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Parametric decay of upper hybrid plasma waves trapped inside density irregularities in the ionosphere

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Abstract

The parametric decay of an upper hybrid (UH) wave into a lower hybrid (LH) wave and a downshifted UH wave inside inhomogeneous ionospheric striations, when both pump and decay UH modes are trapped, is investigated. The threshold and the growth rate of the instability are determined. The possibility of the existence of both main features in the usually observed spectrum of stimulated electromagnetic emission (SEE), downshifted maximum and long continuum tail, is demonstrated. The conditions of the excitation of the second and third decay modes are established. © 1997 Published by Elsevier Science B.V.

1. Introduction

One of the most important new physical phenomena, discovered in the ionospheric modification by powerful radio waves, is the generation of small scale irregularities–striations. Striations are excited in the resonance region in the vicinity of the reflection point of the ordinary heater wave. They determine such processes as strong field aligned scattering and anomalous absorption of radio waves propagating in the disturbed region.

According to the theory the creation of the striations is a result of the local heating of an anisotropic ionospheric plasma by upper hybrid plasma waves (UH) generated by the linear transformation of the pump heater wave on the striation density depletion [1,2]. We see that the excitation of high level UH wave oscillations in the resonance region is deeply connected with the existence of the striations. Directly, the energy of the pump wave which goes to the excitation of the UH wave determines its strong anomalous absorption in the resonance region. On the other hand, a highly excited UH wave through the decay processes generates UH waves with shifted frequencies, which lead to the artificial emission of a wide spectrum of radio waves by the disturbed ionosphere [3]. This process, known as stimulated electromagnetic emission (SEE), is one of the most interesting and intensively studied in ionospheric experimental nonlinear phenomena. Some observed specific features of the SEE spectrum and its dynamic behavior are found to be closely connected with the excitation of striations in the vicinity of a multiple electron gyrofrequency [4–6].

SEE undoubtedly contains fundamental information about the nonlinear processes proceeding in the ionospheric plasma under the action of a powerful electromagnetic wave. But to realize the physical meaning of

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these processes, to understand the language which the ionosphere uses telling us about these nonlinear phenomena, an adequate theory should be developed.

This theory does not exist yet. In previous theoretical papers models were used which discussed processes in homogeneous plasma only [7–11]. This allowed us to obtain a criterium for the beginning of the decay of an UH wave into a low hybrid wave (LH) and an UH wave with shifted frequency or for other decay processes which are significant near the multiple gyrofrequency region. But that gives only the possibility to have a qualitative description of the main physical processes determining the SEE spectrum formation.

The real situation differs significantly from these models, first of all because the decay processes are in the strongly inhomogeneous plasma. The excitation of an UH wave by itself is determined by the existence of plasma inhomogeneities–striations. Excited UH waves are not free waves–they are trapped inside striations. The amplitudes of an UH wave field and its height distribution are fully defined by the pump radio wave and the structure of striations through the process of exciting plasma inhomogeneities. A connection between UH wave field and pump wave was established recently by the authors [1,2]. It was shown that simultaneously with the growth of the density inhomogeneity a strong heating of electrons inside the striations takes place: T_e becomes several times larger than the background value T_∞. This heating is due to a high plasma amplitude of an UH wave excited inside striations. Thus, both plasma and UH wave distribution in the upper hybrid resonance region is strongly inhomogeneous.

To describe SEE emission the decay processes for an UH wave in such an inhomogeneous plasma should be studied first of all. That is exactly the goal of the present paper. We will consider here the decay of an UH wave into a LH wave and a downshifted UH wave inside an inhomogeneous striation when both pump and decay UH wave are trapped. One should note that only trapped modes provide the possibility for the existence of an absolute instability in inhomogeneous media, which is needed to generate SEE emission. Untrapped modes are only amplified passing through inhomogeneous media [12].

We will see that the leakage of UH wave into the Z-mode radiation plays an important role in the process of parametric decay instability for UH waves. This process does not exist in a homogeneous medium at all. But as we will show directly the leakage usually determines the value of a decay critical field E_c which due to this becomes significantly higher than the critical field in a homogeneous plasma. Further, differently from Refs. [7–11], where a UH wave with electric field $\boldsymbol{E}$ perpendicular to the Earth's magnetic field $\boldsymbol{B}$ was considered only, we will discuss a range of angles between $\boldsymbol{E}$ and $\boldsymbol{B}$, which allows taking into account striation height inhomogeneity to determine the main peculiarities of the spectrum of the excited decay modes. That agrees well with the usually observed SEE spectrum. Both main features, the downshifted maximum and long continuum tail, are clearly seen to be in good agreement with observations. The conditions of excitation of the second, third and higher decay modes are established also.

2. Coupled mode equations for inhomogeneous plasma in striations

As in Refs. [1,2] we consider a magnetized ionospheric plasma with a density slowly growing in the vertical direction. Striations are local depletions of plasma density strongly alongated along the Earth magnetic field lines ($N - N_0 = N_1 < 0$). They are created as a result of local heating of electrons ($T - T_\infty > 0$) by the field of upper hybrid plasma waves E_0, generated inside the striations by a powerful pump radio wave P_0. Consequently, the stationary state of striations is described by a system of nonlinear stationary equations for transport of particle density N_1 and electron temperature T coupled with the wave equation for the electric field of plasma waves E_0, excited in striations at the upper hybrid resonance region by the pump wave P_0.

This system has the form [1,13]

$$\nabla\hat{D}\nabla N_1 + \nabla\hat{D}_T\nabla T = \gamma N_1, \qquad \nabla\hat{\kappa}\nabla T + \frac{\boldsymbol{E}_0\hat{\sigma}\boldsymbol{E}_0}{N_0} = \delta\nu(T - T_\infty), \qquad \nabla(\hat{\varepsilon}\boldsymbol{E}_0) = -\nabla(\delta\hat{\varepsilon}\boldsymbol{P}_0). \tag{1}$$

Here $\hat{D}$, $\hat{D}_T$, $\hat{\sigma}$, and $\hat{\kappa}$ are the tensors of diffusion, thermal diffusion, conductivity, and heat conductivity, respectively, γ is the coefficient of recombination, δ is the average part of the electron energy loss under the collision with ions and neutrals, ν is the frequency of electron collision, T_∞ is the temperature of the plasma far from the striation–it is determined by the average heating of the background ionospheric plasma by a powerful radiowave, $\boldsymbol{P}_0$ is the electric field of the pump wave, $\hat{\varepsilon}$ is the tensor of the dielectric primitivity of the plasma, $\delta\hat{\varepsilon}$ is the part of the tensor which is the response to the density perturbation of the plasma.

We will suggest in a further analysis on the basis of previous results [1,2] that the density perturbation N_1/N_0 in the striations is always small, while the temperature of electrons T may be several times greater than the background value T_∞. The functions N_1 and T are assumed to depend on two variables only, on z, which is along, and on x which is perpendicular to the direction of the magnetic field line in the $\boldsymbol{H}$, $\boldsymbol{P}_0$ plane. Following Refs. [1,2] we emphasize that the heat energy source $\boldsymbol{E}_0\hat{\sigma}\boldsymbol{E}_0/N_0$ is concentrated in the layer of the order of L between the upper hybrid resonance level $\omega=\omega_{\rm UH}=\sqrt{\omega_{\rm pe}^2+\omega_{\rm Be}^2}$ and the reflection point of the pump wave $\omega=\omega_{\rm p}$, since only inside this region the pump wave may excite upper hybrid waves within irregularities. This length scale in the F-region of the ionosphere is of the order of 1–3 km and so it is small compared to the characteristic scale of the transport processes along the magnetic field in the F-region: the longitudinal diffusion length $L_N^{\parallel}=\sqrt{D_\parallel/\gamma}$ and the longitudinal heat conductivity length $L_T^{\parallel}=\sqrt{\kappa(T_\infty)_\parallel/\delta\nu}$ [13]. In the F-region at heights $z>200$ km the parameter γ does not depend on the temperature of electrons T and we will assume also that the parameter δ is independent of T.

Under the conditions mentioned above the analysis of the system (1) has been performed in Refs. [1,2]. As it was shown, the system (1) has a stationary solution. It has the form of localized wave. The characteristic depth of the resultant density irregularities is of the order of $N=N_1/N_0\sim 0.012$–0.05. The stabilization comes from a nonlinear growing of the transport coefficients (mostly thermal conductivity κ), with the temperature T_e. That is why the stationary solution exists only for large enough values of $T_e/T_\infty>1.6$ (or for $N_1/N_0>0.012$).

The excited UH wave $\boldsymbol{E}_0$ could be unstable to have the parametric decay into another UH wave $\boldsymbol{E}_1$ with shifted frequency ω_1 and low hybrid wave (LH) $\boldsymbol{E}_{\rm L}$. To take into account this process we have to consider the system (1) together with additional wave equations for the waves $\boldsymbol{E}_1$ and $\boldsymbol{E}_{\rm L}$

$$\hat{\varepsilon}_1(\boldsymbol{E}_1)=\hat{F}_1(\boldsymbol{E}_0,\boldsymbol{E}_L),\qquad \hat{\varepsilon}_L(\boldsymbol{E}_L)=\hat{F}_L(\boldsymbol{E}_0,\boldsymbol{E}_1). \tag{2}$$

Here $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ are the linear tensors of dielectric permitivity of a plasma for a high frequency ω_1 and low frequency $\omega_{\rm L}$ waves. The right hand sides of (2) represent themselves a coupling between UH and LH waves E_0, E_1 and $E_{\rm L}$. The analogous coupling term like in (2), $\hat{F}_0(\boldsymbol{E}_1,\boldsymbol{E}_{\rm L})$, should be added in the right hand side of the last equation of the system (1).

The coupling relations (2) may be derived for a uniform plasma using a kinetic approach which takes into account the finite Larmor radius effect properly. The system of coupled mode equations then may be generalized on an inhomogeneous case. We will use a system of coupled equations for electrostatic plasma oscillations in a uniform plasma obtained in Ref. [10]. The waves are considered to propagate at small enough angles with the magnetic field B such that only the transverse component of the electric field should be taken into account. Below, E_0 is a transverse components of a pump wave, E_1 and $E_{\rm L}$ are the transverse components of decay modes with frequencies ω_0, ω_1, $\omega_{\rm L}$ and wave numbers $\boldsymbol{k}_0$, $\boldsymbol{k}_1$, $\boldsymbol{k}_{\rm L}$, respectively. For the waves moving in a positive direction along the x axis, i.e. $E_0\sim {\rm e}^{-{\rm i}\omega_0 t+{\rm i}k_{0\perp}x+{\rm i}k_{0z}z}$, such that the frequency matching condition $\omega_0=\omega_1+\omega_{\rm L}$ is fulfilled (the z axis is directed along the magnetic field), this system has the form

$$\left[\frac{\omega}{\overline{\omega}_1}-\frac{\omega_{\rm pe}^2}{\overline{\omega}_1^2-\omega_{\rm Be}^2}\left(1+\frac{3k_{1\perp}^2v_{\rm e}^2}{\overline{\omega}_1^2-4\omega_{\rm Be}^2}\right)\right]E_1$$
$$={\rm i}\frac{e}{m_{\rm e}}\left[\frac{\omega_{\rm pe}^2}{\overline{\omega}_1^2-\omega_{\rm Be}^2}\left(\frac{k_{{\rm L}\perp}}{\omega_0^2-\omega_{\rm Be}^2}-\frac{k_{0\perp}}{\overline{\omega}_{\rm L}^{*2}-\omega_{\rm Be}^2}\right)-\frac{k_{1\perp}\omega_{\rm pe}^2}{2\overline{\omega}_1\omega_{\rm Be}^2}\left(\frac{\omega_0}{\omega_0^2-\omega_{\rm Be}^2}-\frac{\overline{\omega}_{\rm L}^*}{\overline{\omega}_{\rm L}^{*2}-\omega_{\rm Be}^2}\right)\right.$$

$$+\frac{k_{1\perp}\omega_{\rm pe}^2}{2\omega_{\rm Be}^2(\overline{\omega}_1^2-4\omega_{\rm Be}^2)}\left(\frac{\overline{\omega}_1\omega_0+2\omega_{\rm Be}^2}{\omega_0^2-\omega_{\rm Be}^2}-\frac{\overline{\omega}_1\overline{\omega}_{\rm L}^*-2\omega_{\rm Be}^2}{\overline{\omega}_{\rm L}^{*2}-\omega_{\rm Be}^2}\right)\Bigg]E_{\rm L}^*E_0, \tag{3}$$

$$\left[1-\frac{\overline{\omega}_{\rm L}}{\omega_{\rm L}}\frac{\omega_{\rm pe}^2}{\overline{\omega}_{\rm L}^2-\omega_{\rm Be}^2}\left(1+\frac{3k_{{\rm L}\perp}^2v_{\rm c}^2}{\overline{\omega}_{\rm L}^2-4\omega_{\rm Be}^2}\right)-\frac{\omega_{\rm pi}^2}{\overline{\omega}_{\rm L}^2}+{\rm i}\gamma_l^2\right]E_L$$

$$={\rm i}\frac{e}{m_{\rm e}}\Bigg[\frac{\omega_{\rm pe}^2}{\overline{\omega}_{\rm L}^2-\omega_{\rm Be}^2}\left(\frac{k_{1\perp}}{\omega_0^2-\omega_{\rm Be}^2}-\frac{k_{0\perp}}{\overline{\omega}_1^{*2}-\omega_{\rm Be}^2}\right)-\frac{k_{L\perp}\omega_{\rm pe}^2}{2\overline{\omega}_{\rm L}\omega_{\rm Be}^2}\left(\frac{\omega_0}{\omega_0^2-\omega_{\rm Be}^2}-\frac{\overline{\omega}_1^*}{\overline{\omega}_1^{*2}-\omega_{\rm Be}^2}\right)$$

$$+\frac{k_{{\rm L}\perp}\omega_{\rm pe}^2}{2\omega_{\rm Be}^2(\overline{\omega}_{\rm L}^2-4\omega_{\rm Be}^2)}\left(\frac{\overline{\omega}_{\rm L}\omega_0+2\omega_{\rm Be}^2}{\omega_0^2-\omega_{\rm Be}^2}-\frac{\overline{\omega}_{\rm L}\overline{\omega}_1^*-2\omega_{\rm Be}^2}{\overline{\omega}_1^{*2}-\omega_{\rm Be}^2}\right)\Bigg]E_1^*E_0. \tag{4}$$

The left hand sides of these equations represent linear dispersion relations for UH and LH decay modes while the right sides are responsible for the coupling between them. Here, $\overline{\omega}_1=\omega_1+{\rm i}\nu_{\rm e}$, $\overline{\omega}_{\rm L}=\omega_{\rm L}+{\rm i}\nu_{\rm e}$, $\nu_{\rm e}$ is the collision frequency of electrons, $\omega_{\rm Be}$ and $\omega_{\rm Bi}$ are electron and ion gyrofrequencies, $\omega_{\rm pe}$ and $\omega_{\rm pi}$ are electron and ion plasma frequencies. The term

$$\gamma_l^2=\frac{\pi}{2}\frac{\omega_{\rm pi}^2\omega_{\rm L}}{k_{\rm L}^3v_{\rm i}^3}\exp\left[-\left(\frac{\omega_{\rm L}}{\sqrt{2}k_{\rm L}v_{\rm i}}\right)^2\right]+\frac{\pi}{2}\frac{\omega_{\rm pe}^2}{\omega_{\rm L}^2}\left(\frac{\omega_{\rm L}}{k_{\rm L}v_{\rm e}}\right)^3\frac{1}{|\cos\theta|}\exp\left[-\left(\frac{\omega_{\rm L}}{\cos\theta\sqrt{2}k_{\rm L}v_{\rm e}}\right)^2\right]$$

represents Landau damping of LH wave by ions and electrons, where θ is the angle between the direction of wave propagation and the magnetic field. Landau damping of an UH wave at small angles with the magnetic field $\cos\theta\sim m_{\rm e}/m_{\rm i}$ may be all the more neglected by comparison with a collisional one if its frequency is not too close to harmonics of the electron gyrofrequency.

Below, the plasma conditions relevant for the F-region of the ionosphere are considered, $\omega_{\rm Be}\simeq 2\pi\times 1.35$ MHz, $\omega_{\rm Bi}\simeq 2\pi\times 50$ Hz, $\omega_{\rm pe}\simeq 2\pi\times 3$ MHz, $\omega_{\rm pi}\simeq 2\pi\times 30$ kHz, ion gyroradius $\rho_{\rm i}\simeq$ 1–4m, electron gyroradius, $\rho_{\rm e}\simeq 1$ cm, $\nu_{\rm e}\simeq 300$ s^{-1} at $T_{\rm e}\sim T_{\rm i}\sim 1000$ K. Then the characteristic frequency of excited UH waves $\omega_{0,1}\simeq 2\pi\times 4$ MHz and LH waves $\omega_{\rm L}=\sqrt{\omega_{\rm Be}\omega_{\rm Bi}}\simeq 2\pi\times 7$ kHz. According to the analysis given in Refs. [1,2], the characteristic wavelength of excited UH waves k_0^{-1}, $k_1^{-1}\sim 10$ cm is less than the striation scale $l_{\rm s}\sim$ 1–5 m, the last one, in the F-region, is of the order of the ion Larmor radius. The wavelength of LH decay waves for the most effective decay process of the UH wave E_0 into the backscatter downshifted UH wave E_1 is of the order of k_0^{-1} [10,14]. Since the wavelength of the LH wave is less than the ion gyroradius $k_{{\rm L}\perp}\gg 2\pi/\rho_{\rm i}$, the ion contribution into dispersion of the LH wave reduces to the term corresponding to the case of a cold unmagnetized plasma.

Using the conditions $\nu_{\rm e}\ll\omega_{\rm L}\ll\omega_{0,1}$ and $\omega_{\rm Be}^2\ll\omega_{0,1}^2$ we can simplify the set of coupled equations (3)

$$\left(\omega_1^2-\omega_{\rm UH}^2-\frac{3k_{1\perp}^2v_{\rm e}^2\omega_{\rm pe}^2}{\omega_{\rm UH}^2-4\omega_{\rm Be}^2}+{\rm i}\Gamma_1\right)E_1=\omega_{\rm pe}^2\left\{\frac{k_{{\rm L}\perp}}{\omega_0^2-\omega_{\rm Be}^2}+\frac{k_{0\perp}-k_{1\perp}}{\omega_{\rm Be}^2}\right\}{\rm i}\frac{e}{m_{\rm e}}E_{\rm L}^*E_0$$

$$\simeq{\rm i}\frac{e}{m_{\rm e}}E_{\rm L}^*E_0\frac{\omega_{\rm pe}^2}{\omega_{\rm Be}^2}(k_{0\perp}-k_{1\perp}), \tag{5}$$

$$\left(\omega_{\rm L}^2-\omega_{\rm LH}^2-\frac{3}{4}\frac{k_{{\rm L}\perp}^2v_{\rm e}^2}{\omega_{\rm Be}^2}\frac{\omega_{\rm pe}^2\omega_{\rm L}^2}{\omega_{\rm pe}^2+\omega_{\rm Be}^2}+{\rm i}\Gamma_{\rm L}\right)E_{\rm L}={\rm i}\frac{e}{m_{\rm e}}E_1^*E_0\frac{\omega_{\rm L}^2}{\omega_{\rm pe}^2+\omega_{\rm Be}^2}(k_{0\perp}-k_{1\perp}), \tag{6}$$

where

$$\Gamma_1 = \nu_e \omega_1 \simeq \nu_e \omega_{UH}, \qquad \Gamma_L = \nu_e \omega_L \frac{\omega_{pe}^2}{\omega_{pe}^2 + \omega_{Be}^2} + \gamma_l^2 \frac{\omega_{pe}^2 \omega_L^2}{\omega_{pe}^2 + \omega_{Be}^2} \simeq \nu_e \omega_L + \gamma_l^2 \omega_L^2.$$

The characteristic frequency of the LH wave ω_{LH} is defined as follows,

$$\omega_{LH}^2 = \omega_{Bi} \omega_{Be} \left(1 + \frac{m_i}{m_e} \cos^2 \theta \right). \tag{7}$$

Formula (7) is valid if the declination angle θ is not too far from $\theta/2$, such that $\omega_{LH} \ll \omega_{Be}$ [14].

Eqs. (5), (6) can be generalized to the case of a weak nonuniform plasma, taking into account that for the characteristic scale of striations in the perpendicular direction to the magnetic field l_s the following relations are fulfilled,

$$l_s \gg k_{0\perp}^{-1}, \; k_{1\perp}^{-1}, \; k_{L\perp}^{-1}. \tag{8}$$

As follows from Refs. [1,2] the inhomogeneity is characterized by temperature and density variations, which are symmetrical around $x = 0$, and does not depend on the coordinate along the magnetic field z [1,2]. The density depletion n is negative and small compared with the background value N_0, $|N| = |n/N_0| \sim 0.01$–0.04, while the temperature variation can be large enough $(T_e - T_\infty)/T_\infty \geqslant 1$. Then, identifying in the l.h.s. of (5), (6) $k_{1,L\perp}^2 = \lambda_{1,L}^2 \mathrm{d}^2/\mathrm{d}x^2$, where

$$\lambda_1^2 = \frac{3\rho_{e0}^2}{l_s^2} \frac{\omega_{pe}^2 \omega_{Be}^2}{\omega_{UH}^2 - 4\omega_{Be}^2}, \qquad \lambda_L^2 = \frac{3\rho_{e0}^2}{4 l_s^2} \frac{\omega_{pe}^2 \omega_L^2}{\omega_{pe}^2 + \omega_{Be}^2},$$

ρ_{e0} is the electron gyroradius at $T_e = T_\infty$, Eqs. (5), (6) can be presented in the form of oscillator equations

$$\frac{\mathrm{d}^2 f_1}{\mathrm{d}x^2} + g_1 f_1 = F_1, \qquad \frac{\mathrm{d}^2 f_L}{\mathrm{d}x^2} + g_L f_L = F_L. \tag{9}$$

To eliminate unwanted numerical factors, the variable x has been normalized to l_s. The right hand sides of (9) just correspond to the right hand sides of (2), the functions

$$F_1 = \mathrm{i} Q_1 \left(g_0^{1/2} + g_1^{1/2} \right) f_L^* f_0, \qquad F_L = \mathrm{i} Q_L \left(g_0^{1/2} + g_1^{1/2} \right) f_1^* f_0$$

represent coupling between interacting modes and correspond to the coupling terms in the left sides of (2),

$$Q_1 = \frac{e}{m_e} \frac{\omega_{pe}^2}{\omega_{Be}^2 l_s \lambda_1^2 \tau^2}, \qquad Q_L = \frac{e}{m_e} \frac{\omega_L^2}{\omega_{pe}^2 + \omega_{Be}^2 l_s \lambda_L^2 \tau^2},$$

where $f_{0,1,L} = \tau E_{0,1,L}$ and $\tau(x) = T_e/T_\infty$ is the normalized electron temperature. The functions

$$g_0 = \frac{\omega_0^2 - \omega_{UH}^2 + \mathrm{i}\Gamma_1}{\lambda_1^2 \tau}, \qquad g_1 = \frac{\omega_1^2 - \omega_{UH}^2 + \mathrm{i}\Gamma_1}{\lambda_1^2 \tau}, \qquad g_L = \frac{\omega_L^2 - \omega_{LH}^2 + \mathrm{i}\Gamma_L}{\lambda_L^2 \tau}$$

correspond to the square of the effective wave vectors of plasma waves in inhomogeneous media. An equation for the pump field E_0 which is excited, in turn, by an incident powerful radio wave P_0, has to be added to close the system (9). According to the fact that the pump wave E_0 has the same type as the decay mode E_1, this equation can be presented in the same form as (9),

$$\frac{\mathrm{d}^2 f_0}{\mathrm{d}x^2} + g_0 f_0 = F_0, \tag{10}$$

$$F_0 = -\mathrm{i} Q_0 \left(g_0^{1/2} + g_1^{1/2} \right) f_L f_1 + \frac{P_0 N(x) \omega_{UH}^2}{\lambda_1^2}, \tag{11}$$

where the function F_0 includes both coupling between modes $E_{0,1,L}$ (the first term) and the source of energy stemming from the action of the incident external radio wave P_0 (the second term). The identification of the terms in (10) and (1) is the same as between (9) and (2).

3. Critical field

The process of parametric instability is well known to have a threshold character. The energy of the pump wave must be large enough to compensate for any losses of the decay modes. The decay process in inhomogeneous media substantially differs from the process in homogeneous space. Generally speaking, in inhomogeneous media the instability may be not absolute, which means that it is impossible to find a mode with positive growth rate. This is because, in the inhomogeneous case, as distinct from a homogeneous one, the process of parametric decay has a local character. It occurs effectively only in the vicinity of the resonance points $x = x_\pm$ where the matching condition

$$g_0^{1/2} + g_1^{1/2} - g_L^{1/2} = 0 \tag{12}$$

is achieved [12,15,16]. That is why, if all modes are convective ones, an unstable region serves as amplifying medium which only increases the intensity of the wave incident on it. The situation completely changes if at least one wave is trapped inside inhomogeneity, i.e. it has cut-off points defined by null points of the effective wave vector. In this case, a trapped mode many times passes through an unstable region, which can lead to an absolute instability if a threshold is exceeded. But the threshold in this process is now governed by the total damping of the trapped mode inside the inhomogeneity (for instance by the leakage at the reflection point) and energy convection of an untrapped mode from the interaction region rather than collisional dissipation only.

To obtain the critical field E_c and the increment of the instability γ, we consider first a solution of (9) in the approximation of a given constant E_0 ignoring its reduction by power transfer to decay modes.

Our prime concern is the case when both the UH waves E_0 and E_1 are trapped inside the inhomogeneity. The LH frequency ω_{LH} slightly depends on the density variation and the LH wave freely propagates leaving density depletion [14]. According to (8) the solution can be found in the WKB approximation. We restrict ourselves to the case of weak coupling, such that the increment of the instability γ is less than the minimal characteristic frequency of the interacting waves $\gamma \ll \omega_{LH}$. In such a case the solution can be found in the form of expansion over eigenfunctions of homogeneous equations.

Let $\pm x_0$ and $\pm x_1$ be the null points of the pump and the decay UH wave, i.e. $g_0(\pm x_0) = 0$, $g_1(\pm x_1) = 0$. According to $\omega_0 = \omega_1 + \omega_L$, $x_0 > x_1$, i.e. the propagation area of the downshifted UH wave E_1 is always incorporated into the pump wave region $-x_0 < x < x_0$. The gap between two null points is of the order of $\Delta x = x_0 - x_1 \simeq 2\omega_L/\omega_0 N_m$, where N_m is the density depth of depletion. For our parameters at $l_s \sim 5$–10 m, $\tau_m \sim 2$ is the maximum electron temperature and $N_m \sim 0.03$, the gap length Δx is greater than the characteristic wavelength of UH waves $\lambda_{UH} \sim r_e \tau_m^{1/2}/l_s N_m^{1/2}$, $\delta x/\lambda_{UH} \sim 10$. Then, the solution is able to be considered only in the region $-x_1 < x < x_1$ assuming formally the point x_0 is at infinity.

Taking into account that in every striation two waves of each type counterpropagate, the solution can be found in the region $-x_1 < x < x_1$ in the following form,

$$f_1 = C_1^1 \psi_1^1 + C_1^2 \psi_2^1, \qquad f_L = C_L^1 \psi_1^L + C_L^2 \psi_2^L. \tag{13}$$

where $\psi_{1,\mathrm{L}}^{1,2}$ are eigenfunctions of the homogeneous part of Eqs. (9)

$$\psi_1^1 = \frac{\exp\left(\mathrm{i}\int_{-x_1}^{x} g_1^{1/2}\,\mathrm{d}x'\right)}{g_1^{1/4}} \qquad \psi_2^1 = \frac{\exp\left(-\mathrm{i}\int_{-x_1}^{x} g_1^{1/2}\,\mathrm{d}x'\right)}{g_1^{1/4}}$$

$$\psi_1^{\mathrm{L}} = \frac{\exp\left(\mathrm{i}\int_0^{x} g_{\mathrm{L}}^{1/2}\,\mathrm{d}x'\right)}{g_{\mathrm{L}}^{1/4}} \qquad \psi_2^{\mathrm{L}} = \frac{\exp\left(-\mathrm{i}\int_0^{x} g_{\mathrm{L}}^{1/2}\,\mathrm{d}x'\right)}{g_{\mathrm{L}}^{1/4}} \tag{14}$$

and $C_{1,\mathrm{L}}^{1,2}$ are slowly varying functions of the spatial variable x.

The right hand sides of the system (9) describing coupling between modes have then the following form in terms of the notion (13)

$$F_1 = \mathrm{i}Q_1\left(g_0^{1/2} + g_1^{1/2}\right)\left(C_0^1 C_{\mathrm{L}}^{1*}\psi_0^1\psi_{\mathrm{L}}^{1*} - C_0^2 C_{\mathrm{L}}^{2*}\psi_0^2\psi_{\mathrm{L}}^{2*}\right), \tag{15}$$

$$F_{\mathrm{L}} = \mathrm{i}Q_{\mathrm{L}}\left(g_0^{1/2} + g_1^{1/2}\right)\left(C_0^1 C_1^{2*}\psi_0^1\psi_1^{2*} - C_0^2 C_1^{1*}\psi_0^2\psi_1^{1*}\right), \tag{16}$$

where $Q_{1,\mathrm{L}}$ are slowly varying functions of the spatial variable x.

As we have already mentioned above, the efficiency of the decay mode excitation is defined by the presence of transformation points $x_{\pm}$, according to the condition (12). If these points are present, then only in the vicinity of this region the right hand sides of (9), F_1 and F_{L}, give a substantial contribution. Below we will assume that condition (12) is always fulfilled inside the region $-x_1 < x < x_1$.

Substituting (13), (16) and (15), we obtain a solution of the system (9) with the natural boundary conditions that there are no waves coming from $\pm\infty$. Taking into account that the main contribution from the inhomogeneous part of Eqs. (9) is given in the vicinity of $x = x_{\pm}$ we get, from the condition that a nontrivial solution of (9) exists, a relation between the pumping UH wave field intensity E_0 and the increment of the instability γ (any functions are given at $x = x_{\pm}$),

$$|E_0|^2 = g_0^{-1/2}\tau^{-2}\frac{|h''|\,g_{\mathrm{L}}^{1/2}}{2\pi Q_1 Q_{\mathrm{L}}}\,\mathrm{Im}\,\tilde{\Phi}_1, \qquad h'' = \frac{\mathrm{d}}{\mathrm{d}x}\left(g_0^{1/2} + g_1^{1/2} - g_{\mathrm{L}}^{1/2}\right), \tag{17}$$

the imaginary part of the phase $\mathrm{Im}\,\tilde{\Phi}_1$ includes both any damping of the UH wave and the instability increment γ. Extracting an imaginary part from g_1 we have

$$|E_0|^2 = g_0^{-1/2}\tau^{-2}\frac{|h''|\,g_{\mathrm{L}}^{1/2}}{2\pi Q_1 Q_{\mathrm{L}}}\left(\tfrac{1}{2}\int_{-x_1}^{x_1}\frac{\Gamma_1 + 2\omega_1\gamma\,\mathrm{d}x}{g_1^{1/2}\lambda_1^2\tau}\right) \tag{18}$$

Setting $\gamma = 0$ the threshold field of the decay process is obtained,

$$|E_{\mathrm{c}}|^2 = g_0^{-1/2}\tau^{-2}\frac{|h''|\,g_{\mathrm{L}}^{1/2}}{2\pi Q_1 Q_{\mathrm{L}}}\left(\tfrac{1}{2}\int_{-x_1}^{x_1}\frac{\Gamma_1\,\mathrm{d}x}{g_1^{1/2}\lambda_1^2\tau}\right) \tag{19}$$

We see that the threshold field is defined by two processes: total damping of the UH wave in the depletion due to collisions (which is proportional to Γ_1) and convection of the LH wave energy from the interaction zone $(g_{\mathrm{L}}^{1/2}|h''|)$.

Previously, we considered only collisional dissipation of UH waves. One should note that for one isolated striation another mechanism of energy losses is dominant, the so-called Z-mode leakage [17,18]. This mechanism relates to the violation of electrostatic approximation in the vicinity of the turning points $x_{0,1}$, where $g_{0,1}$ tends to zero. This fact gives rise to incomplete reflection of a trapped wave near the turning points and so determines the dissipation of UH wave energy. A small part of this energy is transformed outside the turning points and propagates away as long electromagnetic waves. This mechanism remains dominant for a small group of inhomogeneities. The relation between two mechanisms depends on the global Z-mode propagation condition [17,18]. If the number of striations becomes large enough the total Z-mode leakage may be suppressed and damping is due to collisions [18].

The shift in imaginary part of the phase $\tilde{\Phi}_1$ produced by Z-mode emission is defined by

$$\Gamma_z = \frac{2\pi l_n l_s}{\sqrt{N(x_1)}} \left(k_z \frac{\omega_{Be}}{\omega_{UH}} + \frac{\omega_{Be}}{c} \right), \tag{20}$$

where $l_n = N/(\mathrm{d}N/\mathrm{d}x)$ at $x = \pm x_1$, see Ref. [17]. It is seen that the leakage into Z-mode emission strongly depends on the position of the UH wave generation level. Thus, neglecting collisional damping we have

$$|E_0|^2 = g_0^{-1/2} \tau^{-2} \frac{|h''| g_L^{1/2}}{2\pi Q_1 Q_L} \left(\Gamma_z + \omega_1 \gamma \int_{-x_1}^{x_1} \frac{\mathrm{d}x}{g_1^{1/2} \lambda_1^2 \tau} \right) \tag{21}$$

Setting again $\gamma = 0$ a threshold field of the decay process is obtained

$$|E_c|^2 = g_0^{-1/2} \tau^{-2} \frac{|h''| g_L^{1/2}}{2\pi Q_1 Q_L} \Gamma_z. \tag{22}$$

We see that quite analogous to the previous case the threshold field is defined by two processes: total damping of UH wave in the depletion due to Z-mode leakage and convection of LH wave energy from the interaction zone.

Using expressions (19), (22) we can present the increment in terms of the critical field

$$\gamma = \frac{|E|^2 - |E_c|^2 \frac{1}{2} \int_{-x_1}^{x_1} \Gamma_1 \, \mathrm{d}x / g_1^{1/2} \lambda_1^2 \tau}{|E_c|^2 \omega_1 \int_{-x_1}^{x_1} \mathrm{d}x / g_1^{1/2} \lambda_1^2 \tau} \tag{23}$$

for the case of collisional damping and

$$\gamma = \frac{|E|^2 - |E_c|^2}{|E_c|^2} \frac{\Gamma_z}{\omega_1 \int_{-x_1}^{x_1} \mathrm{d}x / g_1^{1/2} \lambda_1^2 \tau}, \tag{24}$$

for Z-mode leakage.

One should note that both relations (19), (22) are obtained in the supposition of low Landau damping $\gamma_l^2 \ll 1$, which is always fulfilled.

4. Stationary state

If the threshold (19) or (22) is exceeded, the instability will evolve gradually up to a stationary state. It is appreciated that the instability will go on up to the moment when the pump UH field E_0 would reach a critical value E_c. In principle, if the pump field on an initial stage of the instability is strong enough, i.e. E substantially exceeds the critical value (19) or (22), the daughter wave E_1 can also exceed a critical value and in turn will decay into the downshifted UH wave E_2. The last one can in turn generate the next daughter wave E_3 and so on, up to the moment when some UH wave will not be able to propagate or its amplitude will be below a critical level. This process is called cascading and results in the generation of a number of almost equidistantly placed downshifted harmonics. It is assumed that a process of such kind takes place in the ionosphere when several (2

DM, 3 DM) downshifted maximums in the SEE spectrum are observed [3,8]. Once cascading takes place the critical field of parametric instability is not longer defined by relation (19) or (22) because the total dissipation of the UH daughter wave E_1 is substantially governed not by dissipative processes but by outgoing energy flux into downshifted modes $E_2, E_3, E_4, \ldots$.

Now we will consider the simplest case of the problem when an excess of a pump field E_0 over a critical one is not very large (the proper criteria will be defined below), such that the outgoing flux into the nearest daughter wave E_2 is negligible in comparison with collisional damping or Z-mode leakage.

To get the amplitude of the first decay mode E_1 or E_L in such a stationary state we have to evaluate Eqs. (9), (10) simultaneously. Using the same procedure as described in the previous section we got after averaging over quick-oscillating phases

$$|E_0|^2 = \frac{P_0^2}{\sqrt{\Gamma^2+\kappa_0^2}} \frac{1}{N(x_0)^{1/2}\delta^2} \frac{\pi}{2} \frac{1}{g_0^{1/2}\tau^2} \left(\frac{1-4Y^2}{3Y^2} \right)^{3/2}, \tag{25}$$

where $Y = \omega_{eB}/\omega_{UH}$, $\Gamma = \Gamma_z$ or $\Gamma = \frac{1}{2}\int_{-x_1}^{x_1} \Gamma_1 \, dx/g_1^{1/2}\lambda_1^2\tau$ and $\kappa_0 = \int_0^{\Delta\omega} 2\pi Q_1 Q_L |C_1^1|^2/g_L^{1/2}|h''|$ is the total, integrated over all possible frequencies of LH decay modes, leakage into downshifted mode E_1 and low frequency wave E_L. Here $\Delta\omega$ is the spectral width of the decay modes, see below. Then letting $|E_0|^2 = E_c^2$, one has

$$|\tilde{E}_1|^2 = \int_0^{\Delta\omega} |E_1|^2 \, d\omega = E_c^2 \sqrt{(E_{00}/E_c)^4 - 1}, \tag{26}$$

where E_{00} is the value of UH wave field given by (25) if $\kappa_0 = 0$. This value corresponds to the UH pump field at an initial stage of the instability. Relation (26) determines directly the full energy of an excited mode E_1 through the initial amplitude of the UH pump wave E_{00} and critical field E_c.

The decay mode E_1 may have a wide spectrum interval beginning from the maximal value $\omega_{1\max} = \omega_0 - \sqrt{\omega_{Bi}\omega_{Be}}$, see (7). The lower boundary is defined by the maximal depth of density inhomogeneities such that downshifted mode can propagate,

$$\omega_{1\min} \simeq \omega_0 (1 - |N_{\max}|/2). \tag{27}$$

So that the spectral width is equal to $\Delta\omega = \omega_{1\max} - \omega_{1\min} \simeq \omega_0 |N_m|/2 - \sqrt{\omega_{Bi}\omega_{Be}}$. For instance, for the parameters we use, $N_m \sim 0.03$, $\Delta\omega \simeq 2\pi \times 100$ kHz.

Thus, the effective UH wave energy corresponding to the same spectral interval that the pump wave E_0 has, $\Delta\omega_0 \simeq 2\pi\nu_e$, is $|E_1|^2 \simeq |\tilde{E}_1|^2 (2\pi\nu_e/\Delta\omega)$.

To define the critical field at which the second daughter decay wave E_2 is generated, one sets $E_1 = E_c$, we get

$$|E_c^{(2)}|^2 = |E_c|^2 \left[1 + (\Delta\omega/\nu_e)^2 \right]^{1/2}. \tag{28}$$

If $\Delta\omega \gg \nu_e$, as is the case for our parameters $\Delta\omega/\nu_e \sim 10^2$, then $|E_c^{(2)}|^2 \simeq |E_c|^2(\Delta\omega/\nu_e)$. So, we see that the critical field $E_c^{(2)}$ could be much higher than E_c.

By analogy with (26)

$$\int_0^{\Delta\omega} |E_2|^2 \, d\omega = E_c^2 \sqrt{(E_1/E_c)^4 - 1}. \tag{29}$$

Then, letting $E_2 = E_c$, we get an estimate for the critical value of the pump field $E_c^{(3)}$ at which the third daughter wave E_3 is generated,

$$|E_c^{(3)}|^2 = |E_c|^2 \left\{ \left[1 + (\Delta\omega/\nu_e)^2 \right] (\Delta\omega/\nu_e)^2 + 1 \right\}^{1/2}. \tag{30}$$

This value also defines the maximal amplitude of the pump field when our approach is valid. If $\Delta\omega \gg \nu_e$ then $|E_c^{(3)}|^2 \simeq |E_c|^2(\Delta\omega/\nu_e)^2$ and we see that the field $E_c^{(3)}$ is so strong that the generation of each further daughter wave becomes impossible in reality.

5. Discussion of the spectrum of decay waves

Now, we consider a spectrum generated due to the decay process of UH waves inside density depletions – striations at the F-region of the ionosphere.

According to the analysis given above, the spectrum of downshifted waves is directly defined by the parameters of the striations, in particular, by its maximal depth N_m, see (27). For the discussed conditions $N_m \sim 0.012$–0.05 the spectrum width may take the values $\Delta\omega \sim 40$–150 kHz, which agrees with the frequency width of the usually observed SEE long continuum tail.

The amplitude of the decay waves E_1 is defined by the ratio of the UH pump wave amplitude E_{00} (which is a function of the pump field P_0) and the critical field E_c. The total power of the decay modes is defined by relation (26).

The critical field E_c is substantially governed by dissipation of decay modes. Following the results [18] we may distinct two limit cases, which differ by the dissipation mechanism and consequently is expected to differ by the resultant critical field of parametric instability. The first one is the case when one isolated striation is present. The dissipation of both pump and decay UH waves E_0 and E_1 is due to Z-mode leakage. The second case occurs if a large group of striations is present. The leakage into Z-mode radiation is suppressed in this case and the dominant dissipation mechanism is thermal damping.

Before considering these cases in detail, one should note that in the inhomogeneous case the critical field in both cases is a function of several variables, frequency offset $\delta\omega = \omega_0 - \omega_1 = \omega_L$ and the position of transformation points $x_\pm$ inside the striation. The position of the transformation points $x_\pm$ is defined by relation (12) as a function of $\delta\omega$. This relation is sensitive to the value of $\delta\omega$. The relative variation of the offset $\Delta\omega_L/\omega_L$, while the point $x_\pm$ is sweeping the whole region $-x_1 < x < x_1$, is of the order of the striation density depth $\Delta\omega_L/\omega_L \sim N_m \ll 1$. Thus, relation (12) can be always satisfied at all $x_\pm$, while ω_L being close to characteristic frequency ω_{LH}. The last one is a function of the declination angle θ (7). As a consequence, the absolute value of the offset $\delta\omega$ is mainly driven by the changing of angle θ.

In stationary conditions, only a minimal value of the critical field has a meaning. To define the critical field value as a function of the frequency offset $\delta\omega$ one needs to minimize this value over the position of the points $x_\pm$ at each value of the offset frequency. This dependence provides a maximal amplitude of the UH waves in the stationary state. These dependences for both considered cases at the plasma conditions in the F-region and the striation parameters $N_m = 0.05$ and $\tau_m = 3.5$ are shown in Fig. 1. It is seen that the critical field in the second case is substantially less than in the first one. In both cases the obtained critical value E_c increases as the frequency offset increases. The minimal value attained in the second case at $\delta\omega = \sqrt{\omega_{Be}\omega_{Bi}}$ is $E_c \sim 50$ mV/m at the characteristic plasma parameters in the F-region. This value is greater than the critical field estimated for such a process in a homogeneous plasma at the same parameters [10]. In the first case, when Z-mode leakage is dominant, the minimal value is significantly higher, $E_c = \sim 200$ mV/m. The critical fields corresponding to the beginning of the second and the third cascade processes, i.e. $E_c^{(2,3)}$ scales as follows, $E_c^{(2)} \sim 10E_c$ and $E_c^{(3)} \sim 10^2 E_c$. So we see that excitation of the second and especially third harmonics asks for a high amplitude of UH waves E_{00}.

The intensity of decay mode emission is defined by the value of the pump field on an initial stage of instability E_{00}. Essentially because the process of striation generation has a threshold character, the pump field E_{00} cannot be less than a minimal value. This value depends on the associated intensity of electromagnetic pump field P_0 and the dominant damping mechanism. According to Ref. [1] the critical value of P_0 is of the order of 100 mV/m at the characteristic parameters in the F-region. Estimating, we get from (25) in case of

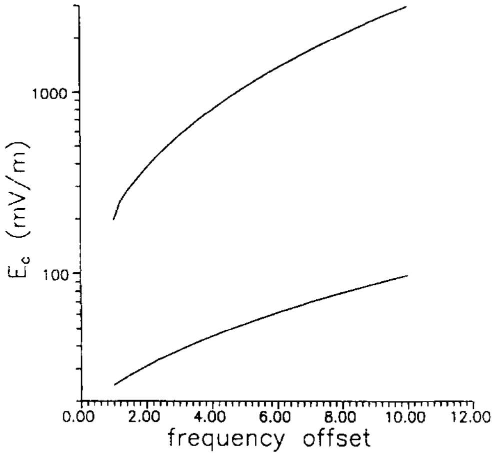

Fig. 1. Dependences of the critical field E_c on the frequency offset $\delta\omega$ at $N_m = 0.05$, $\tau_m = 3.5$ in two cases: (a) the dominant damping is due to Z-mode leakage, upper line; (b) the dominant damping is due to collisions, lower line.

dominant damping is due to Z-mode leakage E_{00} of the order of V/m and in case of collisional dissipation E_{00} an order of magnitude higher. We see from Fig. 1 that in both cases the value of E_{00} substantially exceeds the corresponding critical field level. Thus, at the initial stage of the instability the full spectrum of decay modes is excited in both cases. In the second case, the second and the third cascade processes are possible. While, in the first case, when the dissipation is due to Z-mode leakage, further cascading is more difficult.

As the instability evolves, the amplitude of the pump field E_0 diminishes coming up to a critical value. When its value reaches the maximal amplitude of the critical field, see Fig. 1, the spectrum of the decay modes will become narrowed. In the stationary state, only the first harmonic at $\delta\omega = \sqrt{\omega_{Be}\,\omega_{Bi}}$ is excited, where the critical field is minimal. The characteristic time of spectrum width vanishing can be estimated from (23) or (24), by substituting instead of E corresponding to the maximal value of the critical field E_{cmax} and putting E_c to its minimal value, see Fig. 1. This relaxation time is of the order of $t_{r1} \simeq 10^{-3}$ s or less.

The established frequency spectrum of the first harmonic could result from the density fluctuations determined by the spectrum of emitted LH waves. Detailed theory of this nonlinear process asks for a special analysis.

We emphasize that the excistence and main characteristics of the considered parametric decay processes could be seen in the plasma line of the scattered signal for a high frequency radar in the directions perpendicular to the Earth magnetic field. The possibility to observe this line was previously shown in Ref. [19]. The detailed experimental measurements of plasma line structure in a wide frequency range (~ 100 kHz) and their comparison with SEE data and here the presented theory is of significant interest.

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