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Anomalous absorption of powerful radio waves on the striations developed during ionospheric modification

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Abstract

A nonlinear theory of anomalous absorption of powerful radio waves on small scale irregularities in the ionosphere is constructed. Peculiarities of the absorption near the third electron gyrofrequency are investigated and discussed. The theory is shown to be in agreement with observations. The existence of a maximum in the probe wave absorption is predicted. Its dependence on the shift between the probe wave frequency and the pump wave frequency is determined.

1. Introduction

The generation of the small scale striations is usually considered as one of the most important new physical phenomena discovered in the ionospheric modifications. Striations determine in a large measure such processes as strong field aligned scattering of UHF and VHF radio waves (AFAS). A close connection of striations with the downshifted maximum and some other specific features of high frequency emission from the disturbed ionosphere (SEE) is also established [1-3].

Both anomalous absorption of heater radio waves and wideband attenuation (WBA) of probe waves are connected with the presence of striations. A simple linear model with arbitrary given striations has been considered to explain these processes [4]. In reality, the connection between striations and anomalous absorption is more complicated and should be nonlinear: striations determine the anomalous absorption and that in turn substantially affects the conditions of existence and the structure of striations. These effects could be most pronounced in the vicinity of a multiple gyrofrequency.

According to the theory, the creation of the striations is the result of the local heating of an anisotropic ionospheric plasma by upper hybrid resonance waves, which are generated by the linear transformation of the pump radio wave on striation density depletions. This process leads to a resonance instability which is nonlinear and has an explosive character [5-8]. The instability describes the nonstationary growth of striations. On the contrary, in experimental conditions stationary striations are usually observed and studied, but theoretically the

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steady state solution for striations was not known until recently when the nonlinear theory determining the conditions of existence and the structure of stationary striations was developed [9]. This paper is a prolongation of Ref. [9] – it is devoted to the simultaneous nonlinear description of a set of stationary striations with anomalous absorption taken into account.

2. Stationary state of striations

As in Ref. [9] we consider a magnetized ionospheric plasma with a density slowly increasing in the vertical direction. Striations are local depletions of the plasma density strongly elongated along the Earth magnetic field lines ($N - N_0 = N_1 < 0$). They are created as a result of local heating of electrons ($T - T_\infty > 0$) by the field of upper hybrid plasma waves E_1, generated inside the striations by a powerful pump radio wave E_0. Consequently, the stationary state of striations is described by a system of nonlinear stationary equations for the transport of the particle density N_1 and the electron temperature T coupled with the wave equation for the electric field of plasma waves E_1, excited in striations at the upper hybrid resonance region by the pump wave E_0.

This system has the form [4,9]

$$\nabla \hat{D} \nabla N_1 + \nabla \hat{D}_T \nabla T = \gamma N_1, \qquad \nabla \hat{\kappa} \nabla T + \frac{\boldsymbol{E}_1 \hat{\sigma} \boldsymbol{E}_1}{N_0} = \delta \nu (T - T_\infty), \qquad \nabla(\hat{\varepsilon} \boldsymbol{E}_1) = -\nabla(\delta \hat{\varepsilon} \boldsymbol{E}_0). \tag{1}$$

Here $\hat{D}$, $\hat{D}_T$, $\hat{\sigma}$, and $\hat{\kappa}$ are the tensors of diffusion, thermal diffusion, conductivity, and heat conductivity, respectively, γ is the coefficient of recombination, δ is the average part of the electron energy loss under the collision with ions and neutral particles, ν is the frequency of electron collision, T_∞ is the temperature of the plasma far from the striation – it is determined by the average heating of the background ionospheric plasma by a powerful radiowave, $\boldsymbol{E}_0$ is the electric field of the pump wave, $\hat{\varepsilon}$ is the tensor of the dielectric permittivity of the plasma, $\delta\hat{\varepsilon}$ is a part of this tensor which is the response to the density perturbation of the plasma.

We will suggest in a further analysis on the basis of previous results [9] that the density perturbation N_1/N_0 in the striations is always small while the temperature of the electrons T may be several times higher than the background value T_∞. The functions N_1 and T are assumed to depend on two variables only: on z, which is along, and on x, which is perpendicular to the direction of the magnetic field line in the $\boldsymbol{H}$, $\boldsymbol{E}_0$ plane. Following Ref. [9] we emphasize that the heat energy source $\boldsymbol{E}_1 \hat{\sigma} \boldsymbol{E}_1 / N_0$ is concentrated in a layer of the order of L between the upper hybrid resonance level $\omega = \omega_{\mathrm{UH}} = \sqrt{\omega_{\mathrm{p}}^2 + \omega_{\mathrm{eH}}^2}$ and the reflection point of the pump wave $\omega = \omega_{\mathrm{p}}$, since only inside this region the pump wave may excite upper hybrid waves within irregularities. This length scale in the F-region of the ionosphere is of the order of 1–3 km and so it is small compared to the characteristic scale of the transport processes along the magnetic field in the F-region: the longitudinal diffusion length $L_N^{\parallel} = \sqrt{D_\parallel / \gamma}$ and the longitudinal heat conductivity length $L_T^{\parallel} = \sqrt{\kappa(T_\infty)_\parallel / \delta\nu}$ [4]. In the F-region at heights $z > 200$ km the parameter γ does not depend on the temperature of the electrons T and we will assume also that the parameter δ is independent of T.

Under the conditions mentioned above the analysis of system (1) has been performed in Ref. [9]. As it was shown, using expansion in small parameters $L/L_N^{\parallel}$, $L/L_T^{\parallel}$ and $L_N^{\parallel}/L_T^{\parallel}$ enables one to reduce the first two partial differential equations describing transport processes to a set of ordinary differential equations inside the layer. System (1) then assumes the form

$$\int_0^L \frac{\sigma_\perp E_1^2}{N_0}\,\mathrm{d}z = 2T_\infty \sqrt{k_\parallel \delta \nu_0}\,\sqrt{\tfrac{4}{9}\tau_0^{9/2} - \tfrac{4}{7}\tau_0^{7/2} + \tfrac{8}{63}}\,,$$

$$\gamma N_1 = T_\infty \frac{\mathrm{d}}{\mathrm{d}x}\left(D_\perp^T \frac{\mathrm{d}\tau_0}{\mathrm{d}x}\right) - \frac{D_\parallel^T}{\kappa_\parallel}\frac{1}{L_N^{\parallel}} \int_0^L \frac{\sigma_\perp E_1^2}{N_0}\,\mathrm{d}z, \qquad \hat{\varepsilon}_{xx} E_1 = A - P_0 \frac{N_1}{N_0}, \qquad \frac{\mathrm{d}^2 A}{\mathrm{d}x^2} = \frac{E_1}{d^2}, \tag{2}$$

where $\tau_0 = T_0(x)/T_\infty$, $T_0(x)$ is the temperature inside the striation, $k_\parallel = 5.93 T_\infty/\nu_0$, $\nu_0 = \nu(T_\infty)$, $d = (k_z Y)^{-1}$, $Y = \omega_{eH}/\omega$, $\hat{\varepsilon}_{xx}$ is the transverse component of the dielectric permittivity tensor and $P_0 = E_0(\partial\hat{\varepsilon}/\partial X) X e_x$, $X = \omega_p^2/\omega^2$. The last two equations of system (2) for the plasma wave E_1 have been transformed similarly to Ref. [7].

Usually, the transverse length scale of irregularities l_x is much smaller than $(k_z \omega_{eH}/\omega)^{-1}$ and the term A can be omitted in the first approximation over the parameter $l_x k_z \omega_{eH}/\omega$. Then we can rewrite the wave equation in the following form [9],

$$\hat{\varepsilon}_{xx} E_1 = -P_0 N, \tag{3}$$

where $N = N_1/N_0$ is the normalized density of depletions. In the neighbourhood of the third electron gyrofrequency the above equation has to be written in the form [10]

$$-\delta^4 a_4 \frac{d^4}{d\tilde{x}^4} E_1 + \delta^2 a_2 \frac{d^2}{d\tilde{x}^2} E_1 + E_1(\varepsilon_{xx0} - N - i\Gamma) = -P_0 N, \tag{4}$$

where ε_{xx0} is the unperturbed part of the dielectric permittivity tensor,

$$\Gamma = \frac{\nu}{\omega} \frac{1+Y^2}{1-Y^2}$$

is the contribution from the collisions, $\tilde{x} = \int_0^x \tau^{-1/2}\, dx$, $\delta = \rho_{0eH}/l_x$ (the variable $\tilde{x}$ is renormalized on the characteristic scale l_x), $\rho_{0eH} = \rho_{eH}(T = T_\infty)$ is the electron gyroradius,

$$a_4 = \frac{15Y^4}{(1-4Y^2)(1-9Y^2)} \quad \text{and} \quad a_2 = \frac{3Y^2}{1-4Y^2}.$$

Eq. (4) describes the excitation of high frequency electrostatic oscillations propagating across a magnetic field with regard to thermal corrections. It is valid up to the neighborhood of the third electron cyclotron harmonic. The first term on the left-hand side of (4) is negligible far from the third gyrofrequency and that equation coincides with the one previously derived in Ref. [9].

As is seen from (4) there are two types of electrostatic excitations. One of them is the upper hybrid waves (UH) and the other the Bernstein mode. The analysis of the excitation of waves carried out in Ref. [10] showed that only upper hybrid waves trapped inside striations substantially contribute to the process of plasma heating.

Within the WKB approximation, the solution of (4) taking account of the dominant role of the UH waves was obtained in Ref. [10]. After averaging over the resonance width along z and small-scale UH wave oscillations it has the form

$$|E_1^2| = \frac{(-\epsilon_{xx0})^2}{\delta a_2} \frac{P_0^2}{2} \frac{\pi l}{z_U[1 + (2a_4/a_2) z_U^2]} \frac{1}{\sqrt{\varepsilon_1^2 + \varepsilon_2^2}}, \tag{5}$$

where

$$z_U = \left(\frac{1 - \sqrt{1 + pq}}{-p(\frac{1}{2}a_2)} \right)^{1/2}, \qquad p = \frac{20}{3} \frac{1-4Y^2}{1-9Y^2}, \qquad q = \varepsilon_{xx0} - N - i\Gamma,$$

$$\varepsilon_1 = -\int_{-x_0}^{x_0} \frac{\Gamma}{\delta} \frac{dz_U}{dq}\bigg|_{q = \varepsilon_{xx0} - N} d\tilde{x}, \qquad \varepsilon_2 = \pi \tilde{l} \frac{\omega_{eH}}{c\sqrt{-\varepsilon_{xx0}}},$$

$$l = \left(\frac{dN}{d\tilde{x}} \right)^{-1} \bigg|_{\tilde{x} = x_0} = \frac{\tilde{l} \tau_0^{-1/2}}{l_x(-\varepsilon_{xx0})}, \qquad \tilde{l} = \left(\frac{dN}{dx} \frac{1}{N} \right)^{-1} \bigg|_{\tilde{x} = x_0}.$$

$\pm x_0$ is the turning point of the UH waves trapped inside the striation, where $\varepsilon_{xx0} = N(x_0)$ (for simplicity the striations are assumed to be symmetrical around $\tilde{x} = 0$). The terms ε_1 and ε_2 in the denominator of (5) allow for dissipation of UH waves due to collisions and the so-called Z mode leakage, respectively. The latter effect is due to incomplete reflection of trapped UH waves near the turning points $\pm x_0$ at a finite value of the longitudinal wave number k_z. A small part of the energy is transformed outside the turning points and propagates away as long electromagnetic waves. To determine this damping it is necessary to take into consideration the following corrections over the parameter $l_x k_z \omega_{eH}/\omega$ connected with the neglected term A in Eq. (2) (see Ref. [11]).

Taking the integral over the height z in the heating layer taking account of (5) we arrive at the following relation for the heating source in (2) (for details see Ref. [10]),

$$\int_0^L \frac{\sigma_\perp E_1^2}{N_0}\,\mathrm{d}z = \frac{Q_0}{N_0}\tau_0^{-3/2}\int_0^{-N} \frac{\mathrm{d}(-\varepsilon_{xx0})(-\varepsilon_{xx0})^2\Theta(1+pq)}{\sqrt{1+pq}\left[\left(1-\sqrt{1+pq}\right)/\left(-\frac{1}{2}pa_2\right)\right]^{1/2}} \frac{P_0^2(-\varepsilon_{xx0})}{P_0^2(0)} \frac{l}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}},$$
$$Q_0 = \frac{\nu_0}{4}\frac{P_0^2(0)L_0}{\delta a_2}\frac{1+Y^2}{1-Y^2}. \tag{6}$$

where N is the normalized density of the depletions (3), L_0 is the length scale of the vertical plasma density variation and

$$\Theta(x) = 1 \quad \text{if } x > 0,$$
$$\quad\;\; = 0 \quad \text{if } x < 0.$$

Eq. (6) together with (2) completely define the stationary state of the striations.

For the problem of one isolated striation Eq. (6) can be simplified by neglecting the absorption of the pump wave energy flux due to the conversion into the UH wave energy. In that case the function P_0 is a constant independent of z.

Furthermore, under the ionospheric modification experiment for every isolated striation

$$\varepsilon_2 \gg \varepsilon_1, \tag{7}$$

and the Z mode leakage effect gives the main contribution to the damping mechanism defining the amplitude of the electric field. Within this approximation the stationary state of the striations was analyzed in Refs. [9,10].

The assumption of a single isolated striation makes it possible to find the form of and the conditions for the existence of the striations. To study anomalous absorption the theory should be extended to include the case of a set of striations.

3. Anomalous absorption on a group of striations

The problem of the stationary state of striations is changed substantially if a set of irregularities is present. First of all, the anomalous absorption of the pump wave energy flux due to conversion of part of the energy into UH waves becomes important. Indeed, it is known that the existence of even 1–2% density depletions arranged close together give rises to an almost total absorption of the pump field energy flux [4,7]. The variable P_0 in (6) then is no longer a constant being a function of the height z. Its spatial variation with z is governed by the energy conservation law, averaged over time,

$$\frac{\mathrm{d}S}{\mathrm{d}z} = -\langle S_Z\rangle_x - \langle Q_l(x, z)\rangle_x, \tag{8}$$

where S is the pump field energy flux incident on the heating layer, $\langle S_Z\rangle_x$ and $\langle Q_l(x, z)\rangle_x$ are the horizontal average of the Z mode energy outgoing flux and the UH wave dissipation by collisions, respectively.

Another process which should be revised is the damping of the UH waves. In case of a group of irregularities we deal with the problem of many oscillators coupled through the long wavelength Z mode radiated by themselves. They have to be considered jointly to determine the total outgoing flux per striation. If the number of striations in the group were large, the Z mode radiation would be absorbed deep inside by means of the repeated inverse process of the transformation of the Z mode into the UH waves. It is clear that in such a case the Z mode leakage takes place effectively only at the boundary of the group. That is why the total amount of the UH wave energy radiated from the group of striations may be reduced substantially, so that condition (7) may not be satisfied. In particular, within the one-dimensional model this reduction can be effectively allowed for by the factor $1/N_s$ compared to the case of the isolated striation, where N_s is the total number of irregularities in the group. It can be introduced by substituting $\varepsilon_2 = \pi \tilde{l} \omega_{eH}/c\sqrt{-\varepsilon_{xx0}}$ in (5) by

$$\varepsilon_2 = \frac{\pi \tilde{l}}{N_s} \frac{\omega_{eH}}{c\sqrt{-\varepsilon_{xx0}}}. \tag{9}$$

The factor $\varepsilon_2/\varepsilon_1$ for the ionospheric modification experiments for every single irregularity is of the order of 40. That is, already for $N_s^0 \simeq 40$ both mechanisms of UH waves damping (collisional and Z mode leakage) inside the striation are of the same order and condition (7) is not satisfied. For $N_s \gg N_s^0$, Z mode leakage vanishes and collisional damping dominates. As a consequence, the total losses from one striation are also diminished.

To include the absorption effect Eq. (6) has to be examined together with (8). The relation for the total Z mode leakage energy flux S_Z in the right hand side of (8) can be defined through the UH wave energy flux into the striation S_{UH} as follows: $S_Z = 2S_{UH}T$, where $T = 2\varepsilon_2$ is the coefficient of the transformation of UH waves into the Z mode near the turning points $\pm x_0$ [11]. On the other hand the UH energy flux S_{UH} can be directly obtained from (4) by putting this equation in the form div $\boldsymbol{S}_{UH} = \boldsymbol{j} \cdot \boldsymbol{E}$. If the wavelength of the excited wave is less than the characteristic scale of the striations we have

$$S_{UH} = i\omega l_x \tau_0^{1/2} \left[\delta^4 a_4 \left\{ E_{1+}^{\prime\prime\prime} E_{1+}^* - E_{1+}^{*\prime\prime\prime} E_{1+} - E_{1+}^{\prime\prime} E_{1+}^{*\prime} + E_{1+}^{*\prime\prime} E_{1+}^{\prime} \right\} - \delta^2 a_2 \left(E_{1+}^{\prime} E_{1+}^* - E_{1+}^* E_{1+}^{\prime} \right) \right],$$

where E_{1+} means that we chose one of the two counter-propagating modes. Then in the WKB approximation

$$S_{UH} = \omega l_x \delta a_2 \tau_0^{1/2} z_U \left[1 + (2a_4/a_2) z_U^2 \right] |E_1|^2. \tag{10}$$

From (10) we finally obtain

$$S_Z = \frac{\pi \varepsilon_2 \omega \tilde{l}}{\sqrt{\varepsilon_1^2 + \varepsilon_2^2}} P_0^2(-\varepsilon_{xx0}). \tag{11}$$

The pump field energy flux S can also be expressed in terms of P_0^2, $S = \alpha_s P_0^2$. In the general case α_s is a function of the angle between the direction of the pump wave propagation and the magnetic field. For the ordinary mode propagating along the magnetic field

$$\alpha_s = \frac{(1+Y)^2}{X^2} \frac{1+Y-X}{1+Y} \frac{c}{\pi}.$$

Since all functions in (8) are directly proportional to P_0^2, Eq. (8) presents a simple differential equation. Integrating it we obtain

$$P_0^2(-\varepsilon_{xx0}) = P_0^2(0) \exp\left[-\frac{\alpha L_0 \omega}{\alpha_s} \int_0^{-\varepsilon_{xx0}} d\xi \left(\frac{\tilde{S}_Z}{l_x} + \int_{x_2}^{x_1} dx \frac{\tilde{Q}_l(x,\xi)}{l_x} \right) \right], \quad |x_2 - x_1| \sim l_x, \tag{12}$$

where the constant α represents the density distribution of the striations (occupation number, $\alpha \leqslant 1$),

$$\tilde{S}_Z = \frac{\pi \varepsilon_2 \omega \tilde{l}(-\varepsilon_{xx0})}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}}, \qquad \tilde{Q}_l(x, -\varepsilon_{xx0}) = \frac{\tilde{Q}_0 \tau_0^{-3/2}(-\varepsilon_{xx0})^2 \Theta(1+pq) l}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}\sqrt{1+pq}\left[\left(1-\sqrt{1+pq}\right)/\left(-\frac{1}{2}pa_2\right)\right]^{1/2}}.$$

$$\tilde{Q}_0 = \frac{\nu_0}{4\omega\delta a_2}\frac{1+Y^2}{1-Y^2}.$$

The above equation together with (6) define the heat energy source in every striation of the group. It is

$$\int_0^L \frac{\sigma_\perp E_1^2}{N_0}\,\mathrm{d}z = \frac{Q_0}{N_0}\tau_0^{-3/2}\int_0^{-N} \frac{\mathrm{d}(-\varepsilon_{xx0})(-\varepsilon_{xx0})^2\Theta(1+pq)}{\sqrt{1+pq}\left[\left(1-\sqrt{1+pq}\right)/\left(-\frac{1}{2}pa_2\right)\right]^{1/2}}\frac{l}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}}\exp[\phi(-\varepsilon_{xx0})], \tag{13}$$

The term in the exponent in (12) is exactly the anomalous absorption of an incident wave

$$\phi(-\varepsilon_{xx0}) = -\frac{\alpha L_0 \omega}{\alpha_s}\int_0^{-\varepsilon_{xx0}} \mathrm{d}\xi\left(\frac{\tilde{S}_Z}{l_x} + \int_{x_2}^{x_1}\mathrm{d}x\frac{\tilde{Q}_l(x, \xi)}{l_x}\right). \tag{14}$$

Below we will consider a model problem with a finite scale of the heating zone along the x direction. For simplicity the pump wave is supposed to propagate along the magnetic field.

For further calculations let us introduce the effective heating parameter

$$\eta = \frac{Q_0}{2T_\infty N_0 l_x\sqrt{\delta\nu_0 k_\parallel}} \tag{15}$$

and pass over from x to the dimensionless variable

$$X = x/l_0, \qquad l_0 = \left(\frac{D_\perp^2 k_\parallel}{\delta\nu_{0\gamma} D_\parallel}\right)^{1/4}.$$

The characteristic length l_0 is determined by the processes of transverse diffusion during the characteristic relaxation time of the electron temperature $(\delta\nu_0)^{-1}$ and the plasma density γ^{-1}. For the F-region conditions, the parameter $l_0 \sim 5$–10 m. Here $D_\perp$, $D_\parallel$ and $k_\parallel$ are the transport coefficients calculated for $T = T_\infty$.

As in Refs. [9,10] we put system (2) in the form of a nonlinear oscillator equation,

$$\frac{\mathrm{d}^2 y}{\mathrm{d}X^2} = f(y), \qquad f(y) = -\frac{y^5\sqrt{\frac{4}{9}y^{-9}-\frac{4}{7}y^{-7}+\frac{8}{63}}}{2^{3/2}\sqrt{1+y^{-2}}} - \frac{N}{2\varepsilon}, \tag{16}$$

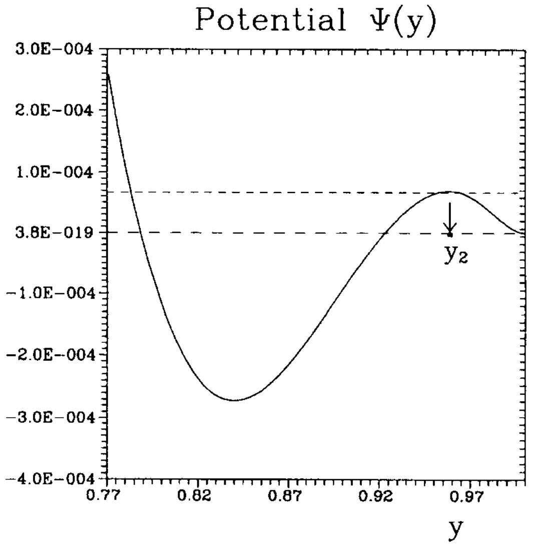
where the function $N(y)$ is implicitly defined by

$$\frac{y^{-4}\sqrt{\frac{4}{9}y^{-9}-\frac{4}{7}y^{-7}+\frac{8}{63}}}{\eta} = \int_0^{-N}\frac{\mathrm{d}(-\varepsilon_{xx0})(-\varepsilon_{xx0})\Theta(1+pq)}{\sqrt{1+pq}\left[\left(1-\sqrt{1+pq}\right)/\left(-\frac{1}{2}pa_2\right)\right]^{1/2}}\frac{\tilde{l}}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}}\exp[\phi(-\varepsilon_{xx0})].$$

Note that y takes values in the interval $0 < y < 1$.

Because the size of the heating zone along x is larger than the characteristic scale of striations l_0 we will seek a solution of (16) on the infinite interval while keeping the number N_s finite. Then, integrating (16) we obtain

$$\frac{1}{2}\left(\frac{\mathrm{d}y}{\mathrm{d}X}\right)^2 = \int_{\tilde{y}}^{y} f(y)\,\mathrm{d}y = -\Psi(y). \tag{17}$$



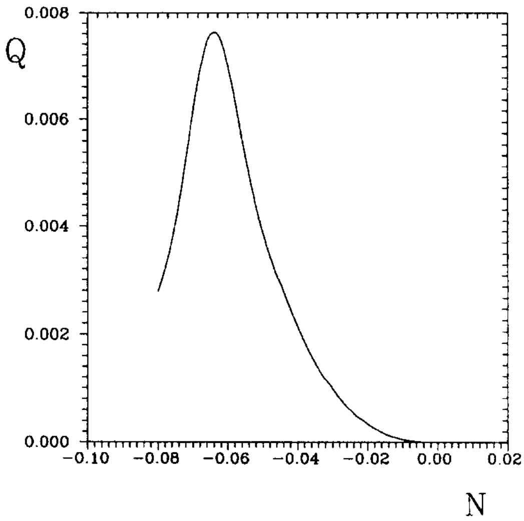


Fig. 1. Dependence of the "potential" $\Psi(y)$ on $y=\tau_0^{-1/2}$. The point $y=y_2$ is shown by an arrow.

Fig. 2. Dependence of the heating power source Q (arbitrary units) defined by (13) on the normalized density of depletions N.

The "potential" $\Psi(y)$ is shown in Fig. 1. The function $\Psi(y)$ reaches its local maximum value at $y=y_2$, where y_2 is the root of the equation $f(y)=0$.

The dotted line in Fig. 1 relates to the soliton-type solution

$$\int_{\tilde{y}}^{y} \frac{\mathrm{d}y_1}{\sqrt{2\int_{\tilde{y}}^{y_1} f(\xi)\,\mathrm{d}\xi}} = X, \tag{18}$$

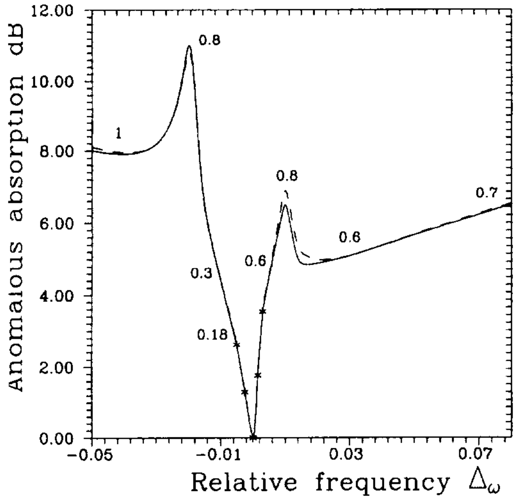
where $\tilde{y} \leqslant y \leqslant y_2$ and the parameter $\tilde{y}$ is determined by the relation

$$\int_{\tilde{y}}^{y_2} f(y)\,\mathrm{d}y = 0. \tag{19}$$

This parameter $\tilde{y}$ corresponds to the maximum temperature in the center of the striation, i.e where $X=0$, $\tau_{max}=1/\tilde{y}^2$.

It should be noted that, as it is seen from Fig. 1 for the existence of a solution, a small average heating is needed, $y \leqslant y_2$. The origin of this additional heating should be connected with the absorption of the pump field due to collisions. It is small compared with UH wave heating inside the striation in the parameter $|E_0|^2/|E_1|^2 \ll 1$. Within this approximation we put $y_2=1$.

In fact, the occupation number α is a free parameter of the problem. It is natural to set it at its maximal value α_{max}. One should remark that this number α_{max} is not always equal to unity. To illustrate this fact let us consider the dependence of the heating power Q, defined by (13), on the normalized density N. This is shown in Fig. 2. The maximum which is observed belongs to the maximal possible value of density N for given α. Such a behavior has a simple interpretation in the frame of the obtained solution (5). The heating peaks near the turning points $\pm x_0$. That is, if absorption is strong or the depth of the striation is sufficiently large, the heating power source concentrates before the striation center, as is shown in Fig. 2. One can now understand that if the stationary value of the density is larger than the maximal one, then there is no solution for this α. Decreasing the occupation number α we at last attain the value at which the stationary solution appears. This is just the maximum of the occupation number α_{max}.



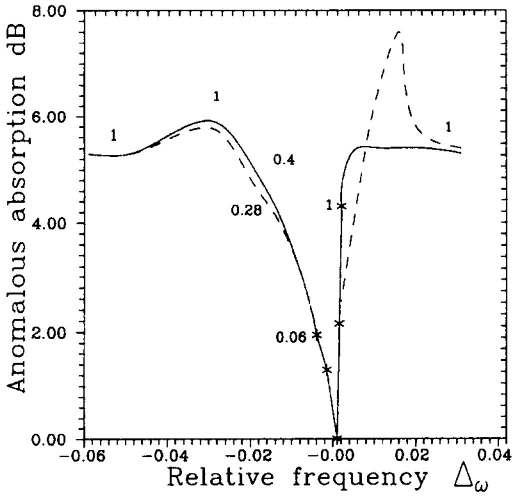


Fig. 3. Dependence of anomalous absorption of the probe (dashed line) and heater (solid line) waves on the relative frequency of the pump field $\Delta_\omega = (\omega - 3\omega_{eH})/3\omega_{eH}$ near the third electron cyclotron gyrofrequency. $N_s = 100$, $\delta_\omega = -0.019$. The maximal occupation number α_{max} at a given frequency is shown by the numbers along the curves. Asterisks show the extension of the theoretical dependences to the point $\omega = 0$.

Fig. 4. Dependence of anomalous absorption of the probe (dashed line) and heater (solid line) waves on the relative frequency of the pump field $\Delta_\omega = (\omega - 3\omega_{eH})/3\omega_{eH}$ near the third electron cyclotron gyrofrequency. $N_s = 1000$, $\delta_\omega = -0.019$. The maximal occupation number α_{max} at a given frequency is shown by the numbers along the curves. Asterisks show the extension of the theoretical dependences to the point $\omega = 0$.

In principle a solution of (16) exists for $\alpha < \alpha_{max}$ but it is natural to expect that for small α the solution is unstable and the number of striations will increase up to the value defined by α_{max}.

There is another parameter of the problem, the total number of striations N_s in a group, which describes the large scale structuring of a plasma in the ionospheric modification. In general this structure is determined by the joint action of the processes of self-focusing of the heater wave and its anomalous absorption [9,12]. We do not consider self-focusing here and take N_s (or the scale of the heating zone) as a free parameter of the problem.

We will consider two limit cases that are distinct in the number of striations in a group: $N_s \simeq N_s^0$ (i.e. both mechanisms of UH wave damping give almost the same contribution) and $N_s \gg N_s^0$ (i.e. collisional damping prevails). As we have already mentioned above these two limit cases vary also in the total UH wave energy damping for each striation, it is smaller in the second case. That is why it is anticipated that anomalous absorption increases with the reduction of the number N_s. Note that another case, $N_s \ll N_s^0$, when Z mode leakage damping dominates, has been considered previously in Ref. [10].

We produce a numerical simulation of Eq. (16) and use the solution obtained to calculate anomalous absorption (14). We have taken into account that the incident wave passes the heating zone twice. The result is, as is expected, that anomalous absorption is smaller in the second case. This is shown in Figs. 3, 4 (solid lines) as a function of the pump wave frequency at a given value of the pump field intensity $E_0 \simeq 150$ mV/m, or in terms of the parameters introduced above $\sqrt{\eta}\,\varepsilon = 8.60$. The characteristic value of anomalous absorption, far from cyclotron resonance, constitutes 8 dB and 5 dB in the first and second cases, respectively.

Of special interest is the frequency range near the multiple electron gyrofrequency where anomalous absorption has a variety of specific features [13]. A decrease of the absorption as the pump frequency increases towards cyclotron frequency $\omega < 3\omega_{eH}$ is seen in both cases; Figs. 3, 4 (solid line). This is mainly dictated by the vanishing of the occupation number, which is shown by the numbers along the curves. The observed maximum in that region corresponds to the point of most effective excitation of UH waves (this effect has been discussed in detail in Ref. [10]). In the range above the cyclotron frequency $\omega > 3\omega_{eH}$ the anomalous absorption versus this frequency has a different behavior in the foregoing two cases; see Figs. 3, 4 (solid line). In the first case it peaks at a maximum value and then drops. As is seen this peculiarity is due to the increase of the parameter α. While in the second case the anomalous absorption is constant almost up to the neighbourhood of cyclotron resonance and then dramatically decreases. For exact cyclotron resonance, i.e. $\omega = 3\omega_{eH}$, the anomalous absorption is precisely equal to zero because of the strong Landau damping of the plasma waves. The theory does not describe the behavior in the immediate vicinity of the resonance point (see Ref. [10]) (where other effects should be taken into account, for instance Z mode leakage) and the curves are schematically extended up to the point $\omega = 3\omega_{eH}$. This part of the dependences is marked by the asterisks in the figures.

The variation of the total number of striations N_s in a group giving rise to the total energy loose variations should also substantially affect the characteristic parameters of the striations such as its maximal temperature. It constitutes $T_c \sim 2.5\tau_0$ in the first case and $T_c \sim 5\tau_0$ in the second one. At the same time one can remark that the density is almost the same in both cases, $N_c \sim 3\%$. The increase of temperature is supposed to be due to reducing the UH wave damping stemming from Z mode leakage when the number of striations increases. One should note that the characteristic depth of the striation in a group N_c is less than for one isolated striation for the same value of the pump field.

In line with the anomalous absorption of the heater wave the wideband attenuation of a probe wave also provides important information [13]. Usually, the diagnostic wave has a frequency 70 kHz below the pump wave frequency [13]. We consider attenuation of a probe wave using a solution of (16) with an arbitrary small frequency shift $\delta_\omega = (\omega_d - \omega)/3\omega_{eH} \ll 1$ of a probe wave of frequency ω_d versus the frequency of the heater wave ω. The attenuation of the probe wave is defined by (14) as well as the anomalous absorption of the heater wave. For the probe wave, however, the function $\phi(-\varepsilon_{xx0})$ in (14) depends both on the probe wave frequency ω_d (explicitly) and implicitly, through the parameters of the resulting striation, on the pump wave frequency ω. It is clear that if the condition $\delta_\omega = 0$ is satisfied exactly, anomalous absorption of the probe wave coincides with anomalous absorption of the pump field. As we will see the differences appear when $\delta_\omega \neq 0$. One can expect that because $\delta_\omega \ll 1$ they should be most pronounced in the vicinity of the electron gyrofrequency. As before we consider two limit cases, $N_s \simeq N_s^0$ and $N_s \gg N_s^0$. As is seen in Fig. 3 (dashed line) the behaviour of the diagnostic wave attenuation slightly depends on the frequency shift δ_ω for small values of $N_s \simeq N_s^0$ and

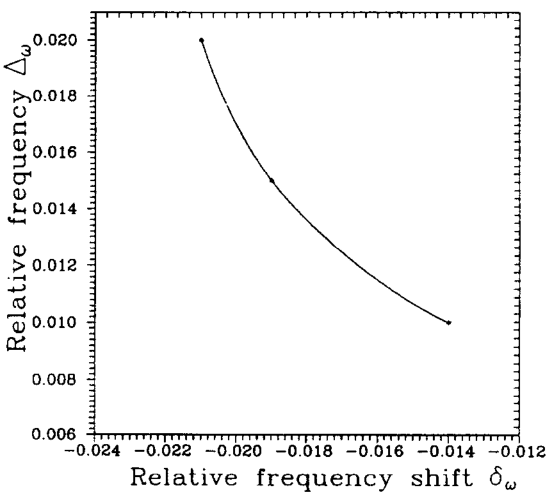


Fig. 5. Dependence of the position of the maximum of the probe wave absorption on the relative frequency shift δ_ω.

follows the dependence obtained at $\delta_\omega = 0$. This means that in this case Z mode leakage which does not have a peculiarity at $\omega = 3\omega_{eH}$ gives a larger contribution to the damping of UH waves. On the other hand, in the opposite case, $N_s \gg N_s^0$, an absorption peak appears; see Fig. 4 (dashed line). This fact is in qualitative agreement with the experimental data [13]. The position of the maximum follows the frequency shift δ_ω of the probe wave. This dependence is shown in Fig. 5. The origin of this peculiarity is due to the relevant choice of the relative frequency shift, which is negative and in our case equal to $\delta_\omega = -0.019$ or $\omega_d - \omega \simeq 70$ kHz. One should note that a shift with the same value has been used in experiment [13]. For a negative shift $\delta_\omega < 0$ it turns out that while the heater wave creates a striation structure at frequencies above cyclotron resonance the probe wave interacts with this structure at frequencies below the resonance value, in the region of the most effective UH wave excitation; see Fig. 4 of Ref. [10]. As is seen from Figs. 3, 4, in this region ($\omega < 3\omega_{eH}$) anomalous absorption also increases. That is why the position of maximal absorption of a probe wave is the displaced maximum of UH wave excitation. One should expect that this phenomenon can be observed only for $\delta_\omega < 0$.

So, we see that nonlinear theory predicts the existence of a maximum in the probe wave absorption. One can suppose that this maximum has already been observed in Ref. [13]. A quite definite motion of this maximum with the probe wave shift is determined. The experimental investigation of the latter effect is of substantial interest.

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