The source surface crudely simulates the effect of the solar wind opening up magnetic field lines and producing coronal holes (Wang and Sheeley, 1990). More details on the construction of the potential field can be found in van Ballegooijen, Cartledge, and Priest (1998). So that the results of the present study can be compared with those of Wang, Sheeley, and Lean (2000) throughout the simulation, we shall determine the same variables, namely, the total (net) surface flux (Equation (5)), the total (net) open flux (Equation (6)), the non-axisymmetric dipole component (Equation (7)) at R_{ss}, and the total dipole component (Equation (8)) at R_{ss},

$$\Phi_{\text{tot}} = R_{\odot}^2 \int \mid B_r(R_{\odot}, \theta, \phi, t) \mid \, d\Omega, \tag{5}$$

$$\Phi_{\text{open}} = R_{ss}^2 \int \mid B_r(R_{ss}, \theta, \phi, t) \mid \, d\Omega, \tag{6}$$

$$\langle b_{11}(R_{ss}) \rangle = \frac{\int \mid b_{11}(R_{ss}, \theta, \phi, t) \mid \, d\Omega}{4\pi}, \tag{7}$$

$$\langle b_1(R_{ss}) \rangle = \frac{\int \mid b_1(R_{ss}, \theta, \phi, t) \mid \, d\Omega}{4\pi}. \tag{8}$$

The non-axisymmetric dipole represents the lowest order east-west dipole component at the source surface, while the total dipole is the sum of the lowest order north-south (b_{10}, axisymmetric dipole) and east-west (b_{11}, non-axisymmetric dipole) component at the source surface. In the next section a single bipole is considered in the northern hemisphere to determine how the open flux varies with the initial tilt angle of the bipole's axis and on the latitude of emergence.

3. Single Bipoles

On the Sun new magnetic flux emerges as bipolar magnetic regions or as magnetic activity complexes (Gaizauskas *et al.*, 1983). This magnetic flux emerges with a wide range of sizes and strengths (Wang and Sheeley, 1989; Harvey and Zwaan, 1993). It varies from large active regions (sunspots) and complexes (sunspot groups) which follow an eleven year cycle and emerge between two latitude belts in each hemisphere to small ephemeral regions (Harvey, 1984), which have a much weaker cycle variation and emerge with random orientations all across the Sun. To consider the evolution of the open flux we are only going to consider the large scale emergences of flux, which produce large unipolar areas as they are transported across the solar surface. The smaller flux elements with random orientations are likely to connect very low down in the solar atmosphere and are unlikely to contribute significantly to the open flux which reaches out to $R_{ss} = 2.5\ R_{\odot}$. The evolution of the surface and open flux of a single large bipolar magnetic region is now considered.

3.1. Variation with Tilt Angle

3.1.1. *Results of Simulation*

To begin with we shall consider how the open magnetic flux depends on the tilt angle of the bipole. A bipole is placed in the northern hemisphere such that the midpoint between its positive and negative polarities lies at longitude $\sim 180°$,

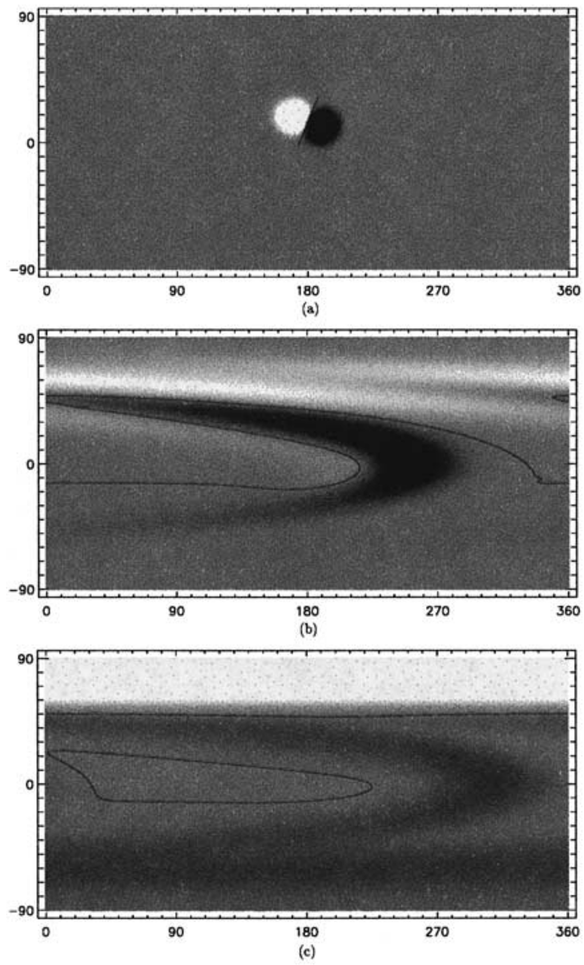

Figure 2. Evolution of the radial component of the magnetic field (B_r) at the solar surface for an initial tilt angle of $\alpha = 20°$ where (a) shows the initial distribution, (b) after 15 rotations and (c) after 30 rotations. *White* represents positive flux and *black* negative flux and the *thin solid line* is the polarity inversion line. The saturation levels for the field are set to (a) 100 G, (b) 10 G and (c) 5 G.

latitude $\sim 15°$. The total (net) flux of the bipole is 5×10^{23} Mx, while the separation in heliospheric degrees of the centres of its polarities is $\beta = 19.7°$. Each flux distribution is chosen with a Gaussian profile where the width of the profile is 0.4β (van Ballegooijen, Cartledge, and Priest, 1998). The tilt angle of the bipole is defined as the angle between the line joining the centres of the polarities and the east-west line. It ranges over $-90° \leq \alpha \leq +90°$, where positive tilt angles denote a leader flux lying equatorward of the follower flux, in agreement with Joy's law. Results by Wang and Sheeley (1989) for cycle 21 and by Tian *et al.* (1999) for cycle 22 show that roughly 70% of all bipoles on the Sun have positive tilt angles, and 80% of all bipoles have tilt angles in the range -10 to $+30°$.

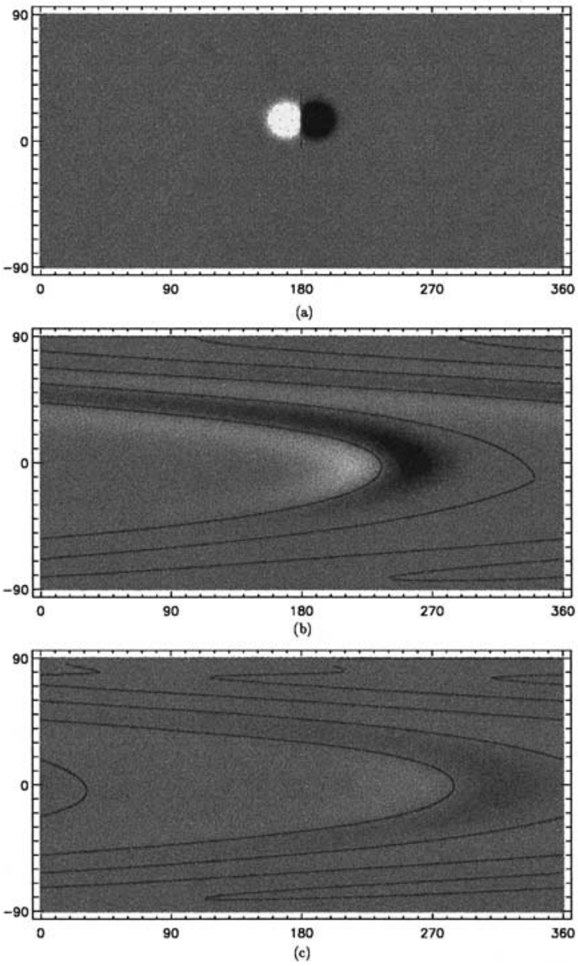

Figure 3. Evolution of the radial component of the magnetic field (B_r) at the solar surface for an initial tilt angle of $\alpha = 0°$ using the same notations as Figure 2.

In Figure 1 the four contour plots show the evolution as a function of tilt angle over a 30 month period of (a) total (net) surface flux ($\Phi_{\text{tot}}/4\pi R_{\odot}^2$), (b) total (net) open flux ($\Phi_{\text{open}}/4\pi R_e^2$), (c) the total dipole ($\langle b_1 \rangle (R_{ss}/R_e)^2$) and (d) the non-axisymmetric dipole ($\langle b_{11} \rangle (R_{ss}/R_e)^2$). The total surface flux is given as an average value over the solar surface (in gauss) while the other three have been scaled to their equivalent strengths (in nano-Tesla, nT) at the radius of the Earth ($R_e = 1$ AU). By expressing the final three values by their equivalent strengths at the radius of the Earth (Wang, Sheeley, and Lean, 2000) it is assumed that beyond the source surface the open flux is distributed uniformly in heliographic latitude and longitude. This assumption is supported by observations from *Ulysses* (Balogh *et al.*, 1995; Lockwood *et al.*, 1999; Smith *et al.*, 2001). Only tilt angles in the range $-60°$ to

$+60°$ are considered, since values outside this range are somewhat unrealistic. In each of the plots it can be seen that the behavior of the four quantities studied depends critically on the tilt of the bipole axis.

From Figure 1(a) it can be clearly seen that for any given tilt angle the net surface flux decreases from rotation to rotation as supergranular diffusion cancels flux at the polarity inversion line (PIL). However, the rate at which the surface flux decreases and the final amount depends on the tilt angle. The smaller the absolute value of the tilt angle, the more flux is canceled during the simulation. The two dotted lines denote the tilt angles considered by Wang, Sheeley, and Lean (2000). In Figure 1(b) the variation of the open magnetic flux with tilt angle is shown. The dashed line denotes the initial value of the open magnetic flux (3.4 nT), which is the same for all tilt angles. As found by Wang, Sheeley, and Lean (2000), it is possible to have the amount of open magnetic flux during the simulation greater than the initial value. Most of the cases where the open flux is greater than the initial value occur for positive tilt angles, but there is a significant range for negative tilt angles. For some tilt angles the open magnetic flux evolves independently of the total flux.

A variety of behavior of the open flux is possible depending on the initial tilt angle. For tilt angles in the range $-60° \leq \alpha \leq -16°$ the open magnetic flux initially decreases and then begins to increase. The increase at a later time may bring the open magnetic flux above its original value ($\alpha \leq -26°$). Eventually, a maximum is reached and the amount of open magnetic flux starts to decrease. The time of maximum in the open flux increases with the magnitude of the tilt angle and occurs anywhere from 6–7 rotations into the simulation. One surprising feature is that if the tilt angle is such that $\alpha \leq -46°$ the amount of open magnetic flux after 30 rotations is greater than the initial value. Even though an increase in open flux can be found for this range of tilt angles, the values lie outside the dominant values (-10 to $+30°$) observed by Wang and Sheeley (1989), so this type of behavior would be rare for newly emerging bipolar magnetic regions. For tilt angles in the range of $-16° \leq \alpha \leq 2°$ the open magnetic flux decreases throughout the simulation. This range does cover a significant fraction of the observed bipole tilt angles, so one would not expect any increase of the open magnetic flux past that of the initial value obtained at emergence. Finally, for the range ($\alpha > 2°$) the open magnetic flux immediately starts to increase at the start of the simulation, reaches a subsequent maximum and then starts to decay. As happens for the negative tilt angles, if the tilt angle increases, the maximum value of the resulting open flux increases. Also, as the tilt angle increases, the time taken to reach the maximum increases, saturating around 6 months for tilt angles greater than 35°. When the tilt angle is such that $\alpha \geq 46°$, the amount of open magnetic flux after 30 rotations is greater than the initial value.

The evolution of the total dipole component (Figure 1(c)) follows closely the evolution of the open flux for all of the tilt angles. In contrast, the non-axisymmetric dipole component only follows the evolution of the open flux at the early stages of evolution (maximum of ten rotations). Initially, the total dipole at the source surface

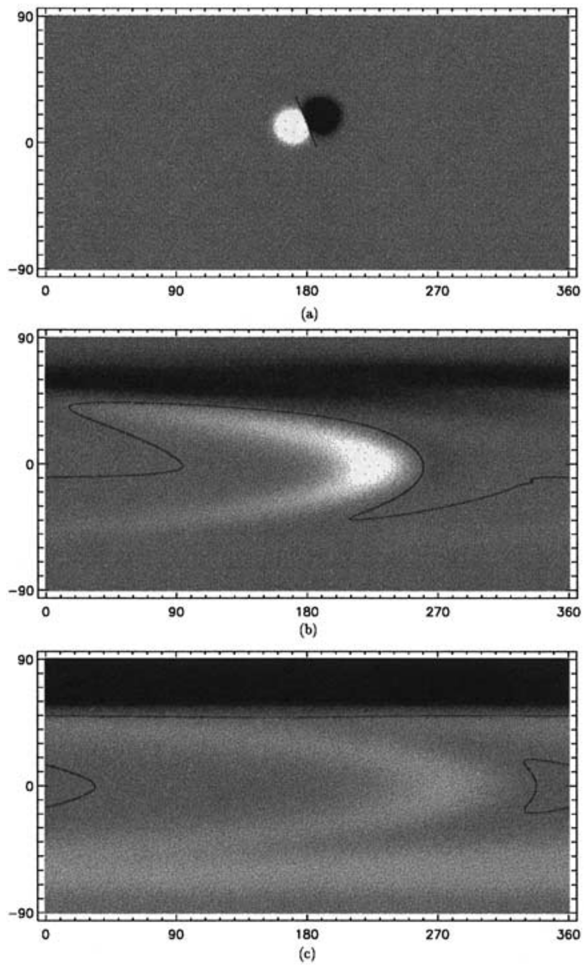

Figure 4. Evolution of the radial component of the magnetic field (B_r) at the solar surface for an initial tilt angle of $\alpha = -20°$ using the same notations as Figure 2.

is dominated by the non-axisymmetric dipole (b_{11}), since the bipole emerges at the equator and has east-west orientation. However, as the flux is pushed poleward by meridional flow and the polar fields are produced, the non-axisymmetric dipole component decays away. At the same time the north-south axisymmetric dipole component (b_{10}) is enhanced and dominates the total dipole, as can be seen in Figure 1.

3.1.2. *Role of Differential Rotation*

To explain why the open magnetic flux can behave independently of the surface flux, the effect of differential rotation on the bipole along with its tilt angle has to be considered. This can be seen through Figures 2–4, which show the evolution of the surface field for tilt angles of $\alpha = 20°$, $0°$, and $-20°$, respectively. In each case, the surface field is shown for (a) the initial configuration, (b) after 15 rotations and (c) after 30 rotations. In each case white represents positive flux, black negative flux and the thin black curve denotes the polarity inversion line (PIL). From the three figures the effect of the surface motions can be clearly seen. Differential rotation extends the bipoles out in an east-west sense while meridional flow pushes the flux poleward. Supergranular diffusion cancels magnetic flux at the PIL and also allows the concentrations of flux to diffuse. As flux is pushed poleward by meridional flow the polarity which is initially at the higher latitude (Figures 2 and 4) reaches the pole first and produces a concentrated polar cap. The other polarity has a preferential diffusion across the equator and produces the southern polar cap which is much more diffuse. If there is no tilt to the bipole (Figure 3) both polarities are transported to the pole at the same rate and a large amount of cancellation takes place.

For a positive tilt angle the lead polarity (black) of the bipole lies at a lower latitude than the following polarity (white, Figure 2(a)). Due to the difference in latitude the centres of the two polarities rotate at different rates, with the lower latitude one rotating faster. As the two poles rotate they move further apart and the coronal flux that connects between them has to rise higher in the corona to connect over. Due to this, the amount of open flux at $R_{ss} = 2.5\ R_{\odot}$ increases, as described in Wang, Sheeley, and Lean (2000). As well as increasing the open magnetic flux, the mechanism of differential rotation also decreases the open magnetic flux indirectly through the process of flux cancellation. The amount of flux cancellation at the polarity inversion line (PIL) depends on the gradient of the radial magnetic field (B_r) across the PIL and also on the length of the PIL. As the magnetic bipole evolves, differential rotation stretches out the PIL in an east-west sense (compare Figure 2(a) with 2(b) and 2(c)) and increases its length. As the length of the PIL increases, the amount of flux cancellation may also increase. This cancels the surface flux and reduces the amount of open flux. Meridional flow enhances this process by pushing the flux to higher latitudes were the differential rotation profile is steeper and the PIL can be stretched out more in an east-west sense. Therefore, differential rotation is the origin of two competing effects which can vary the amount of open magnetic flux.

For positive tilt angles where $\alpha > 2°$, in the first few months of evolution the separation of the centres of the poles (which increases open flux) has a dominant effect over flux cancellation (which decreases open flux). However, after roughly 6 months, differential rotation (which is enhanced as flux is pushed poleward by meridional flow) has stretched out the PIL sufficiently for flux cancellation to take over and the open flux starts to decreases. The larger the tilt angle, the bigger the difference in rotation rate of the poles and the faster the poles move apart. Since the poles separate more quickly, less flux cancels at the PIL after any given time. This produces a larger surface (Figure 1(a)) and more open flux (Figure 1(b)). In addition to the above effects, as the tilt angle increases, less opposite polarity flux lies at a common latitude (compare Figure 2(a) with Figure 3(a)). Therefore,

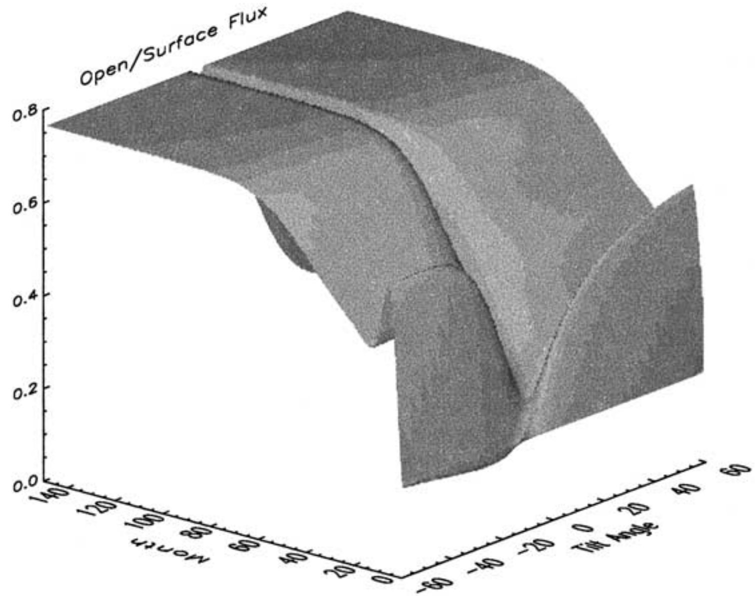

Figure 5. Ratio of the total open flux to the total surface flux ($\Phi_{\mathrm{open}}(t)/\Phi_{\mathrm{total}}(t)$) as a function of the tilt angle of the bipole during the simulation.

as differential rotation stretches out the flux in an east-west sense and meridional flow pushes it poleward, less opposite polarity flux encounters one another and less cancellation takes place. Therefore, when the tilt angle is larger, as the flux is pushed poleward by meridional flow, the surface and open flux may last for a longer time and attain higher values.

For the zero tilt angle case (Figure 3), both of the opposite polarity poles that make up the bipole lie at the same latitude. When differential rotation acts on them they rotate at the same rate, so the distance between their centres remains constant and the open flux cannot increase as the flux is pushed poleward. Since all of the flux lies at the same latitude, there is a large amount of flux cancellation, and the minimum value for both the surface and open fields is found at the end of the simulation. By comparing Figure 3 with Figures 2 and 4, it can be seen that the amount of surface flux is much less and the PIL (where flux cancellation is located) is much longer for the $0°$ tilt angle case. A similar situation occurs for very small tilt angles, where the rate of increase of the open flux due to the bipole polarities moving apart is less that the rate of decrease due to flux cancellation at the PIL, so no increase in open flux is observed.

In Figure 4 the surface configurations can be seen for a negative tilt angle of $-20°$. For negative tilt angles the follower polarity of the bipole lies at a lower latitude than the lead polarity. As differential rotation acts on the bipole, the follower polarity rotates faster than the leader and the two poles are squashed together. This leads to a much faster cancellation of the surface field (compared to positive tilt angles) as can be seen in Figure 1(a). The open flux always initially decreases due to the decreasing surface flux and also the centres of the poles move closer together. If, however, the magnitude of the tilt angle is large enough ($\alpha < -17°$), a significant fraction of the lower latitude pole may pass under the higher latitude one. Once this has occurred the separation of the poles increases along with the open flux. If the magnitude of the tilt angle is sufficiently large ($\alpha < -27°$) that the lower latitude follower bipole may pass under the high latitude one sufficiently fast, then the open flux at later times may be larger than the initial value. This, however, can only occur if the two poles pass one another without too much flux cancellation. Due to the initial convergence of the poles of the bipole and subsequent stronger cancellation of the surface flux, the magnitude of open flux for a given negative tilt angle is less than that for the corresponding positive tilt angle. If the tilt angle is such that $-17 < \alpha < 0$, then there is no increase in the open flux, since the small tilt angle cancels much of the flux before the low latitude pole can pass under the high latitude one.

3.2. Ratio of open to surface flux

In Figure 5 the surface plot shows of the ratio of the total open flux to the total surface flux ($\Phi_{\text{open}}(t)/\Phi_{\text{total}}(t)$) as a function of time and of the tilt angle of the bipoles axis. The same initial location and total flux of the bipole are chosen as in the previous case. It should be noted that this ratio is in fact independent of the initial total surface flux of the bipole. For each tilt angle the ratio is initially 0.193, so roughly $\frac{1}{5}$th of the surface flux is open when the bipole lies at low latitudes. This open flux would represent the coronal holes which are commonly seen at the edges of large concentrations of activity (see Wang and Sheeley, 1990; Wang, Sheeley, and Lean, 2000). As flux is transported across the solar surface the ratio varies strongly with time and tilt angle. After roughly 80 rotations, for the majority of the tilt angles, 75% of the remaining total surface flux is open. At this point the surface flux has been pushed poleward by meridional flow and the open flux is in the form of polar coronal holes. For very small tilt angles this value drops to 50%. A larger fraction of the surface flux is open for large tilt angles because the poles are much more unipolar than for small tilt angles. This can be seen by comparing Figures 2 and 4 with Figure 3. The less mixed the polarities are, the larger the distance between them and the higher the height that magnetic flux has to travel before it connects down. This makes it much easier for a larger amount of flux to reach the source surface and become open.

3.3. Latitude of emergence

Now let us consider the dependence of the total surface flux and open flux on the latitude of emergence of the bipole. A bipole of the same flux and width as before is placed at longitude $\sim 180°$. The evolution of the bipole is then followed from starting latitudes of $10°$, $18°$, $26°$, $34°$, and $42°$. This range covers the dominant

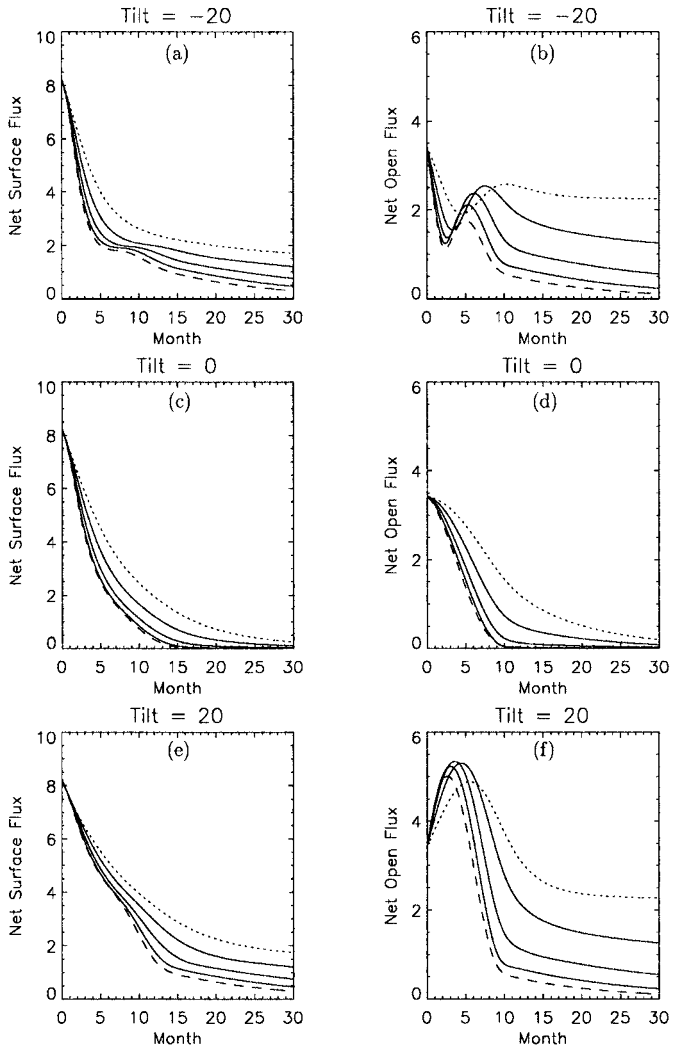

Figure 6. Graphs showing the variation of the total (net) open and total (net) surface flux depending on the latitude of emergence of the bipole. The *left-hand panels* show the surface flux while the *right-hand panels* show the open flux. Results are plotted for tilt angles of −20° (a, b), 0° (c, d) and 20° (e, f), and for latitudes of 10° (*dotted line*), 18°, 26°, 34°, and 42° (*dashed line*) are considered. There is a gradual progression from low to high latitudes.

latitudes at which strong concentrations of flux emerge throughout the 11-year sunspot cycle. At the start of each cycle these flux concentrations emerge at roughly 40° latitude and as the cycle progresses the value drops to roughly 10° as the flux emerges closer to the equator. In Figure 7 the plots show the results for tilt angles of −20° (Figure 7(a, b)), 0° (Figure 7(c, d)) and 20° (Figure 7(e, f)). In each plot the values have been scaled as in Figure 1 and the dotted line represents the lowest latitude of emergence (10°) while the dashed line gives the highest latitude of emergence (42°). The left panels show the total (net) surface flux while the right panels give the total (net) open flux. Between the lowest and highest latitudes there is a smooth transition of the curves.

For all tilt angles it is found that the lower the latitude of emergence, the longer time it takes for the surface flux to decay away. Also, the lower the latitude of emergence, the higher the value of the open flux at the end of the simulation and the longer it takes for the open magnetic flux to reach its maximum value. In general for negative tilt angles as the latitude of emergence decreases, the magnitude of the open flux increases, as shown in Figure 6(b). For positive tilt angles a more complicated behavior is found. Here the maximum in open flux occurs anywhere between emergence latitudes of 14°–24° with the latitude increasing as the tilt angle increases. The evolution of the open flux for positive tilt angles is in agreement with the evolution of the axisymmetric and non-axisymmetric dipole components considered at latitudes of 7, 15, 23° by Wang and Sheeley (1991).

To explain why the total surface and open flux behaves in this manner with respect to the latitude of emergence, the variation of differential rotation with latitude has to be considered. For the latitudes in which bipoles emerge, the higher the latitude of emergence, the larger $|d\Omega/d\lambda|$ becomes. Therefore, the higher the latitude of emergence, the faster the lower latitude pole of the bipole rotates with respect to the higher latitude one for any given tilt angle. This will in general lead to an increase in the amount of open flux. However, with the higher latitude of emergence the bipole is more stretched out by differential rotation. Since the bipole is more stretched out, the PIL becomes longer and after any given time this allows more flux cancellation to occur. This in turn will decrease the total amount of open flux. Therefore, again there are two competing effects and since the amount of open flux decreases with latitude of emergence, the second one can be seen to be the most dominant. As the latitude of emergence changes, meridional flow may also affect the final value of the surface and open flux. For both positive and negative tilt angles as the latitude of emergence increases less of the lower latitude pole of the bipole is able to diffuse across the equator (Wang and Sheeley, 1991) and both polarities are pushed poleward. With this more opposite polarity flux can be sheared by differential rotation and so more flux cancellation occurs. This also leads to less open and surface flux at the end of the simulation.

The variation of the length of the PIL (in solar radii) with respect to month of evolution can be seen in Figure 7 for emergence latitudes of 10° (solid line), 22° (dotted line), and 34° (dashed line). From the graph it can be seen that, at the initial stages where most cancellation occurs, the PIL is much longer the higher the latitude of emergence. It can also been seen that there are three phases to the evolution of the length of the PIL. The first phase is where the PIL increases in length under the effect of differential rotation, which stretches out the bipole in an east-west sense. The second phase in contrast is where the PIL starts to decrease in length. This occurs because, as the higher latitude flux is pushed poleward, a small island of it remains near the equator, as can be seen in Figures 2 and 4. This

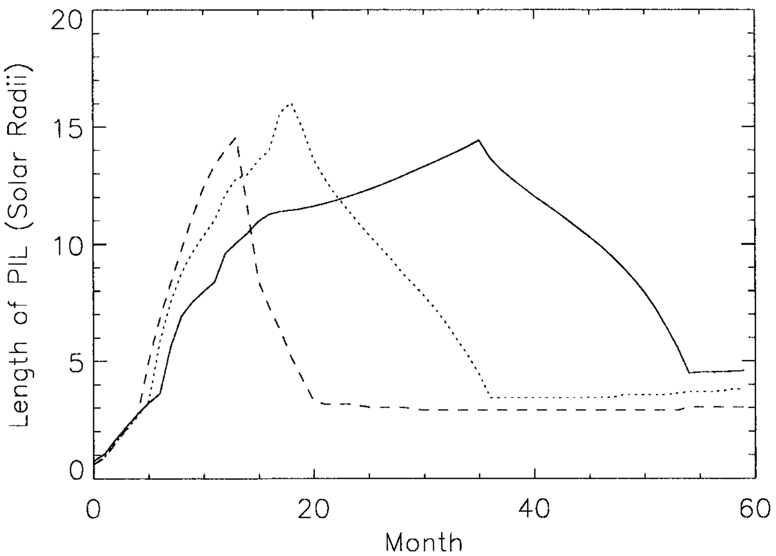

Figure 7. Graph showing how the length of the PIL in solar radii ($R_\odot$) varies with time for different latitudes of emergence. The *solid line* is for a latitude of 10°, the *dotted line* 22°, and the *dashed line* 34°.

flux decays due to cancellation and shrinks in size, reducing the size of the PIL. Finally, the last phase is where the length of the PIL remains roughly constant. This denotes the point where the radial component of the field at the surface forms an axisymmetric top-hat profile and there is a balance between meridional flow and equatorward supergranular diffusion. The top-hat profile can be seen at a latitude of 50° in Figures 2 and 4. These results show that the latitude of emergence can have a very significant effect on the evolution of the open flux through the solar cycle, since, as the cycle progresses, bipoles emerge at lower latitudes where more open flux per bipole flux can remain for a longer time.

4. Discussion

In this paper the origin and evolution of the Sun's open magnetic flux has been considered as single magnetic bipoles are transported across the solar surface. The results, which follow on from the work of Wang, Sheeley, and Lean (2000), consider in detail how both the tilt angle (α) of the bipole axis and its latitude of emergence can affect the amount and variation of the surface and open flux. To follow the evolution of the surface and open flux, a magnetic flux transport model is considered, where the radial magnetic field at the surface of the Sun is evolved under the effects of differential rotation, meridional flow and supergranular diffusion. From this the open magnetic flux is deduced at regular intervals from a reconstruction of potential coronal magnetic fields from the surface field distributions.

It is found that the smaller the absolute value of the tilt angle (α), the quicker the surface flux decays and the less open flux that is left at the end of the simulation. As the absolute value of α increases, the open flux can attain higher values and be maintained for longer times. As found by Wang, Sheeley, and Lean (2000), it is possible for the open magnetic flux during the simulation to be larger than the initial value. The variation of the open flux depends critically on the tilt angle in three different ways depending on the initial tilt angle. For $\alpha > 2$ the open flux evolves independently from the surface flux and initially increases before decaying away. In contrast for $-16^\circ < \alpha < 2^\circ$ the open flux follows closely the surface flux and continually decays. Finally, for $\alpha \leq -16^\circ$ the open flux first decays, then increases in magnitude towards a second maximum before decaying away. For all tilt angles the open magnetic flux follows the total dipole component ($\langle b_1 \rangle$) at the source surface and only follows the non-axisymmetric dipole component ($\langle b_{11} \rangle$) for the first few rotations. As the magnitude of the tilt angle increases, the time taken to reach the maximum also increases, which saturates around 6 months for large positive tilt angles and 7–8 months for negative tilt angles. Larger values of the open flux are possible for positive tilt angles compared to negative ones. For both positive and negative tilt angles, the lower the latitude of emergence, the longer it takes the surface flux to decay and the maximum in open flux to occur. In addition, the lower the latitude of emergence, the larger the open flux at the end of the simulation.

The behavior of the open flux can easily be explained in terms of two competing effects produced by differential rotation. Firstly, when the bipole axis is tilted, differential rotation rotates the centres of each polarity of the bipole at different rates. For positive tilt angles the lower latitude pole, which is the leader, rotates faster than the follower, which is at higher latitudes. With this the poles move further apart and when the potential magnetic field is constructed the field has to rise to higher heights to connect down. This has an effect of increasing the amount of open flux. If on the other hand the bipole has a negative tilt angle the opposite situation occurs, where the leader now lies at a higher latitude than the follower and differential rotation compresses them. This has the opposite effect of decreasing the amount of open flux. In addition, differential rotation also tends to decrease the open flux by increasing the length of the PIL. The longer the PIL, the more surface flux may be canceled and this decreases the open flux. Meridional flow enhances this effect by pushing the bipole to higher latitudes where the gradient in differential rotation is greater. Depending on the exact tilt angle of the bipole and time of the simulation, one of the above effects may be dominant and it is this that leads to a variety of behavior for the open flux.

In the paper by Wang, Sheeley, and Lean (2000) it was hypothesized that the observed lag between the surface and open flux was a result of the fact that at sunspot maximum the activity is distributed more or less uniformly over longitude and magnetic polarities are mixed. These closely mixed polarities limit the amount of open magnetic flux. However, as flux is canceled, activity becomes concentrated in one or two major complexes and longitudinal polarity separations increase. This increase in separation can then lead to an increase in open flux from the value at sunspot maximum. In the future it is planned to carry out full solar cycle simulations to determine whether this hypothesis is correct. Accurate input data on the latitude of emergence of the bipoles and also on the variation of the tilt angle as a function of latitude will be needed to produce realistic simulations.

Acknowledgements

The authors would like to thank Aad van Ballegooijen for helpful suggestions and the UK Particle Physics and Astronomy Research Council for financial support.

References

Balogh, A., Smith, E. J., Tsurutani, B. J., Southwood, D. J., Forsyth, R. J., and Horbury, T. S.: 1995, *Science* **268**, 1007.
Bond, G., Kromer, B., Beer, J. Muscheler, R., Evans, M. N., Showers, W., Hoffman, S., Lotti-Bond, R., Hajdas, I., and Bonani, G.: 2001, *Science* **294**, 2130.
DeVore, C. R., Sheeley, N. R., and Boris, J. P.: 1984, *Solar Phys.* **92**, 1.
Gaizauskas, V., Harvey, K. L., Harvey, J. W., and Zwaan, C.: 1983, *Astrophys. J.* **256**, 1056.
Hathaway, D. H.: 1996, *Astrophys. J.* **460**, 1027.
Harvey, K. L.: 1984, *The Hydromagnetics of the Sun*, Noordwijk, The Netherlands.
Harvey, K. L. and Zwaan, C.: 1993, *Solar Phys.* **148**, 85.
Lockwood, M.: 2001, *J. Geophys. Res.* **106**, 16021.
Lockwood, M.: 2002, *Astron. Astrophys.* **382**, 678.
Lockwood, M., Stamper, R., and Wild, M. N.: 1999, *Nature* **399**, 437.
Lockwood, M., Stamper, R., Wild, M. N., Balogh, A., and Jones, G.: 1999, *Astron. Geophys.* **40**, 4.10.
Leighton, R. B.: 1964, *Astrophys. J.* **140**, 1547.
Mayaud, P. N.: 1972, *J. Geophys. Res.* **72**, 6870.
Priest, E. R.: 1982, *Solar Magnetohydrodynamics*, Geophysical and Astrophysical Monographs, D. Reidel Publ. Co., Dordrecht, Holland.
Schatten, K. H., Wilcox, J. W., and Ness, N. F.: 1969, *Solar Phys.* **9**, 442.
Sheeley, N. R., DeVore, C. R., and Boris, J. P.: 1985, *Solar Phys.* **98**, 219.
Sheeley, N. R, Nash, A. G., and Wang, Y.-M.: 1987, *Astrophys. J.* **319**, 481.
Smith, E. J., Balogh, A., Forsyth, R. J., and McComas, D. J.: 2001, *Geophys. Res. Lett.* **28**, 4159.
Snodgrass, H. B.: 1983, *Astrophys. J.* **270**, 288.
Snodgrass, H. B. and Dailey, S.: 1996, *Solar Phys.* **163**, 21.
Solanki, S. K., Schüssler, M., and Fligge, M.: 2000, *Nature* **408**, 445.
Svensmark, H.: 1998, *Phys. Rev. Lett.* **81**, 5027.
Svensmark, H. and Friis-Christensen, E.: 1997, *J. Atmospheric Solar Terrest. Phys.* **59**, 1225.
Tian, L., Zhang, H., Tong, Y., and Jing, H.: 1999, *Solar Phys.* **305**, 189.
van Ballegooijen, A. A., Cartledge, N. P., and Priest, E. R.: 1998, *Astrophys. J.* **501**, 866.
Wang, Y.-M. and Sheeley, N. R.: 1989, *Solar Phys.* **81**, 124.

Wang, Y.-M. and Sheeley, N. R.: 1990, *Astrophys. J.* **365**, 372.
Wang, Y.-M. and Sheeley, N. R.: 1991, *Astrophys. J.* **375**, 761.
Wang, Y.-M. and Sheeley, N. R.: 2002, *J. Geophys. Res.*, in press.
Wang, Y.-M., Lean, J., and Sheeley, N. R.: 2000, *Geophys. Res. Lett.* **27**, 505.
Wang, Y.-M., Nash, A. G., and Sheeley, N. R.: 1989, *Science* **245**, 712.
Wang, Y.-M., Sheeley, N. R., and Lean, J.: 2000, *Geophys. Res. Lett.* **27**, 621.