KINETIC THEORY OF THE ANOMALOUS TRANSPORT OF SUPERTHERMAL ELECTRONS IN TOROIDAL DEVICES

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ABSTRACT. An investigation has been made of the behaviour of fast electrons in toroidal discharges. The kinetic equation, describing the evolution of the fast particle distribution has been derived and analysed. A semi-analytical solution of the kinetic equation has been obtained in the superthermal energy region. The external electric field, turbulence, the non-uniformity of the mean magnetic field and collisions are shown to be important factors affecting the distribution function. The role of the ambipolar electric field has been established and identified as an essential factor in the process of fast electron diffusion. The strong influence of the density profile on the diffusion of fast particles is clearly demonstrated. A comparison has been made with experimental data obtained on the ZT-40M device. Agreement with the results of these experiments is observed.

1. INTRODUCTION

Plasma confinement in toroidal devices is an important problem of thermonuclear fusion research. It is well known that various instabilities leading to discharge turbulization are readily excited in the plasma. Experiments have shown that electron heat conductivity is several orders of magnitude higher than the limit predicted by neoclassical theory [1]. Such a strong energy transfer to a discharge wall is the main energy loss channel in toroidal systems. This is the reason why this problem is of great interest (see the review in Ref. [2]).

The theory suggests two possible mechanisms of anomalous transport. One of these is due to the turbulence generated by potential electric field oscillations caused by drift instabilities [3–6]. The other mechanism is due to the turbulence induced by magnetic fluctuations. When the threshold value is exceeded (determined by overlapping of mean resonance modes), magnetic field fluctuations lead to a stochastic wandering of the magnetic field lines about the discharge and, as a consequence, to a transport of particles moving along these field lines. The qualitative picture of this phenomenon was first presented in [7] as a stochastic description of the diffusion of magnetic field lines. The theory of this mechanism of anomalous transport was later comprehensively studied by many authors [8–12]. According to the theory, the effective diffusion coefficient in the former mechanism is inversely proportional to the velocity of particle motion

$$D_{\perp e} \sim \frac{\langle e^2 \rangle}{B_0^2} \frac{c^2 L_c}{v_e} \tag{1}$$

whereas in the latter case the particle diffusion coefficient increases with increasing particle velocity

$$D_{\perp b} \sim \frac{\langle b^2 \rangle}{B_0^2} L_c v_e \tag{2}$$

The role of each of these mechanisms in the formation of the energy flux has not yet been finally established. Measurements of the excitation level in tokamaks have shown that the first type of transport mechanism dominates in the vicinity of the discharge boundary [13–16], while inside a discharge (where the level of the magnetic fluctuations is difficult to measure) the transport may be due to both excitation mechanisms. In pinch type devices with a reversed field, where the magnetic fluctuation level is two orders of magnitude greater than in tokamaks [17], transport due to large scale magnetic fluctuations is dominant [18].

Along with a thermal particle flux, there also exists a superthermal fast particle flux. Moreover, if the particle lifetime in a discharge increases, the number of superthermal particles may increase appreciably. This effect is obviously of particular importance in devices in which the plasma is heated ohmically. One such device is a reversed field pinch (RFP) where, according to experimental data, a considerable part of the energy is transported by fast particles [19–21].

To estimate the energy confinement efficiency in RFP type devices, it is necessary to determine the distribution function of fast particles in the discharge.

Furthermore, fast particles carry information on the processes proceeding inside the discharge and so the investigation of their distribution may be used to diagnose the state of the plasma [19, 22].

In [22], the authors formulated a consistent kinetic theory of anomalous transport processes in a turbulized plasma and derived the kinetic equation describing the averaged particle distribution function under these conditions. The relaxed state of a turbulent plasma and the anomalous transport processes under RFP conditions were analysed in Refs [23, 24]. Here we are considering the superthermal electron distribution function in an RFP. As has been mentioned above, anomalous transport due to magnetic fluctuations is predominant in an RFP. Therefore, the influence of this particular transport mechanism on the distribution function of superthermal electrons will be our prime concern in what follows. The anomalous transport produces a strong effect on fast particles because the diffusion coefficient (2) grows linearly with increasing particle velocity. Another factor strongly affecting the electron distribution function is an electric field applied to the plasma.

It is known that even a weak electric field applied to a plasma induces the formation of a tail of runaway electrons in the energy range above the critical value, $\epsilon > \epsilon_c$, where

$$\epsilon_c = \frac{E_c}{E} T_e$$

and E_c is the critical field [25, 26]. In an RFP, the applied field is high, i.e. the ratio E/E_c is significantly larger than that in tokamaks, and therefore the distribution function distortion due to the action of the field E is much stronger. In addition to the vortex electric field applied to the plasma, a potential electric field, the so-called 'ambipolar' field, is generated in the discharge owing to the difference in the ion and electron diffusion coefficients. Producing an immediate effect upon fast electron diffusion, this field is responsible for the dependence between the ejection of fast electrons and the transport of the ion plasma component, which have typically been considered independently. On the other hand, as the number of fast particles increases, they themselves may start affecting the macroscopic state of the plasma, which is for example the case with convective transfer in a rippled field [27]. We shall not consider here the effects due to corrugation, but this role will be played by anomalous transport in a turbulent plasma. Thus, an examination of the electron distribution function is necessary for a correct self-consistent analysis of a macroscopic relaxed state.

Besides the factors listed above, the distribution function is noticeably affected by inhomogeneity of the mean magnetic field and temperature. For example, non-uniformity in the electron temperature results in thermal runaway and the appearance of a hot particle tail in the cold plasma region [28].

It should be noted that the influence of anomalous transport in an RFP on the distribution function of fast electrons was examined in Ref. [29], but the authors proceeded from the model kinetic equation which neglects a number of essential factors affecting the distribution function. In particular, the effect of Coulomb collisions, a potential ambipolar field and inhomogeneity of the mean magnetic field were neglected. It is therefore necessary to investigate the superthermal electron distribution function more thoroughly making allowance for the influence of all essential factors. This is the goal of the present paper. In Section 2 we derive the master kinetic equation with account taken of the influence of the applied electric field, an anomalous diffusion, a potential ambipolar field, collisions and the magnetic field inhomogeneity. The principal parameters determining the fast electron distribution function are discussed. Bearing in mind the complexity of the complete problem, at the beginning of Section 3 we analyse the distribution function of fast electrons in a homogeneous magnetic field. The strong influence of inhomogeneity in the electric field applied to plasma and the dependence of the electron diffusion rate on the thermal particle density profile are discussed. In Section 4 we investigate the influence of inhomogeneity in the mean magnetic field upon the electron distribution function and demonstrate the substantial deformation of this function. Finally, in Section 5 we estimate the influence of the indicated effects under specific RFP conditions and compare the theory developed with available experimental data. The results of experiments are seen to be in agreement with the theory.

2. KINETIC EQUATION FOR SUPERTHERMAL ELECTRONS

Let us consider a magnetized plasma with a magnetic field $\boldsymbol{B}(\boldsymbol{r})$ which has regular $\boldsymbol{B}_0(\boldsymbol{r})$ and fluctuational $\boldsymbol{b}$ components. The amplitude of fluctuations will be assumed small compared to the mean field $\boldsymbol{B}_0$, $|\boldsymbol{b}| \ll |\boldsymbol{B}_0|$. The basic quantities characterizing the fluctuations (the correlation length and the correlation time) will be thought of as large compared to the Larmor radius of the particles and their inverse gyrofrequencies as in [22]. Within such a statement of the problem and disregarding toroidality effects, the authors derived the general kinetic equation for the particle distribution function $f(r, u, \tilde{\mu})$ averaged over the ensemble of fluctuations. By virtue of the cylindrical symmetry about the angle θ and the direction z along the cylindrical axis, the function depends only on the radius r [23]:

$$\frac{\partial f}{\partial t} + \frac{e}{m_e} E_e \frac{\partial f}{\partial u} = St(f) + I(f) \tag{3}$$

where the collision integral of particles with fluctuations is given by

$$I(f) = \frac{u}{B(r)} \frac{1}{r} \frac{\partial}{\partial r}\{rK\} + \frac{\partial}{\partial u}\left\{\left(\frac{e}{m_e}\frac{E_a}{B} - \frac{\tilde{\mu}}{2}\frac{dB}{dr}\frac{1}{B}\right)K\right\}$$

$$K = \frac{|u|}{u}\frac{F}{B}\frac{\partial f}{\partial r} + \frac{F}{|u|}\frac{\partial f}{\partial u}\left\{\frac{e}{m_e}\frac{E_a}{B} - \frac{\tilde{\mu}}{2}\frac{dB}{dr}\frac{1}{B}\right\}$$

Here, the ambipolar electric field

$$eE_a = \left(\frac{dn}{dr}\frac{T_e}{n} + \frac{1}{2}\frac{dT_e}{dr}\right)\{1 - 2\delta_m\}$$

due to the difference between the diffusion rates of electrons and ions, the constant $\delta_m = \sqrt{m_e/m_i} \ll 1$ gives the correction related to the ion diffusion (assuming that the transport of ions is defined by magnetic fluctuations too), $n(r)$ is the particle number density,

$$F = \int_0^\infty dL \langle b_r b_r' \rangle$$

is the correlation function of the fluctuations b_r, the integration over L goes along the trajectory of particle motion, $b_r' = b_r(r'(L), \theta'(L), z'(L))$, u is the particle velocity along the magnetic field line, $\tilde{\mu} = u_\perp^2/B$ is the adiabatic invariant of particle motion, E_e is the external field, $E_e = \boldsymbol{E}_e \cdot \boldsymbol{h}$, $\boldsymbol{h} = \boldsymbol{B}_0/B_0$, $St(f)$ is the particle Coulomb collision integral.

In plasma heating devices, the number of fast particles N_f is always small compared to the concentration N of the main particles. That is why the kinetic equation (3) can be linearized in the small parameter $N_f/N \ll 1$. It is convenient to write Eq. (3) in a spherical co-ordinate system in velocity space ϵ, μ, where $\epsilon = u^2 + u_\perp^2$ is the total energy and $\mu = u/\sqrt{u^2 + u_\perp^2}$ is the cosine of the pitch angle (below for brevity we use simply the pitch angle without the cosine). Linearizing, we obtain

$$\frac{\partial f}{\partial \tau} + E_e(r)\delta_1\left\{2\mu\epsilon\frac{\partial f}{\partial \epsilon} + (1-\mu^2)\frac{\partial f}{\partial \mu}\right\} = 4T(r)\frac{\partial^2 f}{\partial \epsilon^2} + 2\frac{\partial f}{\partial \epsilon} + \frac{Z_{\text{eff}}}{\epsilon}\frac{\partial}{\partial \mu}\left\{(1-\mu^2)\frac{\partial f}{\partial \mu}\right\} + \epsilon\mu\delta_2\left\{\frac{1}{rB}\frac{\partial}{\partial r}\{rK\} + \frac{2E_a}{B}\left\{\frac{\partial K}{\partial \epsilon} + \frac{1-\mu^2}{2\epsilon\mu}\frac{\partial K}{\partial \mu}\right\} - \frac{dB}{dr}\frac{1}{B^2}\frac{1-\mu^2}{2\mu}\frac{\partial K}{\partial \mu}\right\} \tag{4}$$

$$K = \frac{|\mu|}{\mu}\frac{F}{B}\left\{\frac{\partial f}{\partial r} + 2E_a\left\{\frac{\partial f}{\partial \epsilon} + \frac{1-\mu^2}{2\epsilon\mu}\frac{\partial f}{\partial \mu}\right\} - \frac{dB}{dr}\frac{1}{B}\frac{1-\mu^2}{2\mu}\frac{\partial f}{\partial \mu}\right\}$$

Here, $E_e(r)$ is the profile of the external longitudinal electric field normalized to the critical electron runaway field E_c [25],

$$E_c = \frac{4\pi e^3 \Lambda n}{T_e}$$

Z_{eff} is the effective ion charge, Λ is the Coulomb logarithm, $\delta_1 = E_{e0}/E_c$ is the dimensionless parameter characterizing the magnitude of the longitudinal field E_{e0} relative to the critical field E_c, $\delta_2 = \nu_a/\nu_0$ is a dimensionless parameter characterizing the particle fluctuation collision frequency

$$\nu_a = \frac{F_{\max}}{a^2 B_0^2}\sqrt{\frac{T_e}{m_e}}$$

as compared with the Coulomb collision frequency of electrons

$$\nu_0 = \frac{4\pi e^4 n \Lambda}{m_e^{1/2} T_e^{3/2}}$$

a is the characteristic system dimension and $T(r)$ is the profile of the electron temperature normalized to the temperature at the centre T_e. Furthermore, we have introduced the dimensionless quantities $r = \tilde{r}/a$, $F = \tilde{F}/F_{\max}$, $B = \tilde{B}/B_0$, $E_a = \tilde{E}_a ea/T_e$, $\epsilon = \tilde{\epsilon}/(T_e/m_e)$, $\tau = \nu_0 t$; δ_1 and δ_2 are small parameters in the problem. In what follows we shall consider steady state solutions of Eq. (4) to determine the established distribution function of superthermal particles. We shall assume the distribution function of the main particles in the plasma to be stationary and in equilibrium. In such a statement, it will be a source of superthermal plasma particles. As boundary conditions for the distribution function f it is natural to require that f be regular as $r \to 0$ and that all the particles die on the boundary for $r = 1$, i.e.

$$\left.\frac{\partial f}{\partial r}\right|_{r=0} = 0, \quad f|_{r=1} = 0$$

The investigation of the complete problem is difficult because of the simultaneous action of such factors as the applied electric field E_e, the anomalous diffusion and the inhomogeneity of the mean magnetic field. We shall therefore begin by examining the joint effect of the external electric field E_e and the fluctuations, assuming the magnetic field gradient to be small

$$\frac{\mathrm{d}B}{\mathrm{d}r}\frac{a}{B} \ll 1 \tag{5}$$

and then proceed to the case of an inhomogeneous magnetic field $B(r)$.

3. DISTRIBUTION FUNCTION OF SUPERTHERMAL ELECTRONS IN A HOMOGENEOUS MAGNETIC FIELD

Provided that condition (5) is satisfied, Eq. (4) has the form

$$\begin{aligned} E_e(r)\delta_1 \left\{2\mu\epsilon\frac{\partial f}{\partial \epsilon} + (1-\mu^2)\frac{\partial f}{\partial \mu}\right\} \\ = 4T(r)\frac{\partial^2 f}{\partial \epsilon^2} + 2\frac{\partial f}{\partial \epsilon} + \frac{Z_{\mathrm{eff}}}{\epsilon}\frac{\partial}{\partial \mu}\left\{(1-\mu^2)\frac{\partial f}{\partial \mu}\right\} \\ + \epsilon\mu\delta_2\left\{\frac{1}{rB}\frac{\partial}{\partial r}\{rK\} + 2E_a\left\{\frac{\partial K}{\partial \epsilon} + \frac{1-\mu^2}{2\epsilon\mu}\frac{\partial K}{\partial \mu}\right\}\right\} \end{aligned} \tag{6}$$

$$K = \frac{|\mu|}{\mu}F\left\{\frac{\partial f}{\partial r} + 2E_a\left\{\frac{\partial f}{\partial \epsilon} + \frac{1-\mu^2}{2\epsilon\mu}\frac{\partial f}{\partial \mu}\right\}\right\}$$

$$E_a = \left(\frac{\mathrm{d}n}{\mathrm{d}r}\frac{T(r)}{n} + \frac{1}{2}\frac{\mathrm{d}T}{\mathrm{d}r}\right)\{1-\delta_m\}, \quad \delta_m \ll 1$$

Equation (6) describes the established distribution function of electrons in the presence of the field E_e and plasma turbulence. The method of solving Eq. (6) depends on the energy range within which we seek the solution. Therefore we shall first examine the energy range immediately adjoining the equilibrium region, which is henceforth referred to as a polynomial region of solution

$$1 \le \epsilon \le \delta_1^{-1/2}, \quad \delta_1 \ll 1 \tag{7}$$

We shall not consider here the thermal runaway of particles [28, 30]. According to the results of [30] one may estimate the critical value of energy y_k, $y = \epsilon\delta_1^{1/2}$ when the temperature profile will relax to the homogeneous one,

$$y_k = \delta_1^{1/2}\delta_2^{-1/3}\left(\frac{a}{L_{\|}c}\right)^{2/3}$$

where a is the scale of the system and $L_{\|}c$ is the correlation length of fluctuations along a magnetic field line. As usual,

$$\frac{a}{L_{\|}c} \ll 1$$

and we assume that

$$y_k \ll 1 \tag{8}$$

And therefore, in accordance with (8), in Eq. (6) we have put T_e = const and for the sake of simplicity T_i = const, because the ion temperature profile contributes only to a correction term in the ambipolar field. So, as can be seen, the inhomogeneity of the electron temperature contributes only to the ambipolar field.

3.1. Behaviour of the solution in the polynomial region

To begin with, we consider the case of a constant field $E_e(r)$ = const. We intend at first to clarify the effects of the ambipolar field, that

$$\frac{\mathrm{d}T}{\mathrm{d}r}\frac{a}{T} \ll \delta_m$$

and neglect the contribution from the temperature gradient in the ambipolar field. In Eq. (6) we transfer to a new variable $y = \epsilon\delta_1^{1/2}$ and seek the solution as a series of eigenfunctions of the Sturm–Liouville problem

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{rF\frac{\partial \chi_l}{\partial r}\right\} + \eta_l\chi_l = 0 \tag{9}$$

$$\left.\frac{\partial \chi_l}{\partial r}\right|_{r=0} = 0$$

$$\left.\frac{\partial \chi_l}{\partial r}\right|_{r=1} = 0$$

i.e.

$$f = C\,n(r)\exp(-\Psi_0/\delta_1^{1/2})\sum_l \chi_l(r)R_l(y,\mu) \tag{10}$$

As the boundary condition for $y \to 0$ we require continuity with the equilibrium distribution function

$$f_0 = C\,n(r)\exp(-y/\delta_1^{1/2})$$

$$n(r)|_{r=1} = 0$$

Substituting (10) into (6) and collecting terms with the same power $\delta_1^{1/2}$ in the zeroth approximation, we obtain $\Psi_0 = y/2$. In the next order of perturbation theory, neglecting corrections of the order of $\delta_m \ll 1$ in the expression for the ambipolar field (6), we obtain a system of equations for the functions $R_l(y, \mu)$

$$2\frac{\partial R_l}{\partial y} = \frac{Z_{\text{eff}}}{y}\frac{\partial}{\partial \mu}\left\{(1-\mu^2)\frac{\partial R_l}{\partial \mu}\right\} - y|\mu|\beta\eta_l R_l + 2\mu y R_l \tag{11}$$

with the boundary conditions as $y \to 0$

$$R_0(0, \mu) = 1, \quad R_l(0, \mu) = 0, \quad l \neq 0$$

where $\beta = \delta_2/\delta_1$. Since the zero eigenvalue of the problem (9) $\eta_0 = 0$, for the function $R_0(y, \mu)$ we obtain an equation containing no contribution from anomalous diffusion, and it will have a solution similar to the one obtained in Ref. [25], where the distribution function was distorted only by the electric field E_e and had a directed character for $\mu \simeq 1$ (Fig. 1, the dashed line). The solution of the system (11) for $l \neq 0$, which satisfies the boundary conditions (because for $l \neq 0$ all the eigenvalues $\eta_l > 0$ are nonnegative), will be $R_l(y, \mu) = 0$. So, up to terms of the order of $\delta_1^{1/2}$ (terms of the order of $\delta_1^{1/2}\delta_m \ll 1$ and $\delta_1 \ll 1$ are neglected) the initial equilibrium distribution function f_0 will not be distorted by anomalous transport. This means, as is readily seen, that the contribution from the anomalous diffusion is completely compensated by the ambipolar field E_a in exactly the same way as in the case of diffusion of thermal plasma particles. As is well known, electrons and ions diffuse together as a single whole with a doubled ion diffusion coefficient, which in our case is δ_m times smaller than the electron one (provided that the predominant ion diffusion mechanism is also an anomalous transport caused by magnetic fluctuations).

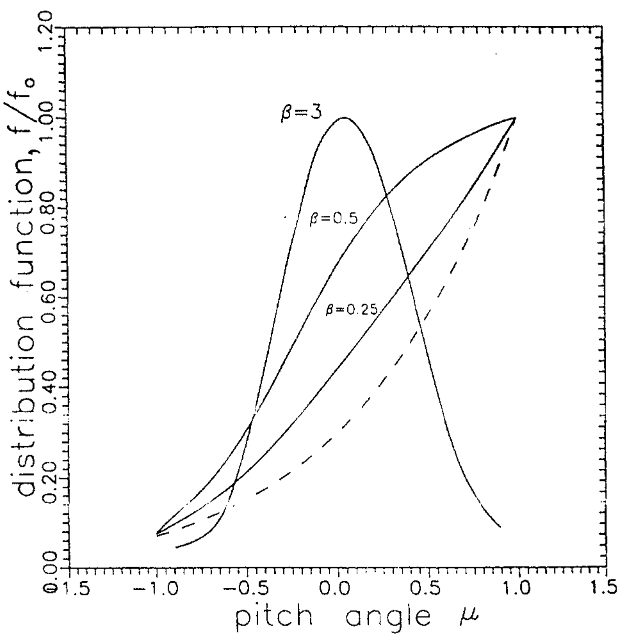

FIG. 1. The dependences of the fast particle distribution function on the pitch angle μ in the polynomial area of solution under various parameters β, y = 2, r = 0.5. The dashed line is the solution obtained in [25] with $\beta = 0$.

To determine the distortion of the distribution function by anomalous transport, we have to examine the solution of Eq. (6) with allowance for terms of the order of $\delta_1^{1/2}\delta_m$ and δ_1, which under certain conditions (to be discussed below) become appreciable. Now we are in a position to consider the case where the parameter δ_m satisfies the condition $\delta_m \gg \delta_1^{1/2}$. This means that we shall take into account the contribution from the anomalous diffusion, proportional to the anomalous ion diffusion coefficient.

With allowance for corrections of the order of δ_m, it is convenient to seek the solution of Eq. (6) as before in the form of an eigenfunction series of the Sturm–Liouville problem

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{rF\frac{\partial g_l}{\partial r}\right\} + \lambda_l g_l = 0 \tag{12}$$

$$\left.\frac{\partial g_l}{\partial r}\right|_{r=0} = 0$$

$$g_l|_{r=1} = 0$$

$$f = C\exp(-\Psi_0/\delta_1^{1/2})\sum_l g_l(r)\Theta_l(y, \mu) \tag{13}$$

$$n(r) = \sum_l A_l g_l$$

with the boundary condition as $y \to 0$

$$\Theta_l(0, \mu) = A_l \tag{14}$$

In order that we might transfer to a system of ordinary differential equations, it is also convenient to expand the functions $\Theta_l(y, \mu)$ in a power series of Legendre polynomials $P_m(\mu)$:

$$\Theta_l(y, \mu) = \sum_m F_l^m(y)P_m(\mu) \tag{15}$$

Substituting (13) into (6) with account taken of (15), we ultimately arrive at a system of linking equations for the functions $F_l^m(y)$, $\Psi_0 = y/2$:

$$2\frac{dF_l^m}{dy} = -\frac{Z_{\text{eff}}}{y}m(m+1)F_l^m + yF_k^n\alpha_{kl}\gamma_{mn}\beta + 2y\left\{\frac{F_l^{m-1}}{2m-1} + \frac{(m+1)F_l^{m+1}}{2m+3}\right\} \tag{16}$$

with boundary conditions

$$F_l^0(0) = A_l$$

$$F_l^m(0) = 0, \quad m \neq 0$$

where

$$\alpha_{kl} = \int_0^1 rg_l\left\{\frac{1}{r}\frac{\partial}{\partial r}\{r\Pi_k\} - E_a\Pi_k\right\}dr \Big/ \int_0^1 rg_l^2 dr$$

$$\Pi_k = F\left\{\frac{\partial g_k}{\partial r} - E_a g_k\right\}$$

$$\gamma_{mn} = (2m+1)\int_{-1}^1 P_m P_n |\mu| d\mu, \quad \beta = \delta_1/\delta_2$$

The system of equations (16) is solved numerically by cutting off the chain of equations on the term L in the expansion in g_l and on the term M in the expansion in P_m, so that a doubling of L and M changes the solution by less than 10%. Clearly, strong distribution function distortions by anomalous transport will take place only when the parameter $\Delta = 2\beta\lambda_m\delta_m$ is of the order of unity $\Delta \sim 1$, where λ_m is the maximum eigenvalue corresponding to the eigenfunctions $g_l(r)$ which contribute to the expansion (14). The smallest distortions may be expected in the case $n(r) = g_0(r)$.

We are examining the behaviour of the system (16) in the model case F = const, when the eigenfunctions of the problem (12) are the Bessel functions $g_l(r) = J_0(\xi_l r)$. We begin with examining the case $n(r) = g_0(r)$. The solution of the system (16) depending on the pitch angle μ and the energy y is presented in Figs 1 and 2. Figure 1 shows solutions obtained for various values of the parameter β. For small β, when $\Delta \ll 1$ and the effect of the electric field E_e dominates, the solution is close to that obtained in [25] (the dashed line in Fig. 1). As β increases, the influence of the anomalous transport (when $\Delta \geq 1$) becomes predominant. In this case the solution becomes almost symmetric in μ. As expected, it is concentrated in the vicinity of $\mu = 0$, where the diffusion coefficient (2) vanishes and falls symmetrically for $\mu \Rightarrow \pm 1$, where the diffusion coefficient is maximal. Depending on the energy (Fig. 2), the distribution function falls exponentially $\ln(f/f_0) \sim -\Delta_0 y^2$, where $\Delta_0 = 2\delta_m\beta\lambda_0$ (λ_0 is a non-zero eigenvalue of the problem (12)). The function f, which is shown in Fig. 2, was averaged over μ.

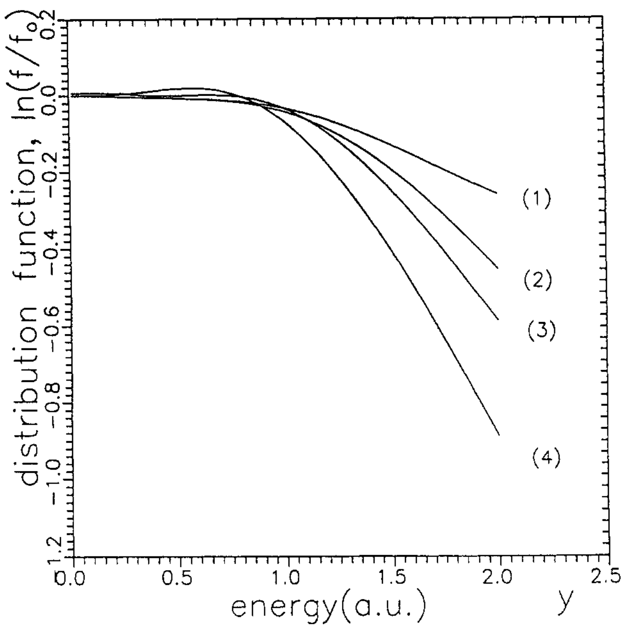

FIG. 2. The dependences of the fast particle distribution function $\ln(f/f_0)$ *averaged over* μ *on energy* y *in the polynomial solution area: (1)* $n(r) = g_0(r)$, $T(r) = const$, $E_e(r) = const$; *(2) in case of non-uniform electric field* $n(r) = g_0(r)$, $T(r) = const$; *(3)* $n(r) = \tilde{n}(r)$, *plot* $\tilde{n}(r)$ *is shown in Fig. 3,* $T(r) = const$, $E_e(r) = const$; *(4)* $E_e(r) = const$, $n(r) = g_0(r)$, $T(r) = 1 - r^2$. $\beta = 0.3$, $r = 0.5$.

It should be emphasized that the initial distribution over the discharge $n(r)$ will not be significantly deformed.

We shall now see what will happen if as the initial profile $n(r)$ we choose an arbitrary function $\tilde{n}(r)$ such that the expansion (14) will involve (and with a substantial contribution) harmonics with $l \neq 0$, as is shown in Fig. 3. The harmonics with eigenvalues $\lambda_l \gg \lambda_0$ should be expected to damp faster than the zero component does already for $y < 1$. As a result, the distribution over the discharge will rather rapidly relax to $g_0(r)$, which will immediately lead to violation of the balance between the ambipolar field and anomalous diffusion. In this case, an effective increase of the diffusion rate may be expected. Indeed, Fig. 3 shows that the distribution over discharge radius for $y = 1$, obtained in the solution of the system (16) with the profile $n(r) = \tilde{n}(r)$, is almost coincident with $g_0(r)$ (dashed line in the same figure). On the other hand, Fig. 2 presents the dependences obtained for one and the same value of β for $n(r) = g(r)$ and the profile $n(r) = \tilde{n}(r)$ depicted in Fig. 3. It is seen that in the latter case the decrement is considerably larger, which testifies to an effective increase of the diffusion rate.

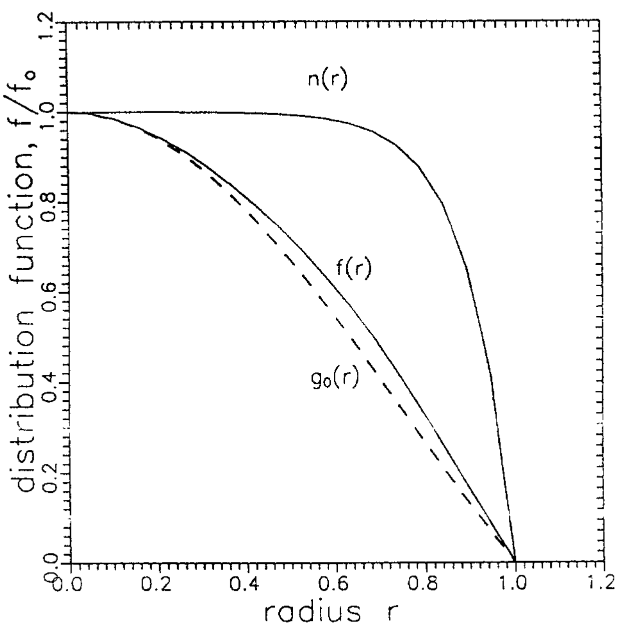

FIG. 3. The distribution over the discharge of the initial density profile ñ(r), the distribution function f(r) at y = 1, obtained in solving the system (16) and the eigenfunction of the problem (12) (dashed line).

The result that was obtained permits us to conclude that the rate and character of the diffusion (and as a consequence the loss of energy due to the diffusion of fast particles) essentially depend on the mean particle density profile. The minimum loss will be when $n(r) = g_0(r)$.

The density profile in a device $n(r)$ is defined by many factors: anomalous transport, neutral particle flow from the chamber wall and external sources, accelerated ion injection and convective transport processes. So by means of injection, for example, one may drive the profile $n(r)$ and thus the process of fast particle transport.

The result obtained suggests that the compensation of anomalous transport by an ambipolar field is a consequence of equilibrium in the leading term of the electron distribution function, that is $\Psi_0(y) = y/2$, and the absence of distortion of the initial electron distribution $n(r)$. As shown above, the strict balance is violated by the anomalous transport itself when $n(r) \neq g_0(r)$. In the presence of an electric field such a balance will also be violated for $y \gg 1$ since according to [25] the electric field E_e induces strong deviations of the electron distribution from equilibrium. We may point out another mechanism of balance violation already for $0 < y < 1$, namely, inhomogeneity of the applied field $E_e(r)$ which in this case plays the role of a fast particle source which is non-uniform in space (note that it is exactly the case realized in an RFP). We shall consider the solution of Eq. (6) in the polynomial region using the methods presented above. As before, we put F = const. Figure 2 presents the dependence of the distribution function $\ln(f/f_0)$ on the energy y for one and the same value of the parameter β and $n(r) = g_0(r)$ in two cases: (a) for E = const and (b) for $E_e(r) = J_0(\kappa_0 r)$, $\kappa_0 = 3.0$ (such a profile is close to the one observed in an RFP discharge). It can be readily seen that in case (b) the fall of the distribution function with energy is much higher than in case (a), which shows an acceleration of the diffusion process. It is noteworthy that the electron temperature inhomogeneity (which we do not consider here) will obviously play a role similar to that played by the inhomogeneity of the external field $E_e(r)$ and will also lead to a violation of strict balance (see Fig. 2).

We have assumed above that $\delta_m \gg \delta_1^{1/2}$, and the correction of the order of $\delta_m \delta_1^{1/2}$ has led to strong distortion of the distribution function when $\Delta \geq 1$. Clearly, in the converse case, that is when $\delta_m \ll \delta_1^{1/2}$, there will be an analogous process which is not distinct qualitatively from the one investigated above apart from the fact that in this case the role of the parameter Δ will be played by the parameter $\Delta_1 = 2\beta\lambda_m\delta_1^{1/2}$. As a result, as before for $\Delta_1 \sim 1$ the distribution function will relax rapidly to the profile $g_0(r)$.

Thus we have completely investigated the behaviour of the solution of Eq. (6) in the polynomial region. We have found that in the special case where the profile $n(r)$ is chosen in the form $n(r) = g_0(r)$ and the external field E_e is homogeneous, the ambipolar field E_a completely damps the anomalous transport of fast electrons with an accuracy of $\delta_m \ll 1$, so that the effective diffusion is determined by the parameter Δ or Δ_1, which is substantially smaller than $\beta\lambda_0$, $\Delta \ll \beta\lambda_0$. On the other hand, such a strict balance in the higher order terms of the expansion may be violated if $n(r) \neq g_0(r)$ or the external field E_e and temperature $T(r)$ are inhomogeneous. We note that the latter always takes place in an RFP. An essential result here is the fact that for $\Delta > 1$ (or $\Delta_1 > 1$) the distribution function of fast particles over the discharge relaxes with increasing energy to the universal profile $g_0(r)$ independently of the initial distribution. This fact will simplify appreciably our analysis in the remaining part of this section, where we consider the energy range $y \gg 1$.

3.2. Behaviour of the solution in the exponential region

We shall now consider the domain of the solution for high energy values $\epsilon \gg \delta^{-1/2}$, where $\delta = \delta_1, \delta_2$. The distortions of the distribution function in this domain are known to be of an exponential character [26]. Therefore, a polynomial expansion is not effective here. As we have seen above, in practice for $\epsilon \gg \delta^{-1/2}$, the distribution function of fast electrons over the discharge is coincident with the zero eigenfunction $g_0(r)$ of the problem (12). Therefore, in this energy range, in the expansion of the distribution function it is natural to make allowance only for terms containing $g_0(r)$:

$$f = C \sum_l g_l \exp(-\Psi_l) \tag{17}$$

Let us consider the case $E_e = 0$. We shall transfer to a new variable $z = \epsilon\delta_2$ and represent Ψ as

$$\Psi = \Psi_0/\delta_2 + \Psi_1/\delta_2^{1/2} + \Psi_2 + \ldots \tag{18}$$

Substituting (17) and (18) into (6) and collecting terms with the same powers of $\delta_2^{1/2}$, we obtain the system of equations

$$\frac{\partial \Psi_0}{\partial \mu} = 0 \tag{19}$$

$$\frac{\partial \Psi_0}{\partial \mu}\frac{\partial \Psi_1}{\partial \mu} = 0 \tag{20}$$

$$4\left(\frac{\partial \Psi_0}{\partial z}\right)^2 - 2\frac{\partial \Psi_0}{\partial z} + \frac{Z_{\text{eff}}}{z}(1-\mu^2)\left(\frac{\partial \Psi_1}{\partial \mu}\right)^2 - z|\mu|\left\{\lambda_0 + B\left(\frac{\partial \Psi_0}{\partial z}\right)^2\right\} = 0 \tag{21}$$

$$B = -4\int_0^1 r g_0^2(r) F(r) E_a^2(r)\mathrm{d}r \Big/ \int_0^1 r g_0^2(r)\mathrm{d}r$$

From (19) it follows that $\Psi_0 = \Psi_0(z)$, and (20) holds identically. From the condition of the absence of a jump in the derivative $\partial\Psi/\partial\mu$ for $\mu = 0$ there follows a natural condition (in view of the symmetry under a substitution of μ for $-\mu$ in (6) for $E_e = 0$)

$$\left.\frac{\partial \Psi_1}{\partial \mu}\right|_{\mu=0} = 0$$

Taking into consideration this condition, as well as the fact that $\Psi_0 = \Psi_0(z)$, from Eq. (21) for $\mu = 0$ we obtain

$$\Psi_0(z) = z/2$$

Substituting the expression found for $\Psi_0(z)$ back into (21), we obtain the equation for determining $\Psi_1(z, \mu)$

$$\frac{\partial \Psi_1}{\partial \mu} = z\sqrt{\frac{|\mu|\lambda_0^*}{z_{\text{eff}}(1-\mu^2)}} \tag{22}$$

where $\lambda_0^* = \lambda_0 + B/4$ is an effective eigenvalue with account taken of the influence of the ambipolar field E_a. From (22) we see, however, that the asymptotical expansion obtained is violated in the vicinity of $\mu = 0$ because the second derivative $\partial^2\Psi/\partial\mu^2$ contains a singularity. Indeed,

$$\frac{\partial^2 \Psi}{\partial \mu^2} \to \infty$$

as $\mu \to 0$, which indicates the presence of a boundary layer near $\mu = 0$. To obtain a correct expansion, it is necessary to investigate the behaviour of the solution in the vicinity of $\mu = 0$. To this end, the small term with a second derivative $(\partial^2\Psi/\partial\mu^2)\delta_2^{1/2}$ in Eq. (21) should be retained. Omitting terms of the order of μ^2 as $\mu \to 0$ in (21) and making the substitution $\Psi_1 = -\ln(\Theta)\delta_2^{1/2}$, we arrive at the Airy equation

$$\frac{\partial^2 \Theta}{\partial \xi^2} = \xi\Theta \tag{23}$$

$$\xi = \frac{\lambda^* z^2 \mu - \varphi(z)}{z_{\text{eff}}^{1/3} z^{4/3} \delta_2^{1/3} \lambda^{*2/3}}$$

$$\varphi(z) = \left\{4\left(\frac{\partial \Psi_0}{\partial z}\right)^2 - 2\frac{\partial \Psi_0}{\partial z}\right\}$$

$$\lambda^* = \lambda_0 + B\left(\frac{\partial \Psi_0}{\partial z}\right)^2$$

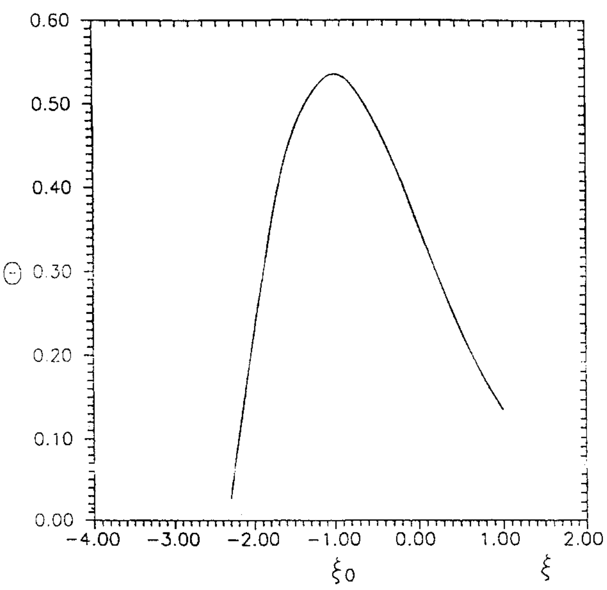

FIG. 4. The solution of Eq. (23) as a function of ξ.

Equation (23) has two linearly independent fundamental solutions, one of which grows exponentially as $\xi \to \infty$ and may be discarded for being limited. The other solution $\Theta = C_0 Ai(\xi)$ is shown in Fig. 4, where ξ_0 is the point where the function Θ and Ψ_1 has an extremum,

$$\frac{\partial \Psi_1}{\partial \mu} = 0$$

Then from the condition

$$\left.\frac{\partial \Psi_1}{\partial \mu}\right|_{\mu=0} = 0$$

we obtain an additional relation that allows us to determine the function $\Psi_0(z)$

$$4\left(\frac{\partial \Psi_0}{\partial z}\right)^2 - 2\frac{\partial \Psi_0}{\partial z} = |\xi_0| z_{\text{eff}}^{1/3} \lambda^{*2/3}(z) \delta_2^{1/3} z^{1/3} \qquad (24)$$

In the general case, to find $\Psi_0(z)$ it is necessary to solve the non-linear equation (24), but for $z \ll \delta_2^{-1}$ expanding Eq. (24) in the small parameter δ_2 we obtain

$$\Psi_0(z) = \frac{z}{2} + \frac{3}{4}|\xi_0| z_{\text{eff}}^{1/3} \lambda_0^{*2/3} \delta_2^{1/3} z^{4/3} \qquad (25)$$

$$\lambda_0^* = \lambda_0 + B/4$$

Taking into account (25), we may determine the angular dependence of the distribution function

$$\Psi_1(\mu, z) = \int_{\mu_0}^{\mu} \sqrt{\frac{z\{\lambda^*(z) z|\mu| - |\xi_0| z_{\text{eff}}^{1/3} \lambda^*(z)^{2/3} z^{1/3} \delta_2^{1/3}\}}{z_{\text{eff}}(1-\mu^2)}}\, d\mu + \tilde{\Psi}_1(z) \qquad (26)$$

where $\tilde{\Psi}_1(z)$ is an unknown function. As $\mu \to 0$, it is necessary to match (26) with the solution of Eq. (23) to find the constant C_0. It is readily seen that as $\delta_2 \to 0$, (25) transforms into $\Psi_0(z) = z/2$ and (26) into (22). As $z \to 0$, the solution found here must pass over to the solution obtained in the polynomial region. Figure 5 shows the dependences of the distribution function on the pitch angle in the polynomial region for $y > 1$ and the solution obtained in the exponential region when $z \to 0$. Good agreement between the two solutions can be seen. The solution obtained by us shows that the leading term of the asymptotic expansion for $\delta_2 \to 0$ is close to equilibrium, and deviations from it occur only for $z \sim \delta_2^{-1}$. This result is in close agreement with the results of the direct numerical simulations [31] carried out for tokamak conditions in the absence of an applied electric field E_e and the experiments [32, 33]. It should be noted that the role of an ambipolar field in this range of energy values comes down to renormalization of the eigenvalue λ_0 which determines the anomalous transport, namely the effective eigenvalue with allowance for the influence of the ambipolar field E_a will be $\lambda_0^* = \lambda_0 + B/4$, $\lambda_0^* < \lambda_0$ since $B < 0$. That is, as expected, the ambipolar field E_a damps the anomalous transport of fast electrons. It is clear that this effect is important when the effective temperature of the fast electrons is of the order of the equilibrium temperature at the centre of the discharge.

The field E_a also affects the behaviour of the distribution function in the far region of energy values as $z \to \infty$. So, when $B \neq 0$ one can readily obtain an asymptotical expression for $\Psi_0(z)$ as $z \to \infty$:

$$\Psi_0(z) \simeq z\sqrt{\lambda_0/B}$$

At the same time when $B = 0$ and $z \to \infty$, from (24) we have

$$\Psi_0(z) \simeq \frac{3}{14} z^{7/6} \lambda_0^{1/3} z_{\text{eff}}^{1/6} \delta_2^{1/6}$$

i.e. in the absence of the ambipolar field E_a the distribution function falls more strongly with energy.

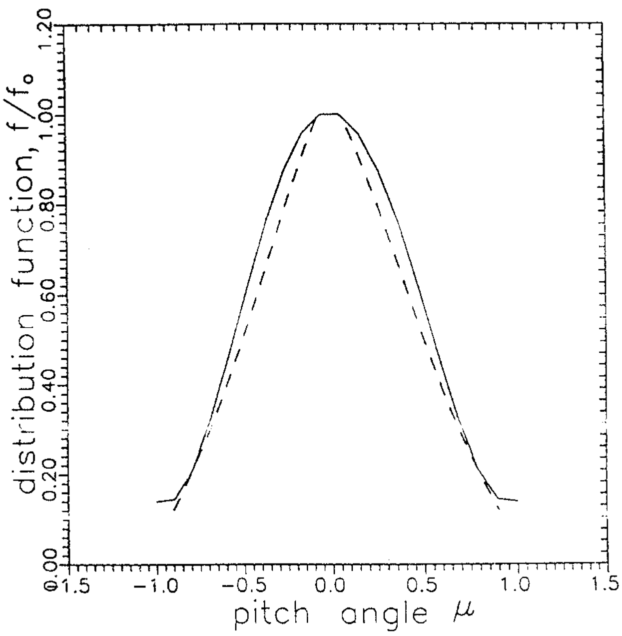

FIG. 5. The fast electron distribution function f/f_0 as a function of μ in the absence of an applied electric field E_e. The solid line is the polynomial solution at $y > 1$ and the dashed line is the exponential solution at $z = y\delta_2^{1/2} \ll 1$.

We shall now consider the case with a non-zero external field E_e. As in the previous case in the expansion (17), we shall take into account only the contribution from the terms $g_0(r)$. In Eq. (6) we transfer to a new variable $z = \epsilon\delta_1$ and represent the index of the exponential in (17) in the form

$$\Psi = \Psi_0/\delta_1 + \Psi_1/\delta_1^{1/2} + \Psi_2 + \dots \tag{27}$$

Substituting (27) into (6) and collecting terms with the same powers $\delta_1^{1/2}$, we obtain a chain of connected equations for the functions Ψ_0, Ψ_1, Ψ_2, ...:

$$\frac{\partial\Psi_0}{\partial\mu} = 0 \tag{28}$$

$$\frac{\partial\Psi_0}{\partial\mu}\frac{\partial\Psi_1}{\partial\mu} = 0 \tag{29}$$

$$4\left(\frac{\partial\Psi_0}{\partial z}\right)^2 - 2\frac{\partial\Psi_0}{\partial z} + \frac{Z_{\text{eff}}}{z}(1-\mu^2)\left(\frac{\partial\Psi_1}{\partial\mu}\right)^2 - z|\mu|\beta\lambda^*(z) + 2\mu zE\frac{\partial\Psi_0}{\partial z} = 0 \tag{30}$$

$$8\frac{\partial\Psi_0}{\partial z}\frac{\partial\Psi_1}{\partial z} - 2\frac{\partial\Psi_1}{\partial z} + \frac{Z_{\text{eff}}}{z}\left\{2(1-\mu^2)\frac{\partial\Psi_1}{\partial\mu}\frac{\partial\Psi_2}{\partial\mu} - \frac{\partial^2\Psi_1}{\partial\mu^2} + 2\mu\frac{\partial\Psi_1}{\partial\mu}\right\} - 2z|\mu|\beta B\frac{\partial\Psi_0}{\partial z}\frac{\partial\Psi_1}{\partial z} + 2\mu zE\frac{\partial\Psi_1}{\partial z} + E(1-\mu^2)\frac{\partial\Psi_1}{\partial\mu} = 0 \tag{31}$$

$$B = -4\int_0^1 rg_0^2(r)F(r)E_a^2(r)\mathrm{d}r \Big/ \int_0^1 rg_0^2(r)\mathrm{d}r$$

$$E = \int_0^1 rg_0^2(r)E_e(r)\mathrm{d}r \Big/ \int_0^1 rg_0^2(r)\mathrm{d}r$$

$$\beta = \delta_2/\delta_1; \quad \lambda^*(z) = \lambda_0 + B\left(\frac{\partial\Psi_0}{\partial z}\right)^2$$

From the first equation (28) it follows that $\Psi_0 = \Psi_0(z)$. Given this, Eq. (29) holds automatically. Then, in the absence of a singularity for $\mu = 1$, from (30) we obtain the equation with the help of which we may determine Ψ_0:

$$4\left(\frac{\partial\Psi_0}{\partial z}\right)^2 - 2\frac{\partial\Psi_0}{\partial z} - z\beta\lambda^*(z) + 2zE\frac{\partial\Psi_0}{\partial z} = 0 \tag{32}$$

This implies

$$\frac{\partial\Psi_0}{\partial z} = \frac{(1-zE) + \sqrt{(1-zE)^2 + z\beta\lambda_0(4-\beta zB)}}{4 - z\beta B} \tag{33}$$

Integrating (33), we arrive at

$$\Psi_0(z) = \frac{Ez}{B\beta} - \frac{\sqrt{1+a_1z+a_2z^2}}{B\beta} - \frac{(8a_2 + a_1B\beta)\ln(a_1 + 2a_2z + 2\sqrt{a_2}\sqrt{1+a_1z+a_2z^2})}{2\sqrt{a_2}B^2\beta^2} + \frac{(4E - B\beta)\ln(S_1)}{B^2\beta^2} \tag{34}$$

$$S_1 = -4a_1B^3\beta^3 - 2B^4\beta^4 - 8a_2B^3\beta^3z - a_1B^4\beta^4z - 2B^3\beta^3(4E - B\beta)\sqrt{1+a_1z+a_2z^2}$$

$$a_1 = 4\beta\lambda_0 - 2E; \quad a_2 = E^2 - \beta^2\lambda_0 B$$

Taking into consideration (33), we obtain from (30)

$$\Psi_1(\mu, z) = -z\int_1^\mu \sqrt{\frac{\beta\left[\lambda_0 + B\left(\frac{\partial\Psi_0}{\partial z}\right)^2\right](|\mu|-1) - 2\frac{\partial\Psi_0}{\partial z}E(\mu-1)}{z_{\text{eff}}(1-\mu^2)}}\mathrm{d}\mu + \tilde{\Psi}_1(z) \tag{35}$$

It should be noted that (35) holds for

$$\beta\{\lambda_0 + B/4\} < E \tag{36}$$

In this case, the solution will not have a singularity for $\mu = 1$. After substitution of (35) into (31), provided that there is no singularity for $\mu = 1$, the unknown function $\tilde{\Psi}_1(z)$ may be determined:

$$\tilde{\Psi}_1(z) = \sqrt{Z_{\text{eff}}}(-1/\sqrt{2} + 1) \times \int_0^z \frac{\sqrt{2\frac{\partial\Psi_0}{\partial z}E - \beta\left[\lambda_0 + B\left(\frac{\partial\Psi_0}{\partial z}\right)^2\right]}}{4\frac{\partial\Psi_0}{\partial z} + z\beta B\frac{\partial\Psi_0}{\partial z} + Ez - 1}\mathrm{d}z \tag{37}$$

The expressions (34), (35) and (37) determine in the exponential approximation the dependence of the distribution function in the exponential region on the energy and the pitch angle μ. As $z \to 0$, the solution obtained must coincide with the solution in the polynomial region for $y > 1$. Figure 6 shows the two solutions; they are seen to be almost coincident.

To establish the asymptotic behaviour of the distribution function as $z \to \infty$, we represent the index of the exponential in (17) in the form

$$\Psi = \Psi_0 z + \Psi_1 + \dots$$

$$\Psi_0 = \varphi_0/\delta_1 + \varphi_1/\delta_1^{1/2} + \varphi_2 + \dots \tag{38}$$

$$z = \epsilon\delta_1$$

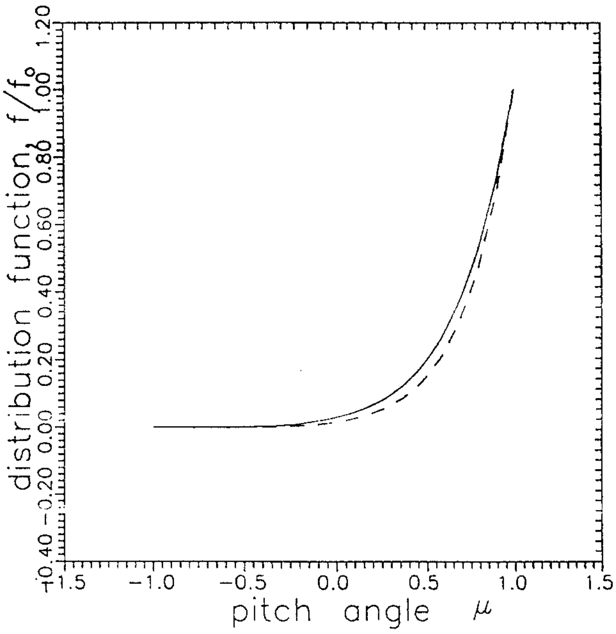

FIG. 6. The fast electron distribution function f/f_0 as a function of μ with an applied electric field E_e present. The solid line is the polynomial solution at $y > 1$ and the dashed line is the exponential solution at $z = y\delta_2^{1/2} \ll 1$.

Substituting (38) into (6) with allowance for (17), keeping terms of the order of z and collecting terms with the same powers $\delta_1^{1/2}$, we obtain the system of equations for φ_0, φ_1, ...:

$$\frac{\partial\varphi_0}{\partial\mu} = 0 \tag{39}$$

$$\frac{\partial\varphi_0}{\partial\mu}\frac{\partial\varphi_1}{\partial\mu} = 0 \tag{40}$$

$$\left(\frac{\partial\varphi_1}{\partial\mu}\right)^2 Z_{\text{eff}}(1-\mu^2) - |\mu|\beta\{\lambda_0 + B\varphi_0^2\} + 2\mu\varphi_0 E = 0 \tag{41}$$

$$2Z_{\text{eff}}(1-\mu^2)\frac{\partial\varphi_1}{\partial\mu}\frac{\partial\varphi_2}{\partial\mu} - |\mu|\beta B\varphi_0\varphi_1 + 2\mu E\varphi_1 + (1-\mu^2)\frac{\partial\varphi_1}{\partial\mu}E = 0 \tag{42}$$

where $\beta = \delta_2/\delta_1$, and B and E are similar to (30), whence $\varphi_0 = \text{const}$. Then in the absence of singularity for $\mu = 1$ we have from (41):

$$\varphi_0 = \frac{\sqrt{E^2 - \beta^2\lambda_0 B} - E}{-\beta B} \tag{43}$$

Knowing φ_0 from (41), we obtain

$$\frac{\partial\varphi_1}{\partial\mu} = 0, \quad \mu > 0$$

$$\frac{\partial\varphi_1}{\partial\mu} = -\sqrt{\frac{\mu(4E/\beta B)(\sqrt{E^2 - \beta^2\lambda_0 B} - E)}{Z_{\text{eff}}(1-\mu^2)}}$$
$$\mu < 0 \tag{44}$$

$$\varphi_1 = \text{const}, \quad \mu > 0$$

$$\varphi_1 = -\int_1^\mu \sqrt{\frac{\mu(4E/\beta B)(\sqrt{E^2 - \beta^2\lambda_0 B} - E)}{Z_{\text{eff}}(1-\mu^2)}}\,\mathrm{d}\mu + \tilde{\varphi}_1$$
$$\mu < 0 \tag{45}$$

where $\tilde{\varphi}_1$ is an unknown constant. Then in the absence of a singularity we find from (42) for $\mu = 1$ that $\tilde{\varphi}_1 = 0$.

The expressions (43) and (45) determine the asymptotic behaviour of the distribution function for $E_e(r) \neq 0$ as $z \to \infty$. It can be readily seen that in the limit $z \to \infty$, (34) goes over to (43) and (35) to (45). So, the solution (34), (35) in the exponential approximation describes the particle distribution in the entire range of energy values for $z > \delta_1^{1/2}$. It should be noted that the asymptotic expression for the distribution function for $z \to \infty$ (43), (45) holds also under the condition inverse to (36), but this asymptote is apparently not described by the simple expansion (17). The solution obtained in this section describes completely the distribution function of fast particles in the presence of an external field E_e and anomalous diffusion in a homogeneous magnetic field. We have investigated here a rather general case in the model assumption $F = \text{const}$. In Section 5 we shall analyse the distributions of fast electrons under specific experimental conditions, and we now proceed to the question of the influence of an inhomogeneity in the mean magnetic field.

4. DISTRIBUTION FUNCTION OF FAST PARTICLES IN THE PRESENCE OF A FINITE MAGNETIC FIELD GRADIENT

We are now ready to consider the influence produced by an inhomogeneous magnetic field on the fast electron distribution function. From the point of view of physics, the occurrence of terms proportional to the magnetic field gradient in the kinetic equation (3) is associated with maintaining the adiabatic invariant $u_\perp^2/B(r) = \text{const}$ upon a diffusion particle motion along a discharge. In this case, particle

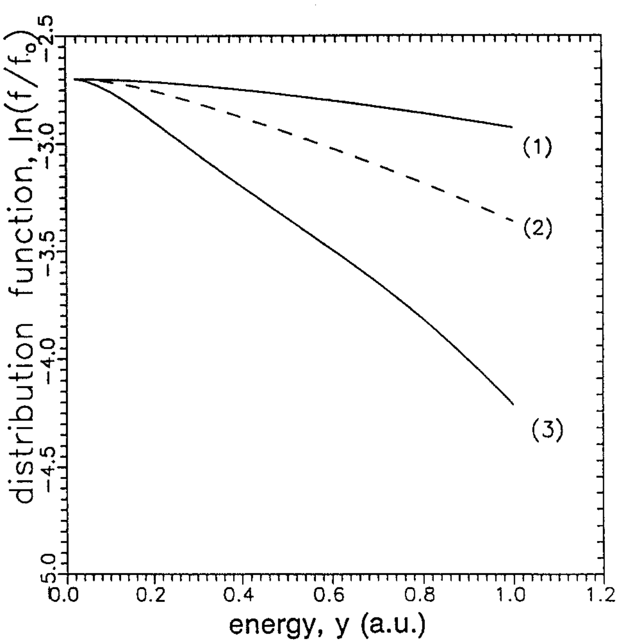

FIG. 7. The dependences of the fast electron distribution function, averaged over μ, $ln(f/f_0)$ on energy y with a finite mean magnetic field gradient present: (1) $dB/dr > 0$, (2) $dB/dr = 0$, (3) $dB/dr < 0$. $r = 0.95$.

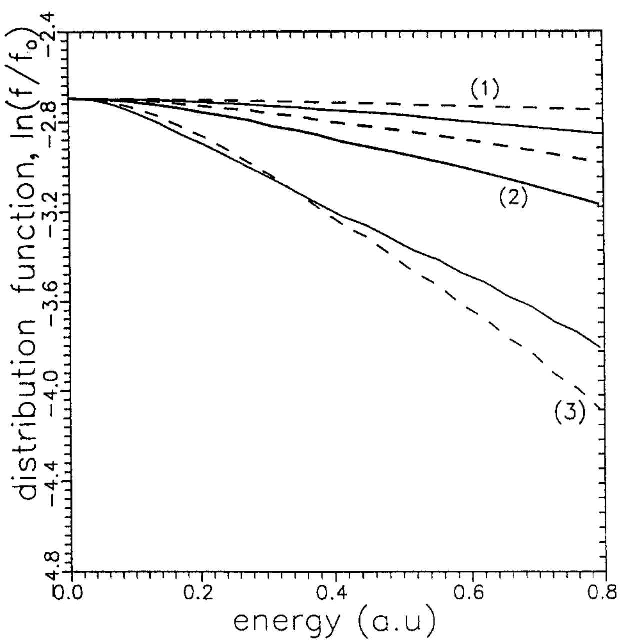

FIG. 8. The dependences of the fast electron distribution function $ln(f/f_0)$ on longitudinal (solid line) and transverse (dashed line) energy, in the presence of a finite mean magnetic field gradient: (1) $dB/dr > 0$, (2) $dB/dr = 0$, (3) $dB/dr < 0$. $r = 0.95$.

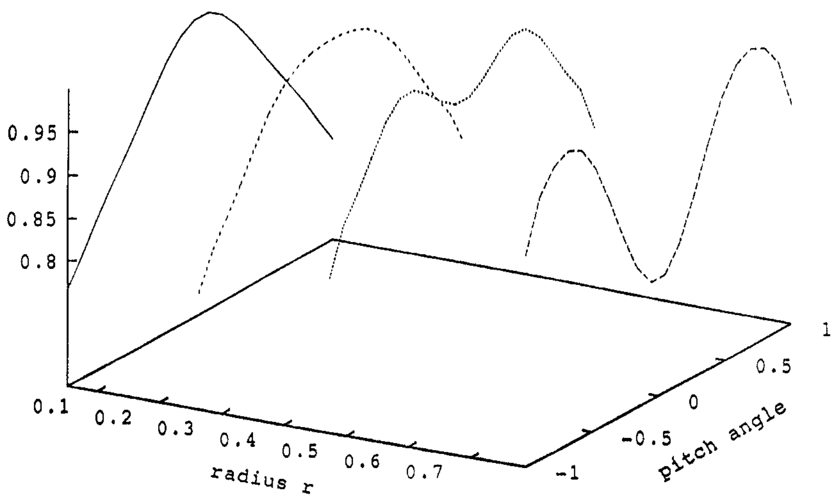

FIG. 9. The dependences of the fast electron distribution f/f_0 on μ at various positions over the radius; $dB/dr < 0$.

diffusion is responsible for a particle energy redistribution from the transverse degree of freedom to the longitudinal one and vice versa, depending on the sign of the derivative dB/dr. Since on average particles move from the centre to the periphery of a discharge, in the case $dB/dr < 0$ part of the transverse particle energy is transferred into the longitudinal particle energy and conversely in the case $dB/dr > 0$. Such dynamics must in turn affect the diffusion process itself. Indeed, if the number of particles with high longitudinal energy increases (when $dB/dr < 0$) or on the contrary decreases (when $dB/dr > 0$), then because the anomalous diffusion coefficient (2) is proportional to the longitudinal velocity, the diffusion rate will respectively either increase or decrease.

Let us consider the stationary solution of Eq. (3) in the range $1 \leq \epsilon \leq \delta^{-1/2}$, where $\delta = \delta_1, \delta_2$. So as not to complicate the picture, we put the external field $E_e(r) = 0$. In order to obtain the solution, we shall use the technique we applied in the solution of Eq. (6) in this range, i.e. the polynomial expansion (13). The function $F(r)$ will put F = const, and $n(r) = g_0(r)$ of the problem (12). Figure 7 shows the dependences of the distribution of fast electrons obtained in the solution of Eq. (6) depending on the electron energy y in two cases: $\mathrm{d}B/\mathrm{d}r > 0$ and $\mathrm{d}B/\mathrm{d}r < 0$. The figure also presents for comparison the result for $\mathrm{d}B/\mathrm{d}r = 0$. As expected, the greatest decrement of $\ln(f/f_0)$ appears in the case $\mathrm{d}B/\mathrm{d}r < 0$, and the smallest in the case $\mathrm{d}B/\mathrm{d}r > 0$. It would also be of interest to trace the behaviour of the solution obtained depending on the longitudinal $u_\parallel^2$ and the transverse $u_\perp^2$ energies. In Fig. 8 we can see the dependences of $\ln(f/f_0)$ on the transverse and the longitudinal particle energies in the cases $\mathrm{d}B/\mathrm{d}r < 0$, $\mathrm{d}B/\mathrm{d}r > 0$ and $\mathrm{d}B/\mathrm{d}r = 0$. We see that both for $\mathrm{d}B/\mathrm{d}r = 0$ and $\mathrm{d}B/\mathrm{d}r > 0$ there exists anisotropy in the particle distribution over transverse and longitudinal energy, so that $T_\perp > T_\parallel$. However, for $\mathrm{d}B/\mathrm{d}r < 0$ the anisotropy changes sign and then $T_\perp < T_\parallel$, which is naturally connected with a substantial energy redistribution from the transverse degree of freedom into the longitudinal one. This effect is most clearly pronounced in the dependence of the distribution function on the pitch angle μ (Fig. 9), which differs substantially from that obtained in the case $\mathrm{d}B/\mathrm{d}r = 0$ (Fig. 1). The fall in the centre (for $\mu \sim 0$) in Fig. 9 and the appearance of maxima near $\mu \sim 1$ is just due to particle efflux from the region $\mu \sim 0$ towards the region $\mu \sim 1$.

5. BEHAVIOUR OF THE DISTRIBUTION FUNCTION OF FAST ELECTRONS IN RFP

In the preceding sections of the paper we have analysed the influence of various factors on the distribution function of fast particles. It should be noted here that just in RFP discharges the contribution from each of them will be substantial because all of them are present in this type of device. An RFP discharge possesses a force free magnetic configuration with components of the mean magnetic field (the toroidal $B_z(r)$ and poloidal $B_\theta(r)$) which are of the same order of magnitude and are determined by one and the same parameter $\Theta = 2\pi a I/\Phi$, where I is the total current, a is the small radius of the torus and Φ is the total toroidal magnetic field flux. Magnetic field compo-

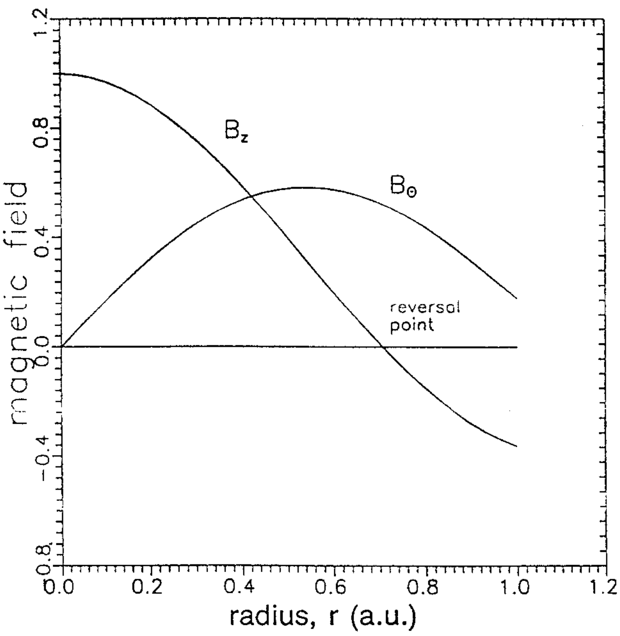

FIG. 10. The radial distribution of magnetic field components along the minor radius.

nents are well enough described by Bessel functions of the form

$$B_z(r) = B_0 J_0(2\Theta r)$$

$$B_\Theta(r) = B_0 J_1(2\Theta r) \tag{46}$$

see Fig. 10 [34]. So, the scale of the magnetic field gradient in an RFP will be of the order of the small radius a of the torus:

$$\frac{\mathrm{d}B}{\mathrm{d}r}\frac{a}{B} \sim 1$$

Since the plasma in an RFP is heated in an ohmic way, there is a rather strong longitudinal electric field E_e in the discharge. Since the applied electric field $\boldsymbol{E} = E\boldsymbol{e}_z$ is toroidal, its projection $E_e(r)$ onto a magnetic field line is described by the relation

$$E_e(r) = E\,\frac{B_z(r)}{B(r)} \tag{47}$$

From (47) it becomes clear that the applied field is strongly inhomogeneous in the space.

Direct measurements of the distribution of superthermal electrons in the steady state of an RFP were carried out on ZT-40M devices [19, 35–37]. For comparison with the theory, we shall consider the experimental data obtained in [35]. Performed in the experiment were direct measurements of the energy distribution of fast electrons in the near-boundary region of a discharge with a temperature $T_w \simeq 20$ eV near the boundary and $T_0 \simeq 220$ eV in the centre of the discharge. The characteristic value of the parameter is $\Theta = 1.4$. Observations were carried out for particles of energy up to 1.5 keV. Against the background of a cold plasma with a temperature $T_w \simeq 20$ eV, there was a tail of energetic electrons which were moving in the direction of the magnetic field and had a Maxwellian distribution with characteristic temperature $T_1 \simeq 530$ eV and a particle flux that moved backward and had a temperature $T_2 \simeq 330$ eV. An anisotropy in the distribution of particles in longitudinal and transverse energies was observed, the temperature being $T_\perp < T_\parallel$. Under the conditions of the experiment described above, the parameters δ_1, δ_2 of our problem were $\delta_1 = 0.21$, $\delta_2 = 0.036$ for $T_0 = 220$ eV, $n = 2.4 \times 10^{13}$ cm^{-3}, $E_e = 10$ V/m and the amplitude of fluctuations $|b/B| \sim 1\%$,

$$\frac{F_{max}}{B_0^2 a} \sim 4 \times 10^{-4}$$

The solution we obtained in Section 3 (Eqs (34), (35) and (37)) gives qualitative agreement with experiment: an exponential distribution of particles in energy and anisotropy in the particle distribution over the forward and backward directions along magnetic field lines.

An important comparison with experimental data is given by modelling the superthermal electron current I_{eea}, collected by the electrostatic energy analyser (EEA), as a function of the applied retarding potential V. The current is related to the electron distribution function $f(r, \mu, \epsilon)$ by the expression

$$I_{eea}(r, V) \propto \int_{v_0}^{\infty} \int_0^{\infty} f(r, \mu, \epsilon) v_\perp dv_\perp v_\parallel dv_\parallel \qquad (48)$$

where

$$v_0 = \sqrt{2eV/m_e}$$

Figure 11 shows the dependences of the current I_{eea} calculated by Eqs (34), (35), (37) and (48) for the given values of the parameters δ_1 and δ_2 for particles travelling along the direction of the magnetic field and in the reverse direction. The profile of the correlation function $F(r)$ shown in Fig. 12 was taken from Ref. [35]. The graph of the zero eigenfunction of the problem (12) $g_0(r)$ for a given $F(r)$ is shown in Fig. 13, where $\lambda_0 = 1.2$. The profile $n(r)$ was chosen in the form $n(r) = g_0(r)$ and $T(r) = 1 - r^2$. The calculated current I_{eea} was normalized to the experimental value. The absolute value of I_{eea} seems to be rather uncertain because of a lack of information on the correlation function near the boundary.

Figure 11 testifies to not only qualitative agreement between the theory and experiment (Maxwellian distribution and anisotropy observed in the distribution of particles travelling in the direction of the magnetic field and backward) but also to quantitative agreement. Thus, the characteristic temperature

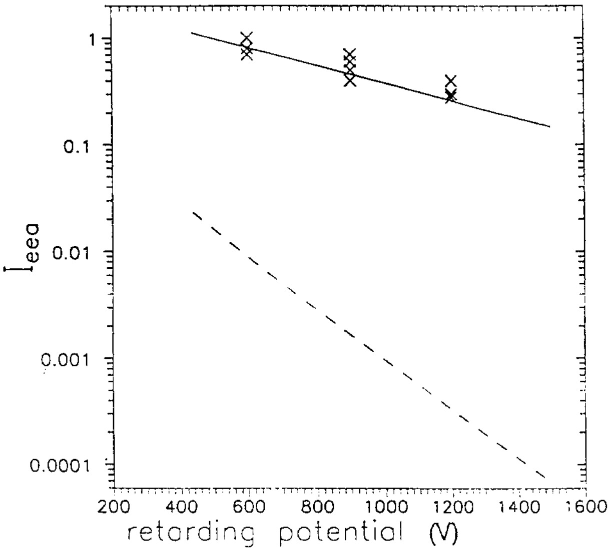

FIG. 11. Comparison of theoretical versus experimental current I_{eea}, collected by an electron energy analyser as a function of the retarding potential V for the case of ZT-40M. The solid line is for particles moving along the magnetic field line and the dashed line for particles moving backward. The experimental data are shown by labels.

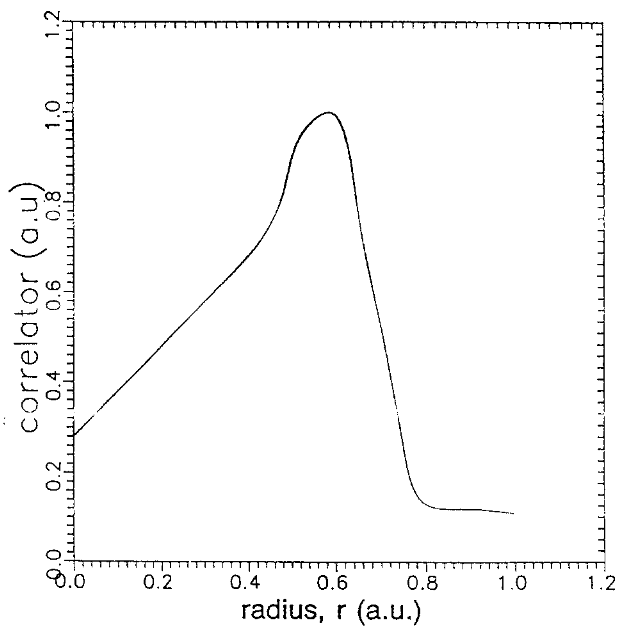

FIG. 12. Correlation function of fluctuations in an RFP.

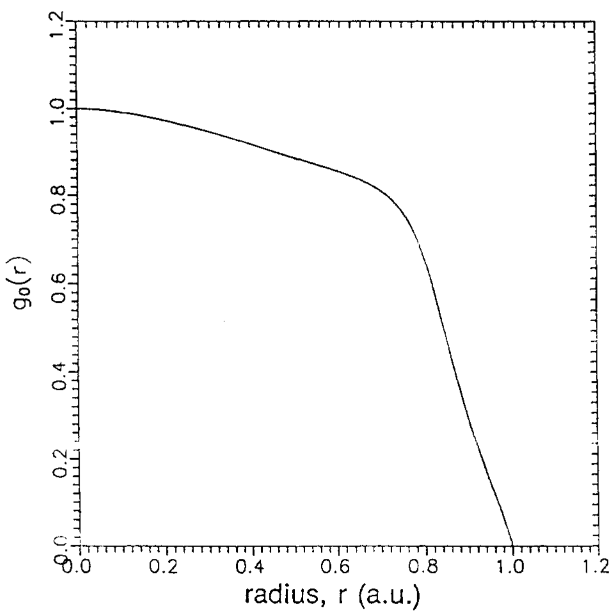

FIG. 13. Eigenfunction of the problem (12) for the correlation function depicted in Fig. 12.

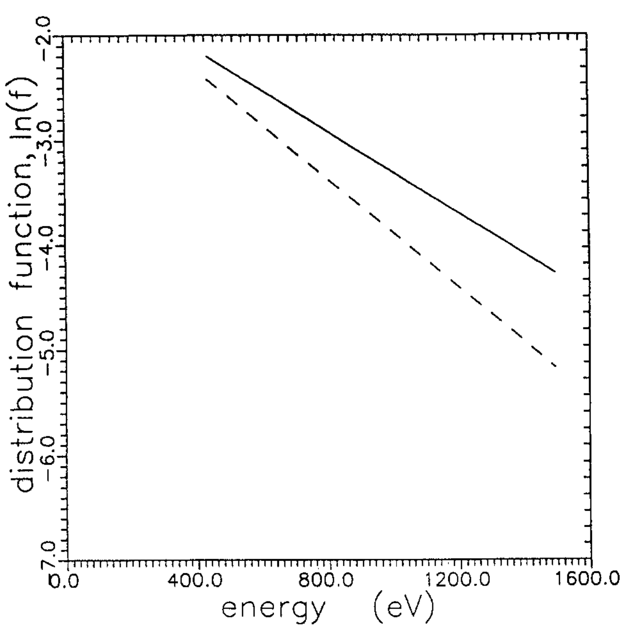

FIG. 14. The dependences of the fast electron distribution function ln(f) on longitudinal (solid line) and transverse (dashed line) energies for particles moving along the magnetic field line; $\beta = 0.17$, $\delta_1 = 0.21$, $\delta_2 = 0.036$.

T_1^{teor} for particles moving along the magnetic field is $T_1^{teor} = 526$ eV. But for particles moving in the backward direction $T_2^{teor} = 183$ eV, i.e. it is less than the experimental value. The backward flowing component of fast electrons is seen to be much smaller than the forward one (less than 10%). This value is also close to the experimental observations [35].

The behaviour of the distribution of fast electrons in longitudinal and transverse energies is shown in Fig. 14 for the parameters mentioned above. In this case for particles moving along the magnetic field line $T_\perp < T_\parallel$; $T_\parallel = 513$ eV, $T_\perp = 385$ eV.

6. SUMMARY

In the entire energy range above thermal energies, we have constructed the solution of the kinetic equation describing the distribution function of fast particles in the presence of an external electric field, collisions and anomalous diffusion due to magnetic fluctuations. The ambipolar electric field resulting from the difference in the ion and electron diffusion rates is shown to play an essential role in the process of diffusion and to be responsible for the fact that the character of the diffusion depends strongly on the density profile of the main plasma particles and on the profile of an external electric field applied to the plasma. The influence of inhomogeneity in the magnetic field of a discharge upon the fast particle distribution function is investigated. The analysis performed suggests that the magnetic field inhomogeneity is an important factor affecting the fast particle distribution function. In the polynomial region of energy values, which is an immediate neighbour of the thermal region, the semi-analytical solution is constructed with allowance for all the factors mentioned above. On the basis of a comparison with experimental data we have shown that the theory describes the basic details of the distribution function of fast particles. We have referred our analysis mainly to the case of RFP discharges, but the results obtained in the absence of a strong electric field seem to be closely connected with anomalous process in tokamak discharges.

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