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## Stationary state of isolated striations developed during ionospheric modification

A.V. Gurevich<sup>1</sup>, A.V. Lukyanov<sup>1</sup>, K.P. Zybin<sup>1</sup>

*Max-Planck-Institut für Aeronomie, D-37191 Katlenburg-Lindau, Germany*

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### Abstract

A nonlinear theory determining for the first time the conditions for existence and the structure of the stationary striations generated in the ionospheric modifications by powerful radio waves is developed. A strong enhancement of the electron temperature inside the striations is predicted. A considerable change of the amplitude of striations and its dramatic diminishing in the narrow region near the electron cyclotron resonance is established. The structure of the density depletions and its characteristic length, width and depth are shown to be in agreement with the observations.

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### 1. Introduction

One of the most essential new physical phenomena, discovered during ionospheric modification by powerful radio waves, was the generation of small scale striations which are plasma density depletions strongly elongated along the Earth's magnetic field. The striations determine both the effective field aligned scattering of UHF and VHF radio waves (AFAS) and the anomalous wideband attenuation of HF radio waves (WBA) in the disturbed region [1,2]. The close connection of the striations with high frequency (SEE) and low frequency (LFE) emission of the disturbed ionosphere [3–5] is also established.

Considerable changes in the structure of SEE and WBA were found under the action at the ionosphere of radio waves with a frequency near the multiple gyrofrequency [6]. Both the downshifted maximum and the anomalous absorption are strongly diminishing in the narrow region of the order of ten kHz [7] and even less [8] in the vicinity of the multiple gyroresonance.

According to the theory, creation of the striations is a result of the local heating of the anisotropic ionospheric plasma by upper hybrid resonance waves, which are generated due to the linear transformation of the pump radio wave at the striation density depletions. This process leads to the resonance (or thermal parametric) instability [9,10]. The resonance instability is nonlinear and has an explosive character [11–14]. These main features of the instability are confirmed in ionospheric experiments [15].

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<sup>1</sup> On leave of absence from P.N. Lebedev Institute of Physics, Russian Academy of Sciences, 117924 Moscow, Russian Federation.

A fundamental problem is the nonlinear stationary state which is established after the full development of the resonance instability. Exactly this stationary state determines the main characteristics of various nonlinear phenomena in the ionosphere, which were explored using different radio methods (AFAS, WBA, SEE, LFE). Moreover, the striations were recently observed directly in experiments in situ on board rockets [16]. The striations have been seen as essentially local stationary depletions of the plasma density, with scales of the order of 10 meters across and several kilometers along the magnetic field lines.

In the present paper the theory of the stationary state of the striations in the ionospheric plasma will be developed.

## 2. Simplification of nonlinear transport equations

As was mentioned above, the striations are local depletions of the density of plasma particles ( $N - N_0 = N_1 < 0$ ), created as a result of local heating of electrons ( $T - T_\infty > 0$ ) by the field of upper hybrid plasma waves  $E_1$ , generated by a powerful pump radio wave  $E_0$ . Consequently, the stationary state of striations is described by a system of nonlinear stationary transport equations for particle density  $N_1$ , electron temperature  $T$  and wave equation for the electric field of plasma waves  $E_1$ , excited in striations at the upper hybrid resonance region by the pump wave  $E_0$ .

This system has the form [17],

$$\nabla \hat{D} \nabla N_1 + \nabla \hat{D}_T \nabla T = \gamma N_1, \quad \nabla \hat{\kappa} \nabla T + \frac{E_1 \hat{\sigma} E_1}{N_0} = \delta \nu (T - T_\infty), \quad \nabla (\hat{\epsilon} E_1) = -\nabla (\delta \hat{\epsilon} E_0). \quad (1)$$

Here  $\hat{D}$ ,  $\hat{D}_T$ ,  $\hat{\sigma}$ , and  $\hat{\kappa}$  are the tensors of diffusion, thermal diffusion, conductivity, and heat conductivity, respectively,  $\gamma$  is the recombination coefficient,  $\delta$  is the average part of electron energy loss under collisions with ions and neutrals,  $\nu$  is the frequency of electron collision,  $T_\infty$  is the temperature of the plasma far from the localized striation (it is determined by the average heating of the ionospheric plasma by a powerful radio wave),  $E_0$  is the electric field of the pump wave,  $\hat{\epsilon}$  is the tensor of the dielectric permittivity of the plasma,  $\delta \hat{\epsilon}$  is the part of this tensor which is the response to the density perturbation of the plasma.

We will suggest in a further analysis on the basis of previous results [17] that the density perturbation  $N_1/N_0$  in the striations is small while the electron temperature  $T$  may be several times higher than the background value  $T_\infty$ .

To begin with, let us consider the case when the functions  $N_1$  and  $T$  depend on two variables only: on  $z$  and on  $x$ , which are along and perpendicular to the direction of the magnetic field line in the  $\mathbf{H}$ ,  $E_0$  plane, respectively. Then the system of equations takes the form

$$\frac{\partial}{\partial x} \left( D_\perp \frac{\partial N_1}{\partial x} \right) + \frac{\partial}{\partial z} \left( D_\parallel \frac{\partial N_1}{\partial z} \right) + \frac{\partial}{\partial x} \left( D_\perp^T \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial z} \left( D_\parallel^T \frac{\partial T}{\partial z} \right) = \gamma N_1, \\ \frac{\partial}{\partial x} \left( \kappa_\perp \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial z} \left( \kappa_\parallel \frac{\partial T}{\partial z} \right) + \frac{\sigma_\perp E_1^2}{N_0} = \delta \nu (T - T_\infty), \quad \hat{\epsilon}_{xx} E_1 = A - P_0 \frac{N_1}{N_0}, \quad \frac{d^2 A}{dx^2} = \frac{E_1}{d^2}, \quad (2)$$

where  $d = (k_z Y)^{-1}$ ,  $Y = \omega_{eH}/\omega$ ,  $\epsilon_{xx}$  is the transverse component of the dielectric tensor,  $\omega_H$  is the electron gyrofrequency,  $\omega$  and  $k_z$  are the frequency and the wave vector of the pump wave,  $\omega_p$  is the Langmuir plasma frequency, and  $P_0 = E_0 (\partial \hat{\epsilon} / \partial X_p) X_p e_x$ ;  $X_p = \omega_p^2 / \omega^2$ . The last equation of system (2) for the upper hybrid wave  $E_1$  was transformed similarly to Ref. [12].

Let us now emphasize that the heat energy source  $\sigma_{\perp} E_1^2$  is concentrated in the layer of order  $L$  between the upper hybrid resonance level  $\omega = \omega_{\text{UH}} = \sqrt{\omega_p^2 + \omega_H^2}$  and the reflection point of the pump wave  $\omega = \omega_p$ , since only inside this region the pump wave may excite upper hybrid waves within irregularities. This length scale in the F-region of the ionosphere is of the order of 1–3 km and so it is small compared to the characteristic scale of the transport processes along the magnetic field in the F-region: the longitudinal diffusion length  $L_N^{\parallel} = \sqrt{D_{\parallel}}/\gamma$  and the longitudinal heat conductivity length  $L_T^{\parallel} = \sqrt{\kappa(T_{\infty})_{\parallel}}/\delta\nu$  [17]. In the F-region at heights  $z > 200$  km the parameter  $\gamma$  does not depend on the electron temperature  $T$  and we will assume also that the parameter  $\delta$  is independent of  $T$ .

We will seek the solution of system (2) in the form of expansion by small parameters  $L/L_N^{\parallel}$  and  $L/L_T^{\parallel}$ ,

$$T = T_0 + T_1 + \dots, \quad N = N_0 + N_1^{(0)} + N_1^{(1)} + \dots, \quad T_1 \ll T_0, \quad N_1^{(1)} \ll N_1^{(0)} \ll N_0.$$

Then, for the heat transfer equation we have in different orders of the parameter  $(L/L_T^{\parallel})$

$$\begin{aligned} (L/L_T^{\parallel})^0: \quad & \frac{\partial}{\partial z} \left( \kappa_{\parallel} \frac{\partial T_0}{\partial z} \right) = 0, \quad T_0 = T_0(x), \\ (L/L_T^{\parallel})^1: \quad & \frac{\partial}{\partial z} \left( \kappa_{\parallel} \frac{\partial T_1}{\partial z} \right) = - \left[ \frac{d}{dx} \left( \kappa_{\perp} \frac{dT_0}{dx} \right) + \sigma_{\perp} \frac{E_1^2}{N_0} \right]. \end{aligned} \quad (3)$$

Assuming that  $z = 0$  is the point of upper hybrid resonance and taking the integral over  $z$  from  $z = 0$  to the reflection point of the pump wave  $z = L$  we obtain from (3) the following relation for the full heat flux flowing out from the heated layer,

$$\kappa_{\parallel} \frac{\partial T_1}{\partial z} \Big|_0^L = -Q, \quad (4)$$

where

$$Q = L \left[ \frac{d}{dx} \left( \kappa_{\perp} \frac{dT_0}{dx} \right) + \frac{1}{L} \int_L^0 \frac{\sigma_{\perp} E_1^2}{N_0} dz \right].$$

Making the same expansion for the density  $N_1$ , one has

$$D_{\parallel} \frac{\partial N_1^{(1)}}{\partial z} \Big|_0^L + D_{\parallel}^T \frac{\partial T_1}{\partial z} \Big|_0^L = -L \left( \frac{d}{dx} D_{\perp} \frac{dN_1^{(0)}}{dx} + \frac{d}{dx} D_{\perp}^T \frac{dT_0}{dx} - \gamma N_1^{(0)} \right) = Q_1. \quad (5)$$

We will consider two separate zones: one of them lies in the heating layer  $x \in [0, L]$  and another one in the region outside it. To match the equations inside and outside the heating area, it is necessary to examine the external problem and match the corresponding particle (5) and heat fluxes (4).

Considering the external problem, it is possible to assume, according to the small parameter  $(L/L_T^{\parallel})$ , that the length scale of the heating layer is negligible. Under this condition, the heat transfer equation attains the form

$$\frac{\partial}{\partial x} \left( \kappa_{\perp} \frac{\partial \tau}{\partial x} \right) + \frac{\partial}{\partial z} \left( \kappa_{\parallel} \frac{\partial \tau}{\partial z} \right) - \delta\nu(\tau - 1) = \frac{Q}{T_{\infty}} \delta(z). \quad (6)$$

Here  $\delta(z)$  is the Dirac delta function and  $\tau = T/T_{\infty}$ . The term on the right hand side,  $Q\delta(z)$ , is responsible for the heat flux (4). The coefficients  $\kappa_{\perp}$  and  $\kappa_{\parallel}$  are functions of the temperature of the electrons. Taking into account that in the F-region the collisions of electrons with ions play a leading role, we have:  $\kappa_{\perp} = k_{\perp} \tau^{-1/2}$ ,  $\kappa_{\parallel} = \kappa_{\parallel} \tau^{5/2}$ ,  $k_{\parallel} = 5.93 T_{\infty} / \nu_0$ ,  $k_{\perp} = 1.77 T_{\infty} \nu_0 / m \omega_H^2$ ,  $\nu_0 = \nu_e(T_{\infty})$ .

It is important that in the stationary solution of Eq. (6) the characteristic transverse length scale  $x_0$  is governed by the dimension of the source  $Q$ , while the longitudinal length scale of striations results from the

balance between the second and third terms in the left hand side of the equation. The last statement is correct for a sufficiently high temperature when

$$\tau_{\max} \gg 1/\xi_0^4, \quad \xi_0 = x_0 \sqrt{\delta\nu_0/k_{\perp}}. \quad (7)$$

As we shall see below, relation (7) is usually satisfied. Neglecting therefore the first term in Eq. (6), we can easily integrate it with respect to  $dz$  with a natural boundary condition  $\partial\tau/\partial z = 0$  at  $\tau = 1$ . Using relation (3), we then obtain the value  $Q$ ,

$$Q = 2T_{\infty} \sqrt{k_{\parallel} \delta\nu_0} \sqrt{\frac{4}{9}\tau_0^{9/2} - \frac{4}{7}\tau_0^{7/2} + \frac{8}{63}}. \quad (8)$$

Substituting this expression into (4) we get the heat balance equation in the layer. It takes the form

$$\frac{d}{dx} \kappa_{\perp} \frac{dT_0}{dx} + \frac{1}{L} \int_0^L \frac{\sigma_{\perp} E_1^2}{N_0} dz = \frac{2}{L} T_{\infty} \sqrt{k_{\parallel} \delta\nu_0} \sqrt{\frac{4}{9}\tau_0^{9/2} - \frac{4}{7}\tau_0^{7/2} + \frac{8}{63}}. \quad (9)$$

The first term in Eq. (9) corresponds to the heat flow due to the transverse heat conductivity, the second one describes the plasma heating in the layer by the electric field of the plasma wave and the right hand side is responsible for the heat losses from the layer determined by the longitudinal heat conductivity.

Let us now examine the particle transfer. In the exterior region Eq. (2) has the form

$$\frac{\partial}{\partial x} \left( D_{\perp} \frac{\partial N_1}{\partial x} \right) + \frac{\partial}{\partial x} \left( D_{\perp}^T \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial z} \left( D_{\parallel} \frac{\partial N_1}{\partial z} \right) + \frac{\partial}{\partial z} \left( D_{\parallel}^T \frac{\partial T}{\partial z} \right) - \gamma N_1 = Q_1 \delta(z). \quad (10)$$

Note that the transverse length scale in (10), as well as in (6) is defined by the right hand side of the equation, the transverse scale of the temperature and density of the striations as follows from (3), (5) are the same. Then for the conditions  $N_1/N_0 \ll 1$  and  $T/T_{\infty} > 1$  the first term in (10) is small in comparison with the second one and can be neglected. Thus Eq. (10) takes the form

$$\frac{\partial}{\partial z} \left( D_{\parallel} \frac{\partial N_1}{\partial z} \right) - \gamma N_1 = Q_1 \delta(z) - \frac{\partial}{\partial x} \left( D_{\perp}^T \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial z} \left( D_{\parallel}^T \frac{\partial T}{\partial z} \right). \quad (11)$$

Solving (11) we have

$$-\gamma N_1 = \frac{1}{2L_N^{\parallel}} \int_{-\infty}^{\infty} \exp\left(-\frac{|z-z'|}{L_N^{\parallel}}\right) \left[ Q_1 \delta(z') - \frac{\partial}{\partial x} \left( D_{\perp}^T \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial z'} \left( D_{\parallel}^T \frac{\partial T}{\partial z'} \right) \right] dz'. \quad (12)$$

The density length scale outside the layer (0,  $L$ ) along the magnetic field is defined by the quantity  $L_N^{\parallel}$ , and the temperature length scale by  $L_T^{\parallel}$ . In the F-region  $L_T^{\parallel} \gg L_N^{\parallel}$  [17]. Then, in a first approximation in the parameter  $L_N^{\parallel}/L_T^{\parallel}$ , the integration over  $dz'$  in Eq. (12) could be performed and it takes the following form (at the point  $z = 0$ ),

$$-\gamma N_1 = \frac{Q}{2L_N^{\parallel}} - \frac{\partial}{\partial x} \left( D_{\perp}^T \frac{\partial T}{\partial x} \right) - \frac{1}{L_N^{\parallel}} D_{\parallel}^T \frac{\partial T}{\partial z} \Big|_{z=0}, \quad (13)$$

where  $z = 0$ ,  $N_1 = N_1^{(0)}$ , and  $T = T_0(x)$ . The last two terms in Eq. (13), if we take into account the small parameters  $L/L_N^{\parallel}$ ,  $(T_{\infty}/T_0)(N_1/N_0)$ ,  $(D_{\parallel}/\kappa_{\parallel})(\kappa_{\perp}/D_{\perp}^T)(L/L_N^{\parallel})$ , can be simplified, and Eq. (13) be presented in the form

$$\gamma N_1 = \frac{d}{dx} \left( D_{\perp}^T \frac{dT_0}{dx} \right) - \frac{D_{\parallel}^T}{\kappa_{\parallel}} \frac{L}{L_N^{\parallel}} \left( \frac{1}{L} \int_0^L \frac{\sigma_{\perp} E_1^2}{N_0} dz \right). \quad (14)$$

Eq. (14) describes the variation of the plasma density in the striations inside the layer. The above equation should be examined together with (9). Note that neglecting the transverse transport in (9), (11), we pass over to

the solutions describing the temperature increase and plasma density reduction initiated by a  $\delta$  heat source under longitudinal spreading of heat only [17].

Analyzing Eqs. (14) and (9) jointly it is easy to see that the term  $(d/d_x)\kappa_{\perp}dT_0/dx$  in (9) is not essential. Indeed, from (14) it follows that

$$\int_0^L \frac{\sigma_{\perp} E_1^2}{N_0} dz \sim \frac{\kappa_{\parallel}}{D_{\parallel}^T} L_N^{\parallel} \frac{d}{dx} \left( D_{\perp}^T \frac{dT_0}{dx} \right).$$

So the first term in Eq. (9) in comparison with the second one is small because of the parameter  $L/L_N^{\parallel} \ll 1$  and can be neglected.

The system of equations in the layer now takes the form

$$\begin{aligned} \int_0^L \frac{\sigma_{\perp} E_1^2}{N_0} dz &= 2T_{\infty} \sqrt{k_{\parallel} \delta \nu_0} \sqrt{\frac{4}{9} \tau_0^{9/2} - \frac{4}{7} \tau_0^{7/2} + \frac{8}{63}}, \\ \gamma N_1 &= T_{\infty} \frac{d}{dx} \left( D_{\perp}^T \frac{d\tau_0}{dx} \right) - \frac{D_{\parallel}^T}{\kappa_{\parallel}} \frac{1}{L_N^{\parallel}} \int_0^L \frac{\sigma_{\perp} E_1^2}{N_0} dz. \end{aligned} \tag{15}$$

### 3. Heating energy source in the striations

To obtain a self-consistent problem, the source of energy  $L^{-1} \int_0^L (\sigma_{\perp} E_1^2 / N_0) dz$  should be determined.

Usually, the transverse length scale of irregularities  $l_x$  is much smaller than  $(k_z \omega_{eH} / \omega)^{-1}$  and the term  $A$  in (2) can be omitted in the first approximation over the parameter  $l_x k_z \omega_{eH} / \omega$ . Then we can rewrite the wave equation in the following form,

$$\hat{\epsilon}_{xx} E_1 = -P_0 N_1 / N_0. \tag{16}$$

The expression for the transverse component of the dielectric permittivity tensor  $\hat{\epsilon}_{xx}$  can be obtained by generalizing the dielectric permittivity tensor of the homogeneous plasma, which has the following structure [18],

$$\epsilon_{ij}(k, \omega) = \delta_{ij} + \frac{\omega_p^2}{\omega^2} \left( \sum_{l=-\infty}^{\infty} Z_0 W(Z_l) \Pi_{ij}^l(\chi, Z_l) + Z_0^2 \hat{\epsilon}_z \hat{\epsilon}_z \right), \tag{17}$$

where  $Z_l = (\omega - l\omega_H) / \sqrt{2} |k_z| v_{\parallel T}$ ,  $\chi = k_x^2 \rho_{eH}^2$  and  $\rho_{eH}$  is the electron gyroradius.

$$\Pi^l = \begin{pmatrix} \frac{l^2}{\chi} A_l & il \frac{dA_l}{d\chi} & \frac{k_z}{|k_z|} \sqrt{\frac{2}{\chi}} l Z_l A_l \\ -il \frac{dA_l}{d\chi} & \frac{l^2}{\chi} A_l - 2\chi \frac{dA_l}{d\chi} & -i \frac{k_z}{|k_z|} \sqrt{2\chi} Z_l \frac{dA_l}{d\chi} \\ \frac{k_z}{|k_z|} \sqrt{\frac{2}{\chi}} l Z_l A_l & i \frac{k_z}{|k_z|} \sqrt{2\chi} Z_l \frac{dA_l}{d\chi} & 2Z_l^2 A_l \end{pmatrix}.$$

The perpendicular dispersion is described by the function  $A_l(\chi) = e^{-\chi} I_l(\chi)$ ,  $I_l(\chi)$  is a modified Bessel function. The parallel dispersion is described by the function

$$W(Z_l) = \exp(-Z_l^2) \left\{ i\sqrt{\pi} \frac{k_z}{|k_z|} - 2 \int_0^{Z_l} \exp(t^2) dt \right\}.$$

Below we will consider the frequency range up to the neighborhood of the third electron cyclotron harmonic, i.e.  $\omega \approx 3\omega_{eH}$ . For this case, using the approximation  $\chi \ll 1$  (i.e. the wavelength of the excited modes is larger than the electron gyroradius) and  $Z_l \gg 1$ , and expanding (17) (retaining the first and resonant thermal corrections, which are proportional to  $\chi$  and  $\chi^2/(1-9Y^2)$ , respectively) we obtain from (17)

$$\varepsilon_{xx} = \varepsilon_{xx0}(z) - N(x) - i\Gamma - \frac{3Y^2}{1-4Y^2}\chi - \frac{15Y^4}{(1-4Y^2)(1-9Y^2)}\chi^2, \quad (18)$$

where  $\varepsilon_{xx0}(z) = 1 - \omega_{p0}^2(z)/\omega_{p0}^2(0)$ ,  $N(x) = N_1(x)/N_0$ ,  $z=0$  corresponds to the upper hybrid resonance point. The contribution from collisions  $i\Gamma$  in (18) was introduced phenomenologically and is equal to  $\Gamma = (\nu/\omega)(1+Y^2)/(1-Y^2)$ .

Replacing as usually  $\chi$  by  $\chi = -\rho_{0eH}^2 d^2/d\tilde{x}^2$ ,  $\rho_{0eH} = \rho_{eH}(T=T_\infty)$ ,  $\tilde{x} = \int_0^x \tau^{-1/2} dx'$ , we come to the wave equation describing electrostatic high frequency oscillations propagating across the magnetic field with regard to the thermal corrections,

$$-\delta^4 a_4 \frac{d^4}{d\tilde{x}^4} E_1 + \delta^2 a_2 \frac{d^2}{d\tilde{x}^2} E_1 + E_1 [\varepsilon_{xx0} - N(\tilde{x}) - i\Gamma] = -P_0 N(\tilde{x}), \quad (19)$$

where  $\delta = \rho_{0eH}/l_x$ ,  $a_4 = 15Y^4/(1-4Y^2)(1-9Y^2)$ ,  $a_2 = 3Y^2/(1-4Y^2)$  and  $l_x$  is the scale of density variation across the magnetic field (above we renormalized the variable  $\tilde{x}$  on the characteristic scale  $l_x$ ). This equation could be analyzed by means of the WKB approximation using the substitution

$$E_1 = e^{i\varphi}, \quad \varphi = \varphi_0/\delta + \varphi_1 + \dots \quad (20)$$

Substituting (20) into (19) in the first order of the WKB approximation we obtain

$$-a_4 z^4 - a_2 z^2 + (\varepsilon_{xx0} - N - i\Gamma) = 0, \quad (21)$$

where  $z = d\varphi_0/d\tilde{x}$ . To the next order of the perturbation theory we find

$$a_4 \left( 6i \frac{dz}{d\tilde{x}} z^2 - 4z^3 \frac{d\varphi_1}{d\tilde{x}} \right) + a_2 \left( i \frac{dz}{d\tilde{x}} - 2z \frac{d\varphi_1}{d\tilde{x}} \right) = 0. \quad (22)$$

From (22) we have the following relation for the function  $\varphi_1$ ,

$$\varphi_1 = \frac{i}{2} [\ln(z) + \ln(1 + Az^2)], \quad (23)$$

where  $A = 2a_4/a_2$ . Thus the general solution to the homogeneous part of Eq. (19) in the WKB approximation has the form

$$E_1^h = \sum_k \frac{c_k}{|z_k|^{1/2} |1 + Az_k^2|^{1/2}} \exp \left( i \int_{x_0}^x \frac{z_k}{\delta} dx' - \frac{1}{2} i \arg z_k - \frac{1}{2} i \arg(1 + Az_k^2) \right), \quad (24)$$

where the  $c_k$  are free constants and the  $z_k$  are the respective roots of Eq. (21), which can be written as

$$z_{1,2} = \pm \left( \frac{1 + \sqrt{1 + pq(\tilde{x})}}{-p(a_2/2)} \right)^{1/2}, \quad z_{3,4} = \pm \left( \frac{1 - \sqrt{1 + pq(\tilde{x})}}{-p(a_2/2)} \right)^{1/2},$$

$$p = \frac{2A}{a_2}, \quad q(\tilde{x}) = \varepsilon_{xx0} - N(\tilde{x}) - i\Gamma. \quad (25)$$

The first two roots,  $z_{1,2}$ , correspond to the Bernstein modes, which propagate in case  $p < 0$ , i.e.  $\omega < 3\omega_{eH}$ . The two other roots are connected with the modified upper hybrid waves (below we will refer to them simply as UH waves). In the limit  $|pq| \ll 1$ ,  $z_{3,4}$  in (25) pass to well known WKB solution describing upper hybrid waves

far away from the cyclotron harmonics  $z_{3,4} = \pm(\varepsilon_{xx0} - N - i\Gamma)^{1/4}$ . As is seen from (25) they can propagate only inside the striations, i.e. when  $q > 0$ , and have quasi cutoffs in the loci of full reflection points where  $z_{3,4} = 0$ . For  $\omega < 3\omega_{eH}$  there is a frequency range near  $\omega \approx 3\omega_{eH}$  where these modes have a second group of cutoff points. It occurs at the condition

$$1 + pg < 0. \tag{26}$$

Thus there are two principally different types of excitations in a plasma with small density irregularities. One of them is untrapped Bernstein modes (which have no cutoff points) and the other one is trapped modes inside the striation UH waves.

As is known there is some difference in the behavior of trapped and untrapped modes if the external source of their excitation exists. The amplitude of untrapped modes is limited by the amplitude of the external source, while the amplitude of trapped modes in the absence of dissipative processes tends to infinity. It is limited by the damping process only. In the case of small damping the amplitude of the trapped modes is substantially larger than the amplitude of the external source. Since we are interested in the heating inside the striations produced by excited waves this fact enables us to restrict ourself by taking into account only trapped modes. This means that only UH waves give a significant contribution to the heating process inside the striation. For this reason we can neglect the contribution from the Bernstein modes in our analysis.

To obtain the general solution of the inhomogeneous equation, the partial solution of Eq. (19) should be added to (24),  $E_1^p = P_0 N(\tilde{x}) / (\varepsilon_{xx0} - N - i\Gamma)$ , and finally we obtain

$$E_1 = E_1^h + E_1^p. \tag{27}$$

The solution we have obtained is valid throughout the whole range of the variable  $\tilde{x}$  excluding the neighborhood of the turning points  $x_{1,2}$  where  $z_{3,4}(x_{1,2}) = 0$  and  $x_{3,4}$  where  $1 + Az_{3,4}^2(x_{3,4}) = 0$ . Inside the regions  $x_1 < \tilde{x} < x_3$  and  $x_4 < \tilde{x} < x_2$  it has an oscillating character and outside it is an exponentially decreasing function. Below we will consider the case when the second group of cutoff points is absent. Later we will take into account the effect of its appearance in the final expression for the integral source of the heating.

The expansion we have used cannot be continued through the points  $\tilde{x} = x_{1,2}$  without breaking the approximation, and the inner solution has to be constructed in the vicinity of the points  $\tilde{x} = x_{1,2}$  to define the constants  $c_{3,4}$ . This can be done by introducing new variables  $\xi^\pm = \pm(l/\delta a_2^{1/2})^{2/3} \{(\tilde{x} \mp x_0)/l \pm i\Gamma\}$  and expanding (19) near  $\xi^\pm = 0$ . After that we come to the equation

$$-\frac{a_4}{a_2^{5/3}} \frac{\delta^{2/3}}{l^{2/3}} u^{IV} + u^{II} - u\xi^\pm = 1 \tag{28}$$

where  $u = (E_1/U_0)(\delta a_2^{1/2}/l)^{2/3}$ ,  $U_0 = -P_0 N(\pm x_0)$ . The characteristic value  $l$  is defined by

$$\begin{aligned} \varepsilon_{xx0} - N &\approx -\frac{\tilde{x} - x_0}{l}, & \tilde{x} &\approx x_0, \\ &\approx \frac{\tilde{x} + x_0}{l}, & \tilde{x} &\approx -x_0. \end{aligned}$$

For simplicity we assume that the striation is symmetrical around  $\tilde{x} = 0$  so that  $x_{1,2} = \mp x_0$ .

Provided that

$$\frac{a_4}{a_2^{5/3}} \frac{\delta^{2/3}}{l^{2/3}} \ll 1 \tag{29}$$

Eq. (28) can be reduced to the inhomogeneous Airy equation,

$$u^{II} - u\xi^\pm = 1. \tag{30}$$

The general solution to (30) is

$$u = c_1^\pm \text{Ai}(\xi^\pm) + c_2^\pm \text{Bi}(\xi^\pm) - \text{Gi}(\xi^\pm) \tag{31}$$

where Ai, Bi are Airy functions, Gi is the adjoint Airy function and  $c_{1,2}^\pm$  are free constants.

Identifying (31) in the limit  $\xi^\pm \rightarrow \pm\infty$  with (27), the constants  $c_3$  and  $c_4$  can be defined. One has

$$c_3 = -\frac{U_0}{2} \left( \frac{\pi l}{\delta a_2} \right)^{1/2} \frac{e^{i\Phi/2 - i\pi/4}}{\cos(\Phi/2)}, \quad c_4 = -\frac{U_0}{2} \left( \frac{\pi l}{\delta a_2} \right)^{1/2} \frac{e^{-i\Phi/2 + i\pi/4}}{\cos(\Phi/2)}, \tag{32}$$

where  $\Phi = \int_{-x_0}^{+x_0} (z_3/\delta) d\tilde{x} + \frac{1}{2}\pi$ .

If the damping term  $i\Gamma$  is omitted in Eq. (19) the resonances which are seen to exist when  $\Phi/2 = \pi/2 + \pi n$  manifest themselves as the unbounded amplitude of the electric field  $E_1$ . To determine the electric field amplitude in the resonance conditions it is necessary to take into account the dissipative processes. If the collisions are most essential the resonances correspond to the maximum amplitude of the electric field  $E_1$ , which scales as  $1/\Gamma$  compared to the nonresonant value and formula (32) passes to

$$c_3 = -\frac{U_0}{2} \left( \frac{\pi l}{\delta a_2} \right)^{1/2} \frac{e^{i\Phi/2 - i\pi/4}}{\cos(\Phi/2) - i\varepsilon_1 \sin(\Phi/2)},$$

$$c_4 = -\frac{U_0}{2} \left( \frac{\pi l}{\delta a_2} \right)^{1/2} \frac{e^{-i\Phi/2 + i\pi/4}}{\cos(\Phi/2) - i\varepsilon_1 \sin(\Phi/2)}, \tag{33}$$

where  $\varepsilon_1 = -(\Gamma/\delta) \int_{-x_0}^{x_0} (dz_3/dq)|_{q=\varepsilon_{xx0}-N} d\tilde{x}$ .

But it is worth noting that the effect of Z mode leakage can also play an essential role in the process of upper hybrid wave damping [12]. This mechanism is related to the finite value of the longitudinal wave number  $k_z$ . This fact gives rise to an incomplete reflection of a trapped wave near the turning points, where  $k_x$  tends to zero, and so determines the dissipation of the upper hybrid wave energy. A small part of the energy is transformed outside the turning points and propagates away as long electromagnetic waves. To determine this damping it is necessary to take into consideration the term  $A$  in Eq. (2). According to Ref. [19] this effect could be expressed in just the same manner as collisional damping and give rise to replacement of  $\varepsilon_1$  in (33) by  $\varepsilon_2$  where  $\varepsilon_2 = \pi l_x \omega_{eH}/c\sqrt{-\varepsilon_{xx0}}$ ,  $l_x = [(dN/dx)(1/N)|_{x=x_0}]^{-1}$ . Under the ionospheric modification experiment for every isolated striation  $\varepsilon_2 \gg \varepsilon_1$  and Z mode, the leakage effect gives the main contribution to the damping mechanism defining the amplitude of the electric field.

The ohmic heating of the plasma by electromagnetic waves per unit volume and time is given by

$$Q_l = 2E^* \hat{\sigma} E, \tag{34}$$

where

$$\hat{\sigma} = \frac{\nu}{4\pi} \frac{\omega_p^2}{\omega^2} \frac{1}{1-Y^2} \begin{pmatrix} \alpha_\perp & i\mu & 0 \\ -i\mu & \alpha_\perp & 0 \\ 0 & 0 & \alpha_\parallel \end{pmatrix},$$

$\alpha_\perp = (1+Y^2)/(1-Y^2)$ ,  $\alpha_\parallel = 1-Y^2$  and  $\mu = 2Y/(1-Y^2)$ . For the present case of longitudinal waves propagating across the magnetic field with  $\omega \approx \omega_{UH}$  from (34) we have

$$Q_l = \frac{\nu}{2\pi} \frac{1+Y^2}{1-Y^2} |E_1^2|. \tag{35}$$

Taking into account (33)

$$|E_1^2| = \frac{U_0^2}{4} \frac{\pi l}{\delta a_2} \frac{2[1 + \cos(\tilde{\varphi})]}{z_3(1 + Az_3^2)} [\cos^2(\Phi/2) + \varepsilon_2^2 \sin^2(\Phi/2)]^{-1}, \tag{36}$$

where  $\tilde{\varphi} = \Phi - \frac{1}{2}\pi + 2\int_{-x_0}^{\tilde{x}} (z_3/\delta) dx'$ . Averaging over the quick-oscillating phase  $\tilde{\varphi}$  and  $\Phi$ , one has

$$|E_1^2| = \frac{(-\epsilon_{xx0})^{3/2}}{2} \frac{P_0^2}{\rho_{0eH} a_2} \frac{1}{z_3(1 + Az_3^2)} \frac{c}{\omega_{eH}}. \quad (37)$$

From (37) we finally obtain

$$Q_l(x, z) = \frac{\nu}{4\pi} \frac{1 + Y^2}{1 - Y^2} \frac{(-\epsilon_{xx0})^{3/2} P_0^2}{\rho_{0eH} a_2} \frac{1}{z_3(1 + Az_3^2)} \frac{c}{\omega_{eH}}. \quad (38)$$

To calculate the whole power generated into the striation by the upper hybrid waves the local power density  $Q_l$  has to be integrated over the heating area,

$$Q(x) = \int_0^L Q_l(x, z) dz = \int_0^L \sigma_{\perp} E_1^2 dz. \quad (39)$$

Since the length scale of the vertical plasma density variation  $L_0$  is larger than the height interval of the heating layer it suffices to use the linear approximation  $\epsilon_{xx0} = -z/L_0$ . We need to remark that the anomalous absorption due to the conversion of the pump wave energy into the upper hybrid waves was not taken into account here and we assumed that  $P_0$  is a constant independent of  $z$ . Within this approximation

$$Q(x) = Q_0 \tau_0^{-3/2} \int_0^{-N} \frac{d(-\epsilon_{xx0})(-\epsilon_{xx0})^{3/2} \Theta(1 + pq)}{\sqrt{1 + pq} [(1 - \sqrt{1 + pq})/(-pa_2/2)]^{1/2}},$$

$$Q_0 = \frac{\nu_0}{4\pi} P_0^2 L_0 \frac{c}{\rho_{0eH} \omega_{eH} a_2} \frac{1 + Y^2}{1 - Y^2}, \quad p = \frac{20}{3} \frac{1 - 4Y^2}{1 - 9Y^2} \quad (40)$$

and

$$\Theta(x) = 1, \quad \text{if } x > 0,$$

$$= 0, \quad \text{if } x < 0.$$

The function  $\Theta(1 + pq)$  which is introduced in the integral (40) effectively includes the appearance of the second group of cutoff points in the striation and consequently a cutting of the integral in accordance with (26).

Eq. (15), together with (40), completely defines our problem. Thus, expanding (3) in the parameters  $L/L_N^{\parallel}$ ,  $L/L_T^{\parallel}$  and  $L_N^{\parallel}/L_T^{\parallel}$  allows us to reduce the full system of equations (2) to a set of ordinary equations of one variable  $x$ . We should emphasize however that Eqs. (15), (40) are essentially nonlinear.

#### 4. Stationary state of isolated striations

For the further calculations let us introduce the effective parameter of heating

$$\eta = \frac{Q_0}{2T_{\infty} N_0 \sqrt{\delta\nu_0} k_{\parallel}} \quad (41)$$

and pass over from  $x$  to the dimensionless variable  $X = x/l_0$ ,  $l_0 = (D_{\perp}^2 k_{\parallel} / \delta\nu_0 \gamma D_{\parallel})^{1/4}$ . The characteristic length  $l_0$  is determined by the processes of transverse diffusion during the characteristic relaxation time of the electron temperature  $(\delta\nu_0)^{-1}$  and plasma density  $\gamma^{-1}$ . For F-region conditions, the parameter  $l_0 \sim 5\text{--}10$  m.

Here  $D_{\perp}$ ,  $D_{\parallel}$  and  $k_{\parallel}$  are the coefficients of transport, calculated for  $T = T_{\infty}$ . Then system (14) can be written in the following form,

$$\eta f_Q(Y, N) = \tau_0^{3/2} \sqrt{\frac{4}{9} \tau_0^{9/2} - \frac{4}{7} \tau_0^{7/2} + \frac{8}{63}}, \quad N = \varepsilon \frac{d}{dX} \left( \tau_0^{-3/2} \frac{d\tau_0}{dX} \right) - \frac{\varepsilon \sqrt{\frac{4}{9} \tau_0^{9/2} - \frac{4}{7} \tau_0^{7/2} + \frac{8}{63}}}{\sqrt{2} \tau_0^{5/2} (\tau_0 + 1)^{1/2}},$$

$$f_Q(Y, N) = \int_0^{-N} \frac{d(-\varepsilon_{xx0})(-\varepsilon_{xx0})^{3/2} \Theta(1 + pq)}{\sqrt{1 + pq} [(1 - \sqrt{1 + pq}) / (-pa_2/2)]^{1/2}}. \tag{42}$$

Here  $\varepsilon = L_{N_0}^{\parallel} / L_{T_0}^{\parallel}$ , and  $\eta$  (41) are dimensionless parameters.

It is convenient to introduce, instead of the temperature  $\tau_0$ , the function  $y = \tau_0^{-1/2}$ . Then, from (42), we get

$$\frac{d^2 y}{dX^2} = f(y), \quad f(y) = - \frac{y^5 \sqrt{\frac{4}{9} y^{-9} - \frac{4}{7} y^{-7} + \frac{8}{63}}}{2^{3/2} \sqrt{1 + y^{-2}}} - \frac{N}{2\varepsilon}, \tag{43}$$

where the function  $N(y)$  is implicitly defined by  $f_Q(Y, N) = y^{-3} \sqrt{\frac{4}{9} y^{-9} - \frac{4}{7} y^{-7} + \frac{8}{63}} / \eta$ . Note that  $y$  takes values in the interval  $0 < y < 1$ . Integrating (43) we obtain

$$\frac{1}{2} \left( \frac{dy}{dX} \right)^2 = \int_y^{y_2} f(\xi) d\xi = -\Psi(y). \tag{44}$$

The ‘‘potential’’  $\Psi(y)$  is shown in Fig. 1. The function  $\Psi(y)$  reaches its local maximum value at  $y = y_2$ , where  $y_2$  is the root of the equation  $f(y) = 0$ . The dotted line in the Fig. 1 is related to the soliton-type solution

$$\int_y^{y_2} \frac{dy_1}{\sqrt{2 \int_{y_1}^{y_2} f(\xi) d\xi}} = X, \tag{45}$$

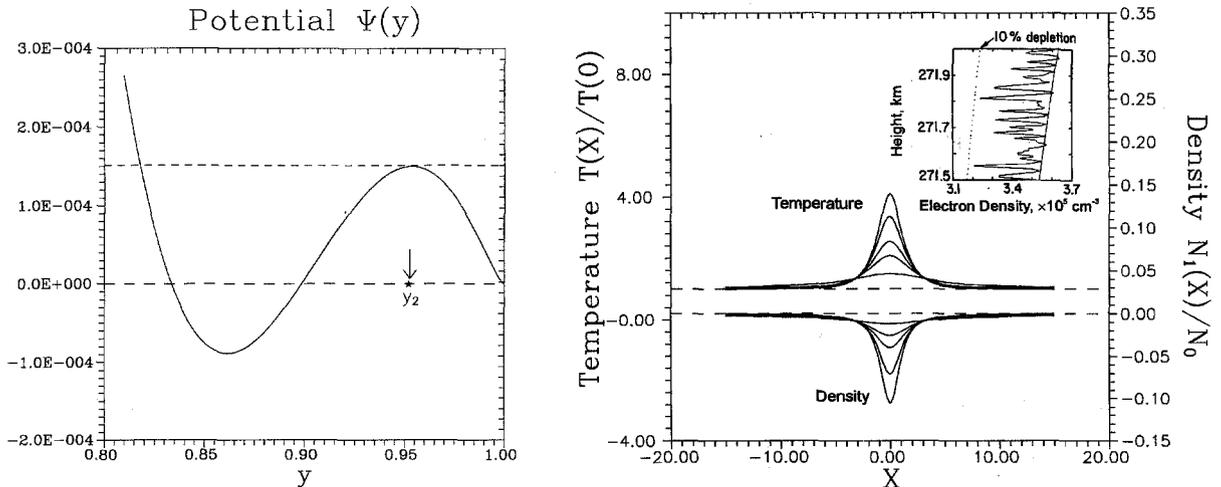


Fig. 1. Dependence of the ‘‘potential’’  $\Psi(y)$  on  $y = \tau_0^{-1/2}$ . The point  $y = y_2$  is shown by an arrow.

Fig. 2. Solution to Eq. (47) for different values of the effective parameter  $\sqrt{\eta} \varepsilon$  as a function of the dimensionless variable  $X$ . The upper bunch corresponds to the temperature dependences and the lower bunch to those for the density. The insert shows several of the plasma depletions observed in experiment [16].

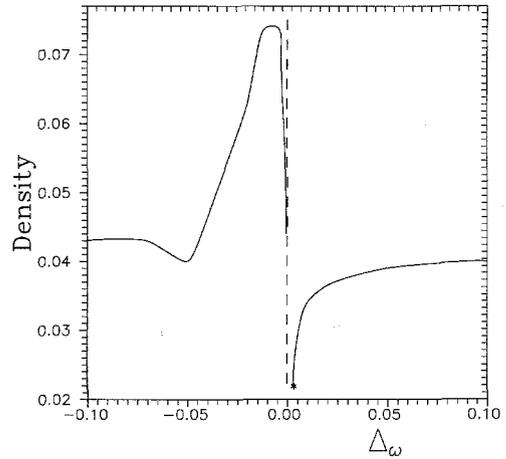
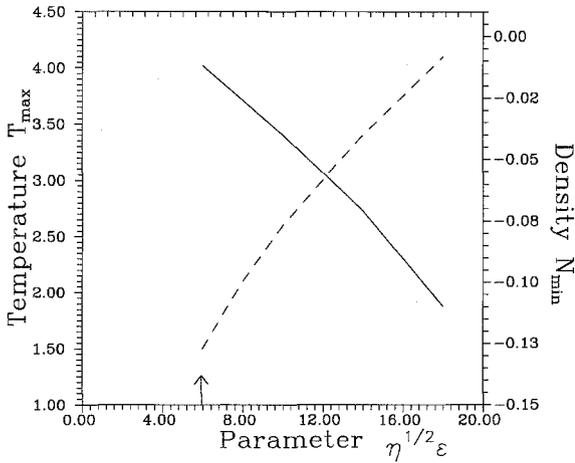


Fig. 3. Dependences of the temperature (dotted line) and density (solid line) on the effective parameter  $\sqrt{\eta} \varepsilon$  at the center of the striation. The arrow displays the minimum value of  $\sqrt{\eta} \varepsilon$  for which the solution exists.

Fig. 4. Dependence of the maximum of the absolute value of the density  $|N(0)|$  on the relative frequency  $\Delta_\omega = (\omega - 3\omega_{eH})/3\omega_{eH}$  near the third gyroharmonic at  $\sqrt{\eta} \varepsilon = 10$ , the critical value is shown by asterisk.

where  $\tilde{y} \leq y \leq y_2$  and the parameter  $\tilde{y}$  is determined by the relation

$$\int_{\tilde{y}}^{y_2} f(y) dy = 0. \tag{46}$$

The parameter  $\tilde{y}$  is directly related with the maximum temperature in the center of the striation, i.e where  $X = 0$ ,  $\tau_{\max} = 1/\tilde{y}^2$ .

We will consider at first the behavior of solution (45) at a fixed frequency far from cyclotron resonance. For that condition the integration in  $f_Q(N, Y)$  can be produced explicitly and system (42) takes a form

$$\frac{d^2 y}{dX^2} = f(y), \quad f(y) = -\frac{y^5 \sqrt{\frac{4}{9}y^{-9} - \frac{4}{7}y^{-7} + \frac{8}{63}}}{2^{3/2} \sqrt{1 + y^{-2}}} + \frac{1}{2\sqrt{\eta} \varepsilon} y^{-3/2} \left(\frac{4}{9}y^{-9} - \frac{4}{7}y^{-7} + \frac{8}{63}\right)^{1/4}. \tag{47}$$

The solution of (47) is shown in Fig. 2. It has the form of a soliton wave and depends on one nondimensional parameter  $\sqrt{\eta} \varepsilon$ . The dependences of the temperature and the density perturbation at the center of the striation on the effective heating parameter  $\sqrt{\eta} \varepsilon$  are shown in Fig. 3. As the effective parameter increases, the temperature and the density perturbation grow effectively. It is easy to see from these figures that the necessary stationary solution exists only for the specific values of the parameter  $\sqrt{\eta} \varepsilon > 5.8$ . There is no solution below the point  $\sqrt{\eta} \varepsilon < 5.8$ . The dependence of the half-width of the soliton  $X_0$  decreases rapidly when the heating parameter varies from 5.8 to 10, but further the fall becomes rather weak and  $X_0 \approx 1$ . A typical magnitude of the parameter  $\xi_0 = X_0(\delta\nu/\gamma)^{1/4}$  in (7) lies inside the range 3–8. Therefore relation (7) is seen to be always satisfied. It should be noted also that, as is seen from Fig. 1 for the existence of a soliton solution, a small average heating is needed:  $y \leq y_2 \approx 0.96$ .

Thus we see that the structure of the striations is defined by the parameter  $E_0/E_0^* = \sqrt{\eta} \varepsilon/5.8$ , where  $E_0$  is the amplitude of the pump wave,  $E_0^*$  is a characteristic field: a stationary solution exists only if  $E_0$  exceeds  $E_0^*$ ,

$$E_0^* = 35.3(\sin \alpha)^{-1}(T_\infty N_0)^{1/2} \sqrt{\frac{L_T^2 \delta}{L_0 L_N}} \sqrt{\frac{v_{Te}}{c}} \frac{(1 - Y^2)^{3/2}}{(1 + Y^2)^{1/2} (1 - 4Y^2)^{1/4}}, \quad (48)$$

where  $v_{Te}$  is the thermal velocity of electrons,  $\alpha$  is the angle between the pump wave electric field  $E_0$  and the magnetic field  $H$ . Estimates show that  $E_0^*$  is of the order of 100 mV/m. This value is easily reached in ionospheric modification experiments.

Let us now analyse the dependence of the maximum of the absolute value of the density perturbation in the striation on the frequency near the third cyclotron harmonic. This dependence is shown in Fig. 4. The strong decreasing of the striation amplitude in the vicinity of the third gyrofrequency is observed. There is a gap in the frequency dependence just in the immediate vicinity of the third gyrofrequency, where no stationary solution exists, see Fig. 4. We see clearly also the asymmetry in the behavior of the maximum of the absolute value of the density on the frequency in the ranges  $\omega < 3\omega_{eH}$  and  $\omega > 3\omega_{eH}$ , in particular the gap in the frequency dependence is narrower below the gyrofrequency than above it. Increasing the amplitude of the striations for  $\omega < 3\omega_{eH}$  and consequently decreasing for  $\omega > 3\omega_{eH}$  is connected with the efficiency of excitation of the UH waves which is defined by (40). The subsequent decreasing of the amplitude of the striation under  $\omega < 3\omega_{eH}$  is connected with the suppression of the UH wave excitation when the threshold defined by condition (26) is exceeded. This threshold in fact determines the limits of integral (40) (and therefore the width of the heating layer at a certain value of  $X$ ), cutting it at the point  $\varepsilon_{xx0} = N - 1/p$ .

So the stationary striations according to our theory are the density depletions elongated along magnetic field lines at the scales  $L_T \sim 10\text{--}15$  km (see Ref. [17]). The characteristic half-width of the striations is  $l \sim 5\text{--}10$  m. The depth of the density depletions  $|N_1/N_0| \sim 1\text{--}10\%$  (Fig. 2). The form, depth and width of the depletions depend on two nondimensional parameters  $\varepsilon$  and  $\eta$  (41), the width increases with decreasing depth. The considered stationary striations exist for a finite value  $N_1 > N_{\min}$  only, where  $N_{\min}/N_0 \approx 0.012$ , see Fig. 2. We emphasize that the structure of the striations (see Fig. 2 of [16]), their elongation ( $\sim 10$  km), the depth of the density depletions (2–10%) and their characteristic half-width scale (4–10 m) observed in experiments [16], correspond well to the present theory. The minimum amplitude of the depletions observed (1–2%) is also in accordance with the theory.

The main prediction of the theory is a strong enhancement of the electron temperature inside the striations  $T/T_\infty \sim 2\text{--}4$  which was not directly observed yet (one can suppose that the optical emission which is usually observed in ionospheric modifications is connected with this temperature enhancement [20]).

We emphasize that the source of the explosive character of the resonance instability is the strong heating inside the striations which is proportional to  $(N_1/N_0)^2$ . The stabilization comes from the nonlinear growing of the transport coefficients (mostly thermal conductivity  $\kappa$ ) with the temperature  $T$ . That is why the stationary solution exists only for large enough values of  $T/T_\infty > 1.6$  (or for  $N_1/N_0 > 0.012$ ). This nonlinear stabilization process was not considered in previous works [9–14].

Note that in the general case the solution of Eq. (15) has the form of a nonlinear wave, so we have a set of striations. However in this case new macroscopic processes, which are beyond the scope of this paper become substantial. First of all it concerns the anomalous absorption of the pump wave. This effect leads to an effective reduction of the heating zone scale  $L$ , and, as a consequence, to a slowing down of the increase of the perturbation magnitude against the amplitude  $E_0$ . The other important process is the self-focusing of the pump wave. The problem is that the density perturbations in striations are always negative  $N_1 < 0$ . Therefore the average electron density is reduced during the excitation of a large number of striations. This fact results in a self-focusing of the pump wave  $E_0$ . But the enhancement of the field  $E_0$  in the focusing region leads in turn to increasing striations. Thus there is a close nonlinear connection between the process of striation formation and focusing: in the focusing zone where the field  $E_0$  is strong, the striations are strong as well, and otherwise

outside of the focusing zone the field is small (it may drop by an order of magnitude [17]). Here striations should be small also or even not excited at all.

## 5. Conclusion

A theory for stationary striations is developed and it seems to be in a good qualitative agreement with the results of ionospheric modification experiments. The theory could serve as background for further theoretical studies of various nonlinear phenomena closely connected with striations: macroscopic processes of anomalous attenuation, scattering and self-focusing of radiowaves, special effects near multiple gyroresonances, generation of clearly pronounced structures in ionospheric radioemission (downshifted maximum, broad upshifted maximum) and so on.

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