Toeplitz and Hankel Operators on Weighted Fock Spaces

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Let \( \varphi \) be a subharmonic function satisfying certain conditions. The weighted Fock space \( \mathcal{F}_\varphi^p \) consists of entire functions \( f \) such that

\[
\|f\|_{p, \varphi}^p = \int_{\mathbb{C}} |f(z)|^p e^{-p\varphi(z)} \ dA(z) < \infty.
\]

The respective Lebesgue \( L^p_\varphi \) spaces and their norms are defined in an obvious way. When \( \varphi(z) = \frac{1}{2} |z|^2 \), we obtain the standard Bargmann-Fock space.

For a symbol \( f \) (satisfying suitable conditions), we define the Hankel operator \( H_f \) by

\[
H_f = (I - P) M_f,
\]

where \( P \) is the orthogonal projection from \( L^2_\varphi \) onto \( \mathcal{F}_\varphi^2 \), and \( M_f \) is the operator of multiplication by \( f \).

There are necessary and sufficient conditions describing when \( H_f \) is bounded or compact, when considered as an operator on the standard Bargmann-Fock space. We will describe these results, as well as consider the case of more general weights.