1. BI3SS16 Frequency Response 2016/17 - Prof Richard Mitchell

Module : State Space and Frequency Response

Overall aim to consider state-space and frequency response modelling and associated control methods

My lectures: Frequency Response

- Review of frequency response of linear systems
- Relating frequency response and time domain.
- Positive and negative feedback.
- System identification from frequency response data.
- Designing Controllers using Frequency Response

Assessment

Examination –

Two hour exam : 2 questions on Frequency Response

Assignments

For Frequency Response :

series of tasks related to specific lectures,

make up the three parts of the coursework.

This should help understanding of lectures.

BUT to understand you MUST do extra work

Web Pages to help – you are advised to see if these help

http://www.reading.ac.uk/~shsmchlr/jsfreqresp/index.htm

Background to course

First given by Peter Fellgett – lots of maths

Dave Keating ↓ maths - emphasise meaning

I mould notes, add MATLAB, and research.

2011/12 : course reduced. 2013/4 joined with state space

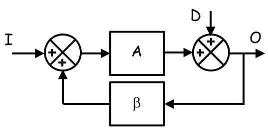
Based on Bode (designed telephone amplifiers) *'The engineer who embarks upon the design of a feedback amplifier must be a creature of mixed emotions. On the one hand, he can rejoice in the improvements in the characteristics of the structure which feedback promises to secure him. On the other hand, he knows that unless he can finally adjust the phase and attenuation characteristics around the feedback loop so the amplifier will not spontaneously burst into uncontrollable singing none of these advantages can be actually realised.'* (1940)

Rest of Lecture

The rest of the lecture comprises

- A reminder of feedback systems
  - Static Analysis (components are constants)
  - Dynamic Analysis and Stability (use of calculus, s and jω)
- Systems and Sinusoids
  - Input and Output Sinusoids
  - Impact of changing gain and phase shift
  - What means re Steady State Output
  - How can assess system stability

General Single Loop Feedback System



A = Forward Transfer Function

β = Feedback Transfer Function

Aβ = Loop Transfer Function

Overall TF : use Forward/1-Loop

If D = 0, $\frac{O}{I} = \frac{A}{1 - A\beta}$

If I = 0, $\frac{O}{D} = \frac{1}{1 - A\beta}$

$$\text{Overall } O = \frac{A}{1 - A\beta} * I + \frac{1}{1 - A\beta} * D$$

$$\text{If loop gain } A\beta \text{ big, } O = \frac{A}{-A\beta} * I + \frac{1}{-A\beta} * D = \frac{1}{-\beta} I$$

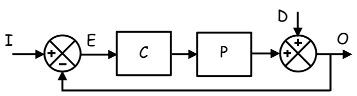
Bode worked on getting maximum possible loop gain

However : can't have high loop gain at all frequencies

Feedback Control System

The above is 'general' single loop feedback system

Control Engineers usually want O = I; so assume β = -1



P is 'process' to be controlled by controller C

A = CP and β = -1; Loop Gain = -CP

$$\frac{O}{I} = \frac{CP}{1 + CP} \qquad \frac{O}{D} = \frac{1}{1 + CP} \qquad O = \frac{CP}{1 + CP} I + \frac{1}{1 + CP} D$$

Still want high loop gain, O ~ I … but cant have at all frequencies

Dynamic Systems & Stability

If loop gain large, O equals I / -feedback value (= I for control)

Implies if I changes O instantaneously changes

BUT, O will take time to change – systems dynamic

As such, O may not reach expected value

it may oscillate away – unstable.

Important to ensure feedback system is stable

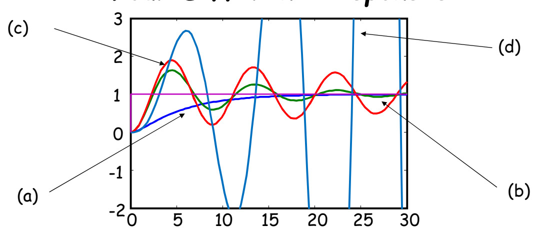
In this course will pay much attention to stability

both absolute and relative

Relative: O reaches final value – so absolutely stable –

but, oscillates too much – how quickly oscillations decay?

Four Different Responses



Overdamped – (a) output not exceed final value

Underdamped – (b) output oscillations decay – poss ok

– (c) oscillations take very long time to decay

Unstable – (d) output oscillates away from final value

We must model Dynamics

Dynamics is about change … so can think of calculus

$\frac{dO}{dt}$ = function of O, I, t, etc. Generate Diff Eqn and solve

In State Space, system modelled by many first order DEs

Use Laplace operator $s \equiv \frac{d}{dt}$. Generate transfer function $\frac{O}{I} = F(s)$

Use partial fractions, Look-up tables -> O(t) = f(t)

If I sinusoid Ksin(ωt), use jω in place of s: $\frac{O}{I} = F(j\omega)$

Simple way of assessing stability, designing controllers…

What we can do with Freq. Resp

We can model dynamics

We can see what happens when diff freq sinusoids are input

This is the Frequency response

We can assess whether system unstable

more easily than using Laplace, partial fractions, etc

We can assess how oscillatory a stable system is – relative stability

We can design controllers to

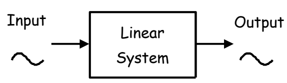
make system stable

Improve response: remove steady state errors; speed it up

From freq resp plot, we can work out model of system (identify it)

Basic Idea on Frequency Response

Uses properties of sinusoids in linear systems



In = K_i sin (ωt)

Out = K_o sin (ωt+ϕ)

ω = 2πf f = freq

O/p sinusoid same freq as I/p – diff amp – delayed (phase lag)

Find Gain = $\frac{\text{Out Amp}}{\text{In Amp}} = \frac{K_o}{K_i}$ and Phase = delay In to Out

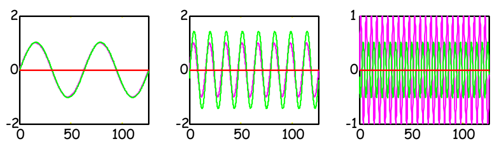
Model sys as TF in jω : a + jb, where a and b functions of ω

Gain = $\frac{K_o}{K_i} = |a + j\,b|$ and Phase $\phi = \angle(a + j\,b)$

NB If TF = $\frac{a + j\,b}{c + j\,d}$; Gain = $\frac{|a + j\,b|}{|c + j\,d|}$ Phase = $\angle(a + j\,b) - \angle(c + j\,d)$

Gain and Phase vary with ω

Plots of Sinusoid I and O at ω = 0.1, 0.4 and 1.2 rad/s



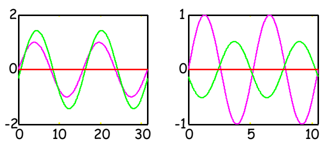
At low f, In ~ Out

At higher f,

different amplitude

bigger delay I to O

Very high f, O/I small



Why these values

Graphs are for $\frac{O}{I} = \frac{5}{10s^2+2s+5} = \frac{5}{-10\omega^2+2j\omega+5}$

(b) On earlier slide

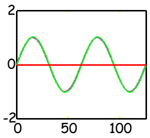
At $\omega = 0.1$ $\frac{O}{I} = \frac{5}{-10*0.01+2j\,0.1+5} = \frac{5}{4.9+j0.2}$

$\left|\frac{5}{4.9+j0.2}\right| = \frac{|5|}{|4.9+j0.2|} = \frac{5}{\sqrt{4.9^2+0.2^2}} = \frac{5}{\sqrt{24.05}} = 1.02$

$\angle\frac{5}{4.9+j0.2} = \angle 5 - \angle 4.9+j0.2$

$= 0 - \tan^{-1}\frac{0.2}{4.9} = -2.34^O$ or -0.04 rads

So, yes O almost same as I

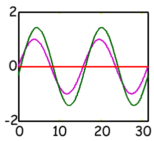


For other two frequencies

At $\omega = 0.4$ $\frac{O}{I} = \frac{5}{-10*0.16+2j\,0.4+5} = \frac{5}{3.4+j0.8}$

$\left|\frac{5}{3.4+j0.8}\right| = \frac{5}{\sqrt{3.4^2+0.8^2}} = \frac{5}{\sqrt{12.2}} = 1.43$

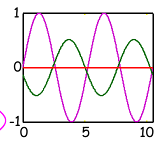
$\angle\frac{5}{3.4+j0.8} = -\tan^{-1}\frac{0.8}{3.4} = -13^O$ or -0.23 rads



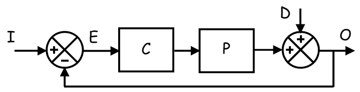
At $\omega = 1.2$ $\frac{O}{I} = \frac{5}{-10*1.44+2j\,1.2+5} = \frac{5}{-9.4+j2.4}$

$\left|\frac{5}{-9.4+j2.4}\right| = \frac{5}{\sqrt{94.12}} = 0.52$

$\angle\frac{5}{-9.4+j2.4} = -\tan^{-1}\frac{2.4}{-9.4} = -166^O$ or -2.9 rads



How affects Feedback System



$\frac{O}{I} = \frac{CP}{1+CP}$ $\frac{O}{D} = \frac{1}{1+CP}$

Key point, gains of C and P change with freq

So might have high gain at low freq, so O/I ~1, O/D ~0

But at high freq, gain low, O/I = small; O/D ~ 1

Note I and D are signals which may be low and / or high freq

Consider Aircraft

Directing commercial aircraft

No quick changes in steering : I is low freq signal

But turbulence – a disturbance – will have higher freq

Fighter aircraft

– need quick changes to avoid missiles – I high freq

So, when designing a system,

need to know what I and D signals are likely to be,

try to arrange loop gain is large at those frequencies.

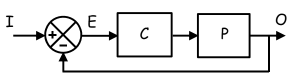
Then (at steady state) system response is ok.

Delay between I and O - Stability

As freq increases, so delay between I and O changes

This delay or Phase shift important … can affect stability

Which we assess by loop transfer function (ie I = 0)



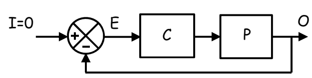
If O is sin(ωt), E = -sin(ωt) = sin(ωt-π) phase lag of 180^O

As ω changes, phase between E and O changes

Key is to find ω such that phase lag of C*P is 180^O

so lag round loop is one complete cycle.

Informal view of stability



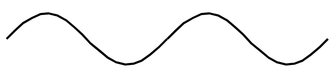
Suppose sinusoid exists s.t. phase(CP) is -180^O & |CP| = 1

Suppose O is a one period sinusoid starting at t=0

By time O completed cycle, sinusoid gone round loop

And hence can continue the sinusoid

At O

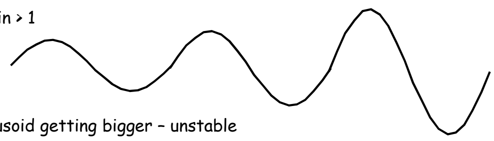


Have oscillator

Now Suppose gain not 1

If Gain > 1

At O



Sinusoid getting bigger – unstable

If Gain < 1

At O

Sinusoid getting smaller – stable

NB strictly analysis incorrect : as signals are sinusoids only at steady state - stability about checking transients!

Key Point

This suggests that (and is true for simple systems)

- Find ω such that phase of C * P is -180^o (phase lag 180^o)
  - (which means phase lag of loop is 360^o)
- Find gain of C * P at that freq
- If gain > 1 then feedback control system is unstable
- If gain = 1 then control system is an oscillator!!!
- If gain < 1 then system is stable

Note can also find ω such that gain of C*P = 1

- Then find phase lag of C*P : stable if phase lag < 180^o

If stable, how close to being oscillator = relative stability

Summary

In lecture we have introduced the frequency response material

Dynamics systems are modelled using jω

Hence transfer function is of form a + jb, functions of ω

Can work out O knowing I is sin(ωt) from |TF| and ∠(TF)

We note how O/I and O/D vary with ω

We note that stability can be assessed using loop transfer func.

Next week we build on this

NB For $\tan^{-1}(y/x)$ cant just calculate y/x and press $\tan^{-1}$ key. Use:

- If x = 0, then $\phi = \pi/2$ else $\phi = \tan^{-1}(\text{abs}(\frac{y}{x}))$;
- If x < 0, $\phi = \pi - \phi$;
- If y < 0, $\phi = 2\pi - \phi$; (gives answer in radians)

Assignment – Part A

Part A of the assignment is associated with lectures 1, 2 and 3.

Go via Blackboard to my web page and download the zip file associated with Part A.

- This has a word doc into which you will put your work
- And an m-file bi3ss1617.m – which you will need from next week

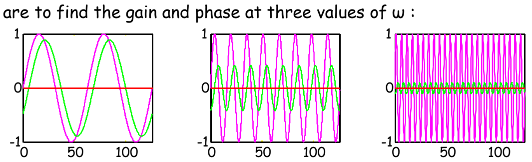
Before next week, reaffirm your knowledge of complex numbers by doing the exercise on the next slide – and on the sheet.

Open the word file, fill in your name and student number, and then find the three gains and phases as stipulated on the next slide and comment on the result.

Save your word file – you will submit it to Blackboard after you have done the exercise associated with the third lecture.

Exercise – System a)

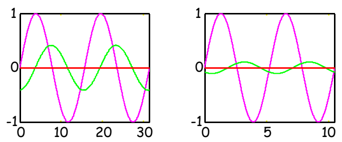
You are to find the gain and phase at three values of ω :



Here $\frac{O}{I} = \frac{1}{-5\omega^2 + 6j\omega + 1}$

Find $\left|\frac{O}{I}\right|$ and $\angle\frac{O}{I}$ at

ω = 0.1, 0.4 and 1.2 rad/s



Find these by hand/calculator: enter in word doc and comment.

2 : Frequency Response of Systems

Last week course introduced

- We showed that we want High Loop Gain for feedback systems
- Loop Transfer function (gain & phase) changes with frequency
- And we had to worry about Stability

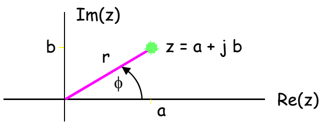
We noted that stability can be assessed by loop transfer function

- If at a frequency loop gain is 1 and phase lag is 360^o = oscillator
- System stable only if gain < 1 for that phase lag
- For C P control system, test gain when phase lag of C * P = 180^o

Such analysis achieved by modelling systems as functions of jω

We will explore more, see how MatLab helps. First, complex numbers

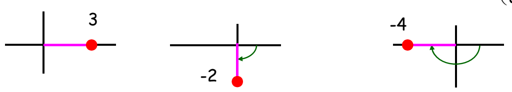
Reminder On Complex Numbers



r, modulus, |z| is distance from org

ϕ, argument, is angle from real axis

$|z| = r = \sqrt{a^2 + b^2}$ $\qquad \angle z = \phi = \tan^{-1}\left(\frac{b}{a}\right)$

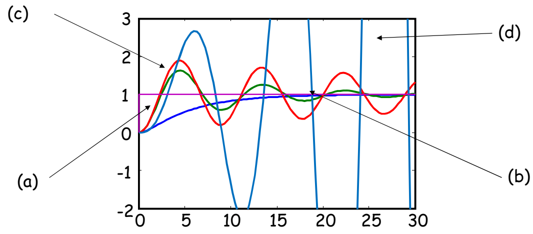


$z = 3 \quad |z| = 3, \ \angle z = 0$

$z = \frac{2}{j} = -2j \quad |z| = 2, \ \angle z = -90$

$z = \frac{4}{j^2} = -4 \quad |z| = 4, \ \angle z = -180$

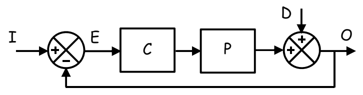
Four Responses – from last week



Closed loop transfer functions, for graphs (a) .. (d)

$$\frac{1}{5s^2+6s+1}; \quad \frac{5}{10s^2+2s+5}; \quad \frac{5}{10s^2+0.5s+5}; \quad \frac{3}{8s^3+6s^2+s+3}$$

Relating to Control System



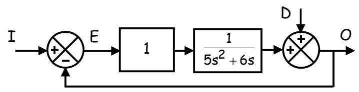
Suppose C = 1 and P = $\frac{1}{5s^2+6s}$, $\frac{O}{I} = \frac{1 * \frac{1}{5s^2+6s}}{1 - -\frac{1}{5s^2+6s}} = \frac{1}{5s^2+6s+1}$

$$\frac{O}{I}(j\omega) = \frac{1}{5j^2\omega^2 + 6j\omega + 1} = \frac{1}{1 - 5\omega^2 + 6j\omega}$$

$$\left|\frac{O}{I}\right| = \frac{1}{\sqrt{(1-5\omega^2)^2 + (6\omega)^2}} \qquad \angle\frac{O}{I} = 0 - \tan^{-1}\frac{6\omega}{1-5\omega^2}$$

System a

In Loop TF terms



(-)LoopTF

$$= \frac{1}{5j^2\omega^2 + 6j\omega} = \frac{1}{-5\omega^2 + 6j\omega}$$

$$\text{LoopGain} = \frac{1}{\sqrt{25\omega^4 + 36\omega^2}}$$

When ω very small Gain $\approx \frac{1}{\sqrt{36\omega^2}} = \frac{1}{6\omega}$

So, Gain = ∞ when ω = 0

Gain = 100 when ω ~ 0.0016 rad/s

At very high freq, LoopGain → 0

In terms of Stability

$$\text{LoopGain} = \frac{1}{\sqrt{25\omega^4 + 36\omega^2}} \qquad \text{LoopPhase} = -\tan^{-1}\frac{6\omega}{-5\omega^2}$$

At very low freq, LoopPhase ~-90°

At very high freq, LoopPhase → -180°

Loop phase -180° only at ω = ∞ when gain = 0 (less than 1)

Therefore, system is stable

Or, Loop Gain is 1 when $25\omega^4 + 36\omega^2 = 1$

or $\omega^2 = 0.027$ or $\omega = 0.165$ rad/s

Then phase = -97.8° so phase lag < 180°, so stable

For the Other Systems

Sys b)

If C = 5 and P = $\frac{1}{10s^2+2s}$; $\frac{CP}{1+CP} = \frac{\frac{5}{10s^2+2s}}{1+\frac{5}{10s^2+2s}} = \frac{5}{10s^2+2s+5}$

Sys c)

If C = 1 and P = $\frac{5}{10s^2+0.2s}$; $\frac{CP}{1+CP} = \frac{\frac{5}{10s^2+0.2s}}{1+\frac{5}{10s^2+0.2s}} = \frac{5}{10s^2+0.2s+5}$

Sys d)

If C = 1 and P = $\frac{3}{8s^3+6s^2+s}$; $\frac{CP}{1+CP} = \frac{\frac{3}{8s^3+6s^2+s}}{1+\frac{3}{8s^3+6s^2+s}} = \frac{3}{8s^3+6s^2+s+3}$

For freq resp, replace s by jω

Showing d) is Unstable System

For $\frac{O}{I} = \frac{3}{8s^3 + 6s^2 + s + 3}$,

$$Loop(j\omega) = \frac{3}{8j^3\omega^3 + 6j^2\omega^2 + j\omega} = \frac{3}{-6\omega^2 + j(\omega - 8\omega^3)}$$

$$Gain = \frac{3}{\sqrt{36\omega^4 + (\omega - 8\omega^3)^2}} \quad Phase = 0 - \tan^{-1}\frac{\omega - 8\omega^3}{-6\omega^2}$$

Can show (using MATLAB for instance) that

at ω = 0.3536 rad/s, Phase -180^o, Gain = 4

Thus system is unstable.

Using MatLab

In this course we will get MatLab to simulate and analyse

Combination of GUIs and code you use – you may write some

In MATLAB:

$5s^2 + 6s + 1$ represented by polynomial vector [5 6 1]

Can 'multiply' polynomials

conv([1 1], [8 -2 3]) ie $(s + 1)(8s^2 - 2s + 3)$

ans = [8 6 1 3] i.e. $8s^3 + 6s^2 + s + 3$

Transfer functions : use two polynomials

num = 1; den = [5 6 1];

Or num = 5; den = conv([2 1], [1 3 4]);

Getting Step Response in MatLab

MATLAB's control toolbox has [y,x,t] = step (n, d, t)

Step resp. for n/d over (optional) time t (times/final t)

Returns response in y; x has 'state variables'; t has time

Code for Graphs, … where assume I is unit step

```
[y(:,1), dummy, t] = step(1, [5 6 1], 30);     % calc a, set t as 0..30
y(:,2) = step(5, [10  2  5], t);               % calc b, use t
y(:,3) = step(5, [10  0.5  5], t);             % calc c
y(:,4) = step(3, [8 6 1 3], t);                % calc d
plot (t,y, [min(t), min(t) max(t)], [0 1 1]);  % plot graphs + I=step
```

In above, each column in y has values for one system

How Find Frequency Response

Dynamic systems – transfer functions - polynomials in jω.

System described by complex number at each ω.

Find Gain and Phase of each.

If $P(j\omega) = a + jb$, $\quad Gain = |a + jb| = \sqrt{a^2 + b^2}$, $\quad Phase = \angle a + jb = \tan^{-1}\frac{b}{a}$

If $G(j\omega) = C(j\omega)P(j\omega)$,

$$|G(j\omega)| = |C(j\omega)||P(j\omega)| \quad \angle G(j\omega) = \angle C(j\omega) + \angle P(j\omega)$$

If $G(j\omega) = \frac{n(j\omega)}{d(j\omega)}$, $\quad |G(j\omega)| = \frac{|n(j\omega)|}{|d(j\omega)|} \quad \angle G(j\omega) = \angle n(j\omega) - \angle d(j\omega)$

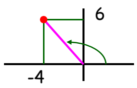
Using MATLAB for Freq Resp

\>> polyval([5 6 1], j*w) returns $-5w^2 + 6jw + 1 = 1-5w^2 + 6jw$

Returns a number if w number, or a vector if w a vector

Functions abs and angle return gain and phase (in rads)

```
>> r = polyval([5 6 1] ,j*1)
r =
 -4.0 + 6.0i
>> [ abs(r) angle(r)*180/pi ]
ans =
   [7.2111      123.6901]        %phase +ve as lead
```



NB Phase in range -180^o to + 180°

So Gain & Phase $\frac{1}{[5\ 6\ 1]}$ @ 1 rad/s = 1/7.2111 -123.6901

Finding and Plotting Freq Response

We find, analyse and often plot how Loop TF varies with ω

Suppose LoopTF defined by two polys num / den

RJM's Matlab Function to evaluate num/den at ang freqs in ω:

```
function resp = doFreq (num, den, w);
% RESP = DOFREQ(NUM, DEN,W)
% calc freq resp of NUM/DEN for all ang freqs in W  by RJM
resp = (polyval(num, j*w) ./ polyval(den, j*w));
```

NB [x1 x2 x3] ./ [y1 y2 y3] = [x1*y1 x2*y2 x3*y3]

Get gain and phase by : abs(resp) & angle(resp)*180/π

If w a vector (diff freqs), resp is (complex) vector

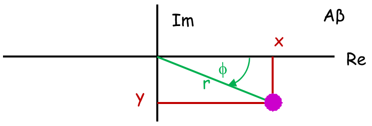
Plot how gain and phase vary with w: Bode or Nyquist … Here Nyquist

Nyquist Diagram

Strictly plot of Aβ or CP (ie – Loop TF) on Argand diagram

Calc gain & phase of Aβ(jω) or CP(jω)

At each frequency, gain round loop is r, phase lag is ϕ;



In Cartesian terms: $x = r \cos(\phi)$, $y = r \sin(\phi)$

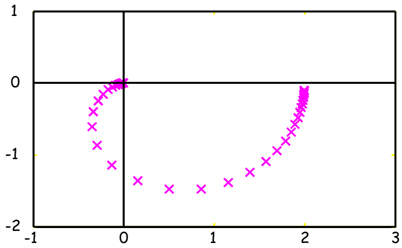
Do such calcs at lots of frequencies…

multiplicatively spaced ie at ω, ω*fac, ω*fac², … fac>1

Nyquist – Gain vs Phase One Graph

```
resp = doFreq (2, [10 5 1], logspace (-2, 1));
plot (real(resp), imag(resp), 'x', …
    [-1,3], [0 0], 'k-', [0 0], [-2 1], 'k-');
```

Calc at log spaced (multiplicative) ω's from 10^{-2} .. 10^{1}

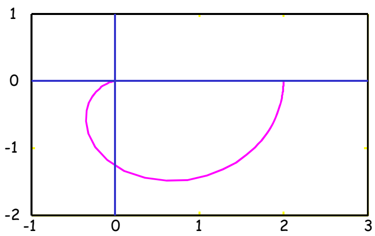


plots real vs imag - crosses at each freq; + axes

Or Nyquist Command – & Draw Lines

Convention : draw lines between points – get smooth 'locus'

```
[rp ip] = nyquist ([2], [10 5 1]);   %nyquist works out suitable w's
plot (rp, ip, 'k-', [-1,3], [0 0], 'k-', [0 0], [-2 1], 'k-');
```



If just call nyquist :

get plot for both +ve and -ve freqs

(and not show axes)

On High and Low Freq Values

In all systems have polynomials in jω

For low freq, ω→0, if ω < 0, ω² smaller, ω³ even smaller etc.

So just use lowest order polynomial term (may be ω⁰)

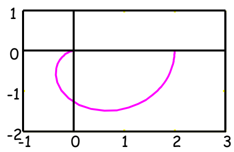
For high freq, ω→∞, if ω > 0, ω² bigger, ω³ even bigger, etc

So just use highest order polynomial term

$$\frac{2}{10j^2\omega^2 + 5j\omega + 1}$$

Low $\omega \to \frac{2}{1} = 2\angle 0$

High $\omega \to \frac{2}{10j^2\omega^2} = 0\angle{-180}$



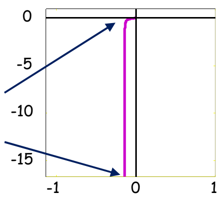
Gives start and end of Nyquist plots. May have ω's in numerator

Our First Example System (a)

$$\text{Loop}(j\omega) = \frac{1}{5j^2\omega^2 + 6j\omega}$$

High freq $\omega \to \frac{1}{5j^2\omega^2} = 0\angle{-180}$

Low freq $\omega \to \frac{1}{6j\omega} = \infty\angle{-90}$



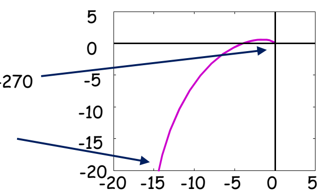

```
[rp ip] = nyquist ([1], [5 6 0]);  %nyquist works out suitable w's
plot (rp, ip, 'k-', [-1.2,1], [0 0], 'k-', [0 0], [-17 1], 'k-');
set(gca, 'xlim', [-1.2 1], 'ylim', [-17, 1]);    % set range of graph
```


And On Unstable System (d)

$$\frac{3}{8j^3\omega^3 + 6j^2\omega^2 + j\omega}$$

High freq $\omega \to \frac{3}{8j^3\omega^3} = 0\angle{-270}$

Low freq $\omega \to \frac{3}{j\omega} = \infty\angle{-90}$




```
[rp ip] = nyquist ([3], [8 6 1 0]);
  %nyquist works out suitable w's
plot (rp, ip, 'k-', [-20,5], [0 0], 'k-', [0 0], [-20 5], 'k-');
```

« with similar set command »

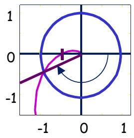
Assessing Stability on Nyquist

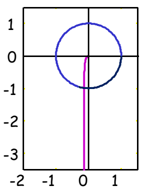
Find where locus meets –ve Re axis (phase -180)

If < 1 from origin, then stable

Find where meet unit circle (gain is 1)

If phase lag < 180, then stable

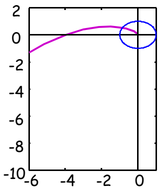




$\frac{1}{5j^2\omega^2 + 6j\omega}$

Gain < 1

PLag < 180



$\frac{3}{8j^3\omega^3+6j^2\omega^2+j\omega}$

Gain < 1

PLag < 180

Summary

In this second lecture we have

Looked more at frequency response

Examples of loop transfer function and assoc closed loop TF

Both as functions of s or of jω

We have seen can calculate gain and phase at diff ang freq

We have seen how MatLab can represent transfer functions

and calculate gain and phase at different angular frequencies

and plotted Nyquist diagrams – where can see if system unstable

Next week we look at Bode diagrams – for showing same info

Exercise

Extract the bi3ss1617.m from the zip file you downloaded last week and store in a folder MatLab can access.

It returns systems with parameters set by your student number.

Log into MatLab and enter the 8 digits of your student number by:

```
>> mystnum = 'xxxxxxxx';     % replace xxxxxxxx by your number.
>> resp = bi3ss1617 (1, mystnum)
```

This returns a struct resp where fields resp.cnum, resp.cden, resp.pnum, resp.pden are polynomials for controller C and Process P

Find the low and high frequency gains and phases for C*P and the gain and phase at ω = 1 rad/s.

Write commands to plot the Nyquist diagram for C*P.

Paste your work as instructed into the doc for later submission

Lecture 3. Bode Diagrams

Have seen that systems can be modelled as functions of jω

Frequency response is seeing what happens as ω varies

By looking at the transfer function of a feedback loop

we can assess whether feedback system is stable

Is gain < 1 when phase lag round loop is 360°?

(for Control System, |C * P| < 1 when ∠ C*P is =180°)

Last week we saw how could plot frequency response

on Argand Plane – as a Nyquist Diagram

and assess stability

This week we show how Bode diagrams can be used.

Bode Diagrams Key Points

Instead of one graph, have two

one shows how gain varies with ω, one how phase varies with ω

Different way of representing SAME info you see on Nyquist

So can also see if system is unstable

Can also approximate graphs by series of straight lines

These are called asymptotes

Very useful for estimating aspects of system

By looking at both graphs can derive model of system

So called System Identification

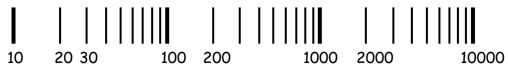
You will use two of my GUIs for plotting and identification

Log Scales and Bode Diagrams

Over large frequency range, eg 10^1 to 10^4, BUT SYSTEM DEPENDENT

Info in each decade just as relevant – so need same space on graphs

Thus plot the graphs with logarithmic scales for frequency



Means same size when go from freq f to f * constant (multiplicative)

Gain also varies over large range, so

Plot $\log_{10}$(Gain) vs $\log_{10}$(Freq)   sometimes 20$\log_{10}$(Gain)   (in dB)

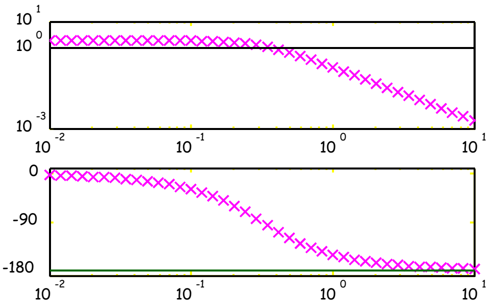
And Phase is plotted linearly vs $\log_{10}$(Freq)

Bode used $\log_e$ … gain in nepers …

MATLAB Code for Bode Diagram

```
function SimpleBode;
% MATLAB code to do simple Bode diagram of system
% uses RJMs doFreq function …  cf with code for Nyquist
% whose loop transfer function is 2/(10s^2 + 5s + 1)
w = logspace (-2, 1);             % 50 ang freqs from 0.01 to 10
resp = doFreq (2, [10 5 1], w);   % denom = 10s^2 + 5s + 1
subplot(2,1,1);                   % plot gain (& G=1) on one graph
loglog (w, abs(resp), 'x', [min(w), max(w)], [1 1], 'k');
subplot(2,1,2);                   % plot phase (&P=180⁰) on other
semilogx (w, angle(resp)*180/pi, 'x', …
        [min(w), max(w)], [-180 -180], 'k');
                                  % note 180/pi so in degrees
```


Bode Diagram Generated



Note – is same info as on Nyquist – diff format

Or using MatLab's Bode command

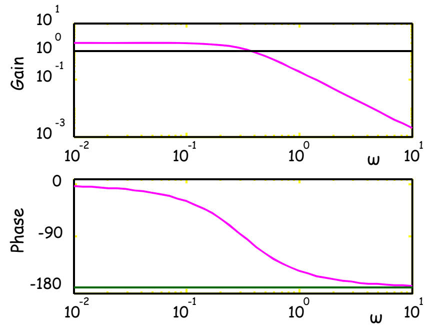
Again more conventional to join dots

```
[m,p,w] = bode(2, [10, 5, 1]);          % Better use bode find w
subplot(2, 1,1);                        % set 1st of 2 rows of plots
loglog(w,m,[min(w), max(w)], [1, 1], 'k');
set(gca, 'ylim', [0.001, 10], 'ytick', [0.001, 1, 10]);
subplot(2,1,2);
semilogx (w, p, [min(w), max(w)], [-180 -180], 'k');
set(gca, 'ylim', [-190, 10], 'ytick', [-180, -90, 0]);
```

Use set command for scale (ylim) & labels (ytick) of axis

Can do [m,p,w] = bode(num,den) finds suitable values of ω

Resultant Diagram



Low ω

Gain =

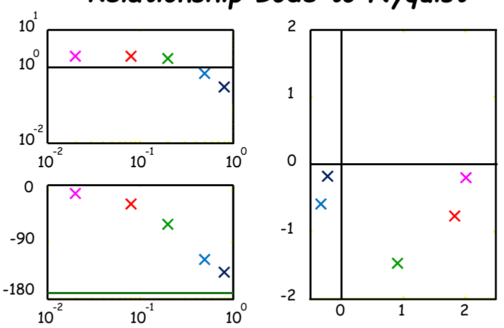
Phase =

High ω

Gain =

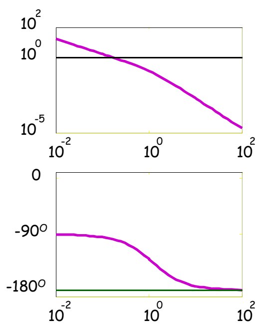
Phase =

Relationship Bode to Nyquist

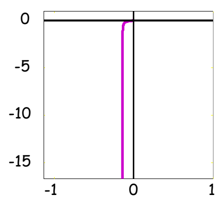


Plots at [0.02 0.08 0.2 0.5 0.8] rad/s

System a) … cf with Nyquist



$$\text{Loop}(j\omega) = \frac{1}{5j^2\omega^2 + 6j\omega}$$

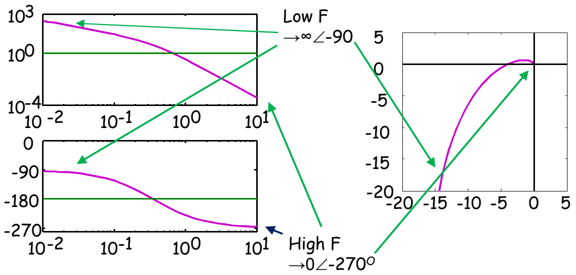


Low ω : gain Inf; phase -90

High ω : gain 0; phase -180 ..

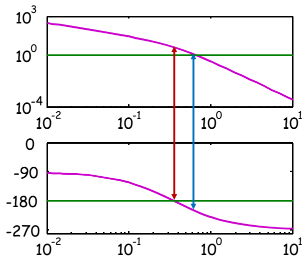
And Unstable System d)

$$Loop(j\omega) = \frac{3}{8j^3\omega^3 + 6j^2\omega^2 + j\omega}$$



Assessing Stability

As per Nyquist, find gain when phase -180° or phase when gain 1



Clearly

At ω where phase is -180

The gain > 1

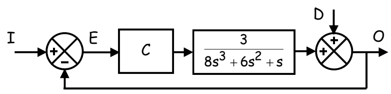
So unstable

At ω where gain is 1

Phase lag > 180

So unstable

Stabilising the System

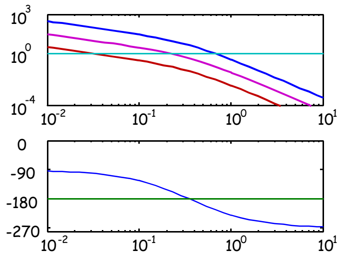


We have analysed the above system where C = 1

It is unstable, when phase lag of C * P is 180, gain ~ 4

What is simplest way of stabilising it?

Bode Plots for C = 1, 0.1, 0.01



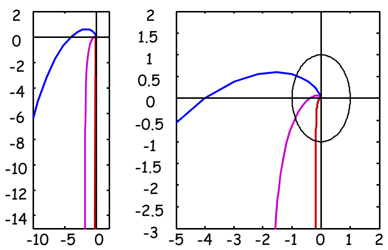
C changes

Gain plot up/down

Phase plot unchanged

as phase of C is 0

Nyquist

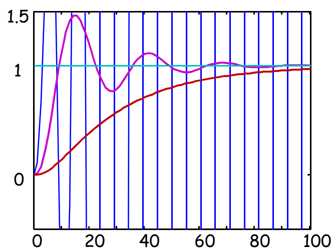


C changes

Plot same shape

But distance from 0,0 changes

Closed Loop Step Response



None that good!

Strategy for Simple P Controller

So we can reduce C to make system stable, but to what?

One strategy is to design so O overshoots final value by 20%

Often : set loop gain = 1 when loop phase is -135^O : (explain later)

Suppose P under control defined by polynomials num/den

Then do the frequency response

```
[m,p,w] = bode (num, den);
```

Look in p vector : find location where phase is -135^O

Find corresponding location in m : is P gain when phase -135^O

As want gain C * P = 1 then, so C = 1 / gain.

But

m, p, w are vectors with 50 gains, phases and ang freqs

Probably not have phase = -135 :in fact at locations 15, 16, 17:

m	p	ω
27.3134	-123.1113	0.1000
21.4863	-129.7019	0.1219
16.3012	-137.8965	0.1509

To find m where p is -135, MatLab has interpolate function

```
>> interp1(p, m, -135, 'spline')  % fits curve (spline) to p
   ans =          17.9373
```

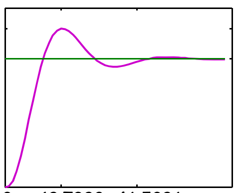
Looks in p for p[index] closest to -135, but notes value not -135

Then finds m[index], but first spline curve to estimate m wanted

% nb strictly p vector be monotonic … we investigate this later

Example

```
[m, p, w] = bode(num, den);
c = 1/interp1 (p, m, -135, 'spline');
[y, dum, t]=step(num * c, [0 0 0 num * c]+den);  %CLOSED LOOP
plot(t,y, [min(t) max(t)], [1 1])
```



C = 1/17.9373
= 0.0557

Note labels for Peak and Steady state values, and peak time and settling time

How find …

How found key values of y and t

y and t are vectors: y has output values at times in t

yss found using c * P(0) /(1 + c*P(0))   P(0) = num(end) / den(end)

yss = (num(end)*c) / (num(end)*c + den(end))

ypk = max(y);   tpk = t (y == max(y));

NB y == max(y) returns locations in y where max(y) stored

%o/s is 100* (ypk – yss) / yss

Settling time : scan y vector, find max time where y > 2% from yss

Hint use max of t ((abs(y – yss) > 2% of yss))

Assumes y reached steady state .. Might need to run step longer?

Can mark on plot: set(gca, 'xtick', [0, tpk, tset]);

set (gca, 'ytick', sort (unique(([yss, 1, ypk]));

In these set commands the vectors must be in ascending order

Summary

We have looked further into Frequency Response

- using Bode plots log gain vs log ω and phase vs log ω
- we have seen how to assess stability
- and how to stabilise by having simple controller

Next week, will investigate Bode plots further

- Estimating low/high freq response
- Approximating response by straight lines
  - Key to work in the course

Exercise – Lecture 3

Do the following, adding to the word document you have been using.

First run resp = bi3ss1617(1, mystnum); to get the system whose Nyquist plot you found. Now plot the Bode diagram.

Next run resp = bi3ss1617(2, mystnum); to get a 3rd order system.

Follow instructions in the word document to use MatLab to design a Proportional controller (so C*P has phase -135^O when its gain is 1) :

Here you load resp.cnum with the value of the controller gain

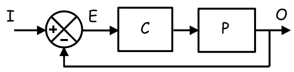
Then use resp = bi3ss1617(0, mystnum, resp)

Sets resp fields clnum, clden with the closed loop transfer function

Plot the resultant step response, labelling peak value, time to peak and settling time. Copy your code and the graph to the doc.

4: Frequency Responses Asymptotes

We have seen how MatLab can be used for frequency response – assessment of stability, and simple design

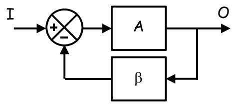


As dynamic, C and P are functions of jω:

Gain = |C*P (jω) |

Phase = ∠ C *P (jω)

Similarly could have system as



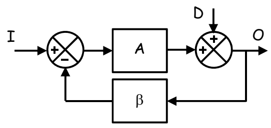
Gain = |A β (jω) |

Phase = ∠ A β (jω)

Graduate engineers need to be able to do more than use MatLab

So will plot approximate responses and do system identification

Key Points on Systems



$O = \frac{A}{1+A\beta}*I + \frac{1}{1+A\beta}*D$

If $A\beta$ high : $O = \frac{1}{\beta}*I + \frac{1}{large}*D$

If $A\beta$ low : $O = \frac{A}{1}*I + \frac{1}{1}*D$

If no D : high loop gain, O set by β; low loop gain, O set by A

Loop gain high at low freqs only: loop gain → 0 as ω→∞

I and D are signals with frequency content

Aim for frequencies in I and D, loop gain is high

So lets start thinking about loop transfer function at diff freqs

Estimating/Approximating Responses

Fundamental concept – for both plotting and later identification

Divide frequency range suitably – starting from low freqs

Can then estimate / approximate in these ranges

How divide – use 'corner frequencies' …

Consider $\frac{K}{1+j\omega/CF}$; Gain = $\frac{K}{\sqrt{1+\omega^2/CF^2}}$; Phase = $-\tan^{-1}(\omega/CF)$

If $\omega/CF \ll 1$; Gain $\approx \frac{K}{\sqrt{1+buggerall^2}} = K$ Phase $\approx -\tan^{-1}(ba) = 0$

If $\omega/CF \gg 1$; Gain $\approx \frac{K}{\sqrt{\omega^2/CF^2}} = \frac{K}{\omega/CF}$ Phase $\approx -\tan^{-1}(big) = -90$

Corner Freq, where ω/CF = 1 or ω = CF

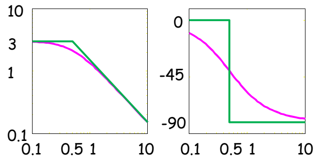
Hence - asymptotes

So we approximate TF before and after CF

Before CF, 1+jω/CF = 1; After CF: 1+jω/CF = ω/CF

Plots for $\frac{3}{1+j\omega/0.5}$

Actual plot moves between asymptotes



Equations for asymptotes:

$\omega < 0.5$, TF $\approx \frac{3}{1}$ G = 3; P = 0

$\omega > 0.5$, TF $\approx \frac{3}{j\omega/0.5} = \frac{1.5}{j\omega}$ $G = \frac{1.5}{\omega}$; P = -90

At ω = 0.5

Gain = 3

or 1.5/0.5 = 3

Works with Multiple Poles …

We model each pole/zero in form 1+jω/CF in two halves,

before ω = CF it is 1: so gain is 1; phase is 0°

after it is jω/CF : so gain is ω/CF; phase is -90°

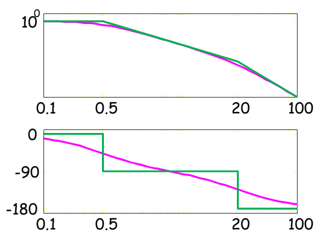
e.g. $TF = \frac{1}{(1+j\frac{\omega}{0.5})(1+j\frac{\omega}{20})} = \frac{1}{1+j\frac{\omega}{0.5}} * \frac{1}{1+j\frac{\omega}{20}}$

if $\omega < 0.5$: $TF \approx \frac{1}{(1)(1)} = 1$; gain is 1, phase is 0

$0.5 < \omega < 20$: $TF \approx \frac{1}{j\frac{\omega}{0.5}} * 1$; gain is $\frac{0.5}{\omega}$, phase is -90

if $\omega > 20$: $TF \approx \frac{1}{j\frac{\omega}{0.5}} * \frac{1}{j\frac{\omega}{20}} = \frac{10}{j^2\omega^2}$; gain is $\frac{10}{\omega^2}$, phase is -180

For this System



$TF = \frac{1}{(1+j\frac{\omega}{0.5})(1+j\frac{\omega}{20})}$

$\omega < 0.5$: TF is 1

$0.5 < \omega < 20$: $TF \approx \frac{0.5}{j\omega}$

$\omega > 20$: $TF \approx \frac{10}{j^2\omega^2}$

At $\omega = 0.5$: $|TF| = 1$ or $\left|\frac{0.5}{j0.5}\right| = 1$;

At $\omega = 20$: $|TF| = \left|\frac{0.5}{j20}\right|$ or $\left|\frac{10}{j^2 20^2}\right| = \frac{1}{40}$

Example with Zero and Poles

$H = \frac{4(1 + j\frac{\omega}{20})}{(1 + j\frac{\omega}{0.5})(1 + j\frac{\omega}{300})}$

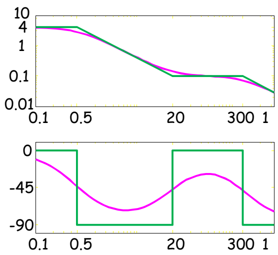
CFs 0.5, 20 and 300

$\omega < 0.5: \ H \approx \frac{4*1}{1*1} = 4$

$0.5 < \omega < 20: H \approx \frac{4*1}{j\frac{\omega}{0.5}*1} = \frac{2}{j\omega}$

$20 < \omega < 300: H \approx \frac{4*\frac{\omega}{20}}{j\frac{\omega}{0.5}*1} = 0.1$

$\omega > 300: \ H \approx \frac{4*\frac{\omega}{20}}{j\frac{\omega}{0.5}*j\frac{\omega}{300}} = \frac{30}{j\omega}$



Pure Integrator and Pole

$H = \frac{3}{j\omega(1 + j\frac{\omega}{7})}$

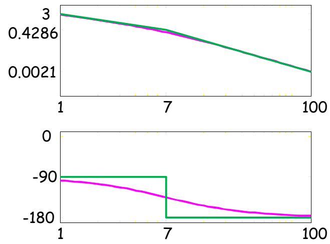
$\omega < 7: \ H \approx \frac{3}{j\omega}$

$\omega > 7: \ H \approx \frac{3}{j\omega * j\frac{\omega}{7}} = \frac{21}{j^2\omega^2}$

At ω = 1, |H| = 3;

At ω = 7, |H| = 3/7

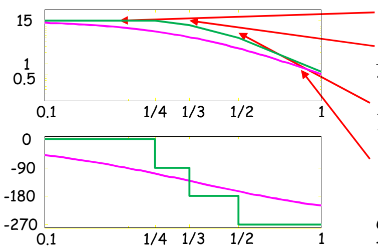
At ω = 100, |H| = 0.0021



Example with Close CFs

$\frac{15}{(2j\omega+1)(3j\omega+1)(4j\omega+1)}$

CFs 1/4, 1/3 and 1/2



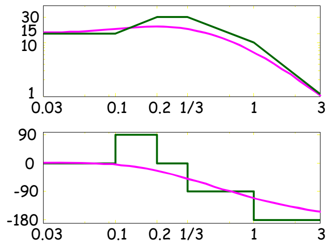
ω's	Model	Gain
$< \frac{1}{4}$	15	Gain = 15
$\frac{1}{4} < \frac{1}{3}$	$\frac{15}{4j\omega}$	Slope -1
$\frac{1}{3} < \frac{1}{2}$	$\frac{15}{12j^2\omega^2}$	Slope -2
$> \frac{1}{2}$	$\frac{15}{24j^3\omega^3}$	Slope -3

CFs close, actual resp far from asymptotes

Another Example

$\frac{15(10j\omega+1)}{(j\omega+1)(3j\omega+1)(5j\omega+1)}$

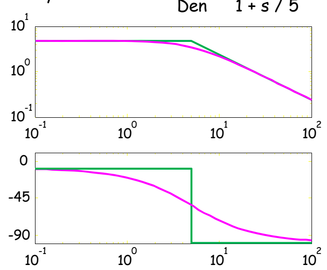
CFs 1/10, 1/5, 1/3 and 1
Plot from 0.03 to 3



ω's	approxTF	Gain
$< \frac{1}{10}$	15	15
$\frac{1}{10}$ to $\frac{1}{5}$	$150j\omega$	15 to 30
$\frac{1}{5}$ to $\frac{1}{3}$	$\frac{150j\omega}{5j\omega}$	30
$\frac{1}{3}$ to 1	$\frac{150j\omega}{15j^2\omega^2}$	30 to 10
1 to 3	$\frac{150j\omega}{15j^3\omega^3}$	10 to $\frac{10}{9}$

MATLAB GUI To help Learn

System 1 $\frac{Num}{Den} = \frac{7}{1 + s / 5}$



Select ω range

0.1 100

For each asym $K(j\omega)^n$ enter ω range, K and n

0.1 5 7 0

5 100 35 -1

Done

See also, on my web page, FreqRespAsyms.html

Key Point

We take a transfer function

We note the corner frequencies,

and so divide the frequency range

For each frequency range :

TF approxed as $K*(j\omega)^n$ ← n could be negative

So Gain = $K*\omega^n$

and Phase = $n*90^O$ or $n*\frac{\pi}{2}$ rad

When plot gain, plot log(K) + n log(ω) vs log(ω)

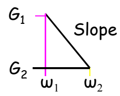
ie is straight line of gradient (slope) n

On Gain Asymptotes

Consider gain asymptote of the form $K\,\omega^m$ (often m is negative)

Suppose asymptote from ω_1 to ω_2 and gain is G_1 at ω_1

What is gain at ω_2?



$$\log(G_2) - \log(G_1) = m\,(\log(\omega_2) - \log(\omega_1))$$

$$\log\left(\frac{G_2}{G_1}\right) = m\,\log\left(\frac{\omega_2}{\omega_1}\right) = \log\left(\frac{\omega_2}{\omega_1}\right)^m$$

$$\frac{G_2}{G_1} = \left(\frac{\omega_2}{\omega_1}\right)^m$$

So if gain 5 at 0.1, when slope -2, gain at 0.5 is $5*\left(\frac{0.5}{0.1}\right)^{-2} = 0.2$

Estimating ω where gain unity

This is often needed … important for stability, transients, etc

$$|A\beta(j\omega)| = \left|\frac{15}{(2j\omega+1)(3j\omega+1)(4j\omega+1)}\right| = \frac{15}{\sqrt{(1+4\omega^2)(1+9\omega^2)(1+16\omega^2)}}$$

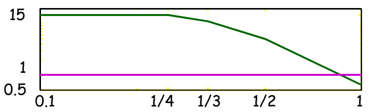
To find where gain is unity solve

$$15 = \sqrt{(1+4\omega^2)(1+9\omega^2)(1+16\omega^2)}$$

Can do in MatLab, using interps, – but in an exam?

Can approximate … using asymptotes

Use Asymptote Models



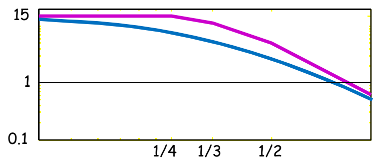
Use models in each range and see if answer consistent with range

$\frac{1}{4} \le \omega \le \frac{1}{3}$ Gain = $\left|\frac{15}{4j\omega}\right| = \frac{15}{4\omega}$; = 1 when $\omega = \frac{15}{4}$; out of range

$\frac{1}{3} \le \omega \le \frac{1}{2}$ Gain = $\frac{15}{12\omega^2}$; = 1 when $\omega = \sqrt{\frac{15}{12}}$; out of range

$\frac{1}{2} \le \omega$ Gain = $\frac{15}{24\omega^3}$; = 1 when $\omega = \sqrt[3]{\frac{15}{24\omega^3}}$ = 0.855; in range

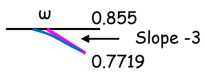
Iteration – for better estimate



Actual gain not return to last asymptote til later ω

Actual Gain at 0.8550 is 0.7719 (In fact Gain 1 at 0.7707)

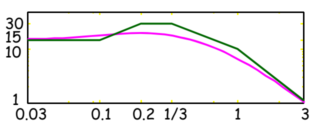
Can iterate to get better estimate : use gains at asymptote ends



Next Est use $\frac{1}{0.7719} = \left(\frac{\omega}{0.855}\right)^{-3}$

Estimate is 0.784

Another Example – from earlier



$$\frac{15(10j\omega+1)}{(j\omega+1)(3j\omega+1)(5j\omega+1)}$$

ω range	approx TF	gain in range
> 1	$\frac{150j\omega}{15j^3\omega^3}$	10 and less

Est where gain = 1, solve $\left|\frac{10}{j^2\omega^2}\right| = 1$ ie $\omega = \sqrt{10} = 3.16$

Gain at 3.16 is actually 0.9468; so iterate

Solve $\frac{1}{0.9468} = \left(\frac{\omega}{3.16}\right)^{-2}$ ie $\omega = 3.077$

Gain then 0.9971
Iterate ω = 3.0725

Summary

Have looked more at plotting frequency responses

In particular have seen how asymptotes can be used to approximate Bode graphs and to estimate key freqs.

Important you understand – will use in course a lot, so look at

http://www.reading.ac.uk/~shsmchlr/jsfreqresp/index.htm

It also helps for identification ..

Working out from Bode plot the structure of the system

Deducing relevant corner frequencies, gains, etc.

This is introduced in two weeks.

Next week we look at second order poles and zeros

Exercise – Lecture 4

This is start of Part B of assignment. Runs for next few weeks.

Download the zip file for Part B. This has two files for the plotting GUI : FreqAsymPlot.m and FreqAsymPlot.fig. Put in suitable folder.

Also included is Word file for Part B. Load your name etc.

Go to MatLab and run >> FreqAsymPlot

In the GUI enter your student number

Use the GUI to draw the asymptotes for system 1. When done, press Done, and copy to the word doc.

Repeat for systems 2, 3, 4 and 5.

For system 5, you also have to identify the asymptote where the gain is 1, and hence estimate ω where this happens.

Follow instructions in the word file on finding the actual value.

5 : More On Bode Plots

We have looked at

- Plotting Freq Resp of Loop TF – using Bode/Nqyuist
- Assessing absolute and relative stability
- Seen how to design a Proportional Controller

You have plotted asymptotes – using a GUI

You have estimated freq where gain unity

In this lecture we will

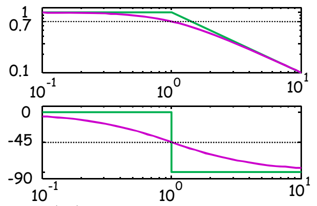
- Look at Bode Plots some more – more examples
- Better Phase asymptotes
- And Second Order Elements
  - And you will use the GUI in a different mode

Asymptotes for First Order System

$\frac{K}{1+sT}$ Gain at $\omega = \frac{K}{\sqrt{1+\omega^2T^2}}$ Phase at $\omega = -\tan^{-1}\omega T$

Asymptote model = K for $\omega < \frac{1}{T}$ and $\frac{K}{j\omega T}$ for $\omega > \frac{1}{T}$

So low freq : Gain = K, phase = 0; high freq, Gain = $\frac{K}{\omega T}$, phase = -90



Here where T = 1

Corner Freq, CF = 1/T

Here CF = 1

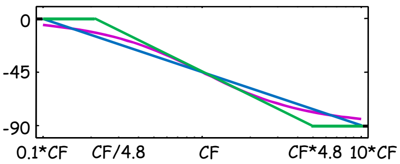
Gain asyms ok, but step change in phase problematic

So …

For better phase sketches

Books recommend phase asymptote from CF/10 to CF*10

But slope is wrong at corner freq:



Better asymptote: 0 : -π/2 or 0 : -90 as ω goes from CF/4.8 ..CF*4.8

But why 4.8? If interested, look at next few slides

(also on my web page with demonstrations of freq resp)

Note – analysis finds freq range 4.8^2 which equals e^π !

Finding slope for better asymptote

For $\frac{K}{1+j\omega T}$; phase $\phi = -\tan^{-1}\omega T$ So $\frac{d\phi}{d\omega} = -\frac{T}{1+(\omega T)^2}$

But use logarithmic scales for ω

As $\frac{d\ln(\omega)}{d\omega} = \frac{1}{\omega}$; $\frac{1}{d\ln(\omega)} = \omega\frac{1}{d\omega}$ so $\frac{d\phi}{d\ln(\omega)} = \omega\frac{d\phi}{d\omega}$

$$\frac{d\phi}{d\ln(\omega)} = \omega\frac{d\phi}{d\omega} = -\frac{\omega T}{1+(\omega T)^2} = -\frac{1}{2} \text{ @ } \omega T = 1$$

But use log based 10: log(x) = ln(x) * log(e)

$$\frac{d\phi}{d\log(\omega)} = \omega\frac{d\phi}{d\omega}\frac{1}{\log(e)} = -\frac{1}{2*\log(e)} \text{ @ } \omega T=1$$

Continued

$$\frac{d\phi}{d\log(\omega)} = -\frac{1}{2*\log(e)} \text{ @ } \omega T=1$$

Defines slope of line. Phase goes 0 to - π/2

ω goes CF/√r to CF*√r

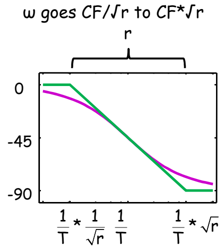
NB: Phase linear, ω logarithmic

So $\frac{0-\frac{\pi}{2}}{\log(r)} = -\frac{1}{2*\log(e)}$

Thus $\frac{\pi}{\log(r)} = \frac{1}{\log(e)}$

Or $\pi\log(e) = \log(r)$

So $\log(e^\pi) = \log(r)$ or $r = e^\pi$



r = 23.14; √r = 4.8

System with Two Lags

$\frac{5}{(1+s)(1+s/3)}$

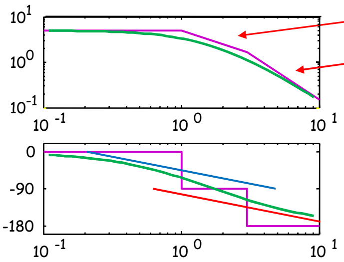
Slope 0 before 1, -1 then til 3, -2 after

Low freq gain found by setting s to $\frac{5}{j\omega}$, $\frac{15}{j^2\omega^2}$

Each extra lag term

decr slope by 1,

decr phase by 90°.



Asym Gain
5/3 at 3rad/s
0.15 at 10rad/s

'extra' asyms help phase sketch

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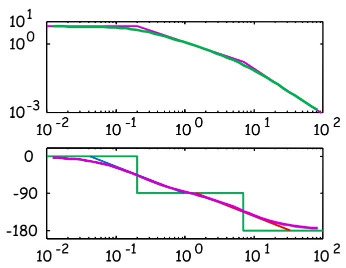
When CFs further apart

$\frac{6}{(1+s/0.2)(1+s/7)}$

Corner freqs in rad/s: 0.2 7



Asym Models

$6,\ \frac{1.2}{j\omega},\ \frac{8.4}{j^2\omega^2}$

Here extra phase asyms dont overlap.

So not interacting

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System with Two Lags and Lead

Corner freqs in rad/s: 0.4 1 3

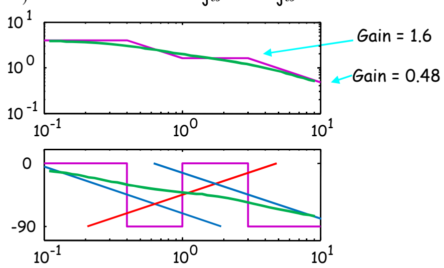
$\frac{4(1+s)}{(1+s/0.4)(1+s/3)}$

Models 4, $\frac{1.6}{j\omega}$, 1.6, $\frac{4.8}{j\omega}$

Lead terms increase

slope by 1

phase by +90°



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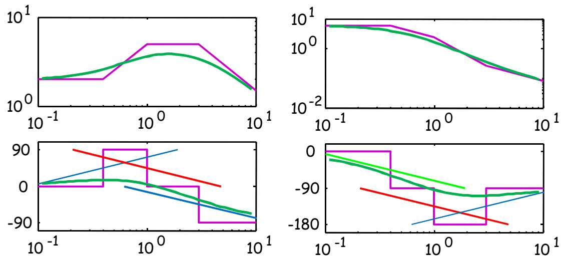
Two Similar, CFs in different order

$\frac{2(1+s/0.4)}{(1+s)(1+s/3)}$ $\frac{6(1+s/3)}{(1+s)(1+s/0.4)}$



Lead first, so G and P up Lead last, so P < -90 then up

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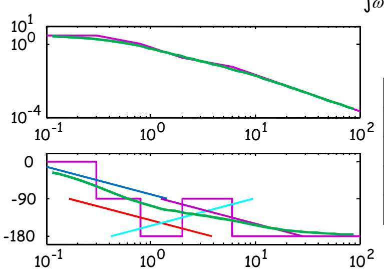
Another Example

Corner frequencies 0.3 0.8 2 6

$\frac{3(1+s/2)}{(1+s/0.3)(1+s/0.8)(1+s/6)}$

Models 3, $\frac{0.9}{j\omega}$, $\frac{0.72}{j^2\omega^2}$, $\frac{0.36}{j\omega}$, $\frac{2.16}{j^2\omega^2}$



asymptotically

ω	Gain
0.3	3
0.8	0.9/0.8 = 1.25
2	0.72/4 = 0.18
6	0.36/6 = 0.06

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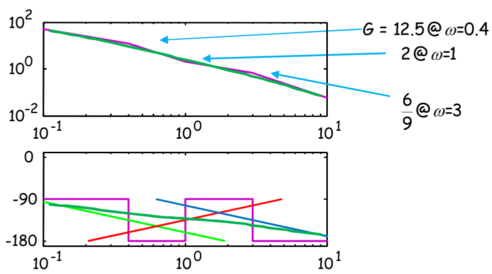
What if Infinite d.c. gain

$\frac{5(1+s)}{s(1+s/0.4)(1+s/3)}$ CFs 0.1, 1, 3 Models $\frac{5}{j\omega}$, $\frac{2}{j^2\omega^2}$, $\frac{2}{j\omega}$, $\frac{6}{j^2\omega^2}$



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Second Order Pole and Asymptotes

$$P(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2} \text{ or } \frac{\frac{K}{\omega_n^2}}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1} \quad P(j\omega) = \frac{K}{\omega_n^2 - \omega^2 + j2\zeta\omega_n\omega}$$

$$|P(j\omega)| = \frac{K}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}; \angle P(j\omega) = -\tan^{-1}\left(\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}\right)$$

Here corner freq is $\omega = \omega_n$. Asymptotes before/after meet at ω_n

For $\omega_n^2 - \omega^2 + j2\zeta\omega_n\omega$, before ω_n is ω_n^2: after ω_n is $-\omega^2 = (j\omega)^2$

So asym for $\omega < \omega_n$ is $\frac{K}{\omega_n^2}$ so Gain constant $\cong \frac{K}{\omega_n^2}$; Phase is 0

Asym for $\omega > \omega_n$ is $\frac{K}{j^2\omega^2}$ and so Gain is $\frac{K}{\omega^2}$; Phase is -180^O

Asymptotes and Actual Response

$\frac{K}{s^2 + 2\zeta s\omega_n + \omega_n^2}$ ζ affects response:

$\zeta > 1$ den = $(1 + s/\omega_1)(1 + s/\omega_2)$:

two separate corner freqs

$\zeta = 1$, factorises as $(1 + s/\omega_n)^2$:

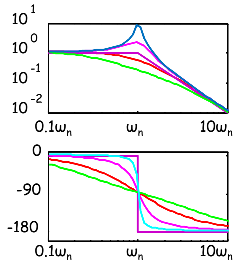
one corner freq at ω_n

$\zeta < 1$, den not factorise,

but has corner freq at ω_n

Smaller ζ peakier response

If $\zeta < \sqrt{½}$ Gain Peaks at $\omega_n\sqrt{(1 - 2\zeta^2)}$



Smaller ζ, quicker phase change

Extra asym range is $e^{\zeta\pi}$

Why 'Extra' Phase Asymptote Range = $e^{\zeta\pi}$

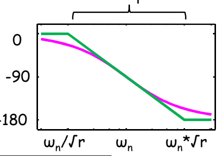
Phase $\phi = -\tan^{-1}\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}$

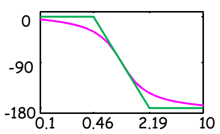
Can show $\omega\frac{d\phi}{d\omega} = -\frac{1}{\zeta}$ at $\omega = \omega_n$

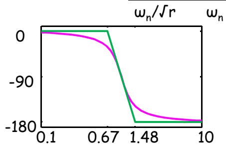
So $\frac{0 - \pi}{\log(r)} = -\frac{1}{\zeta * \log(e)}$

Or $\pi\zeta\log(e) = \log(r)$

So $\log(e^{\zeta\pi}) = \log(r)$ or $r = e^{\zeta\pi}$







$\zeta = 0.5$

$r = 2.19^2$

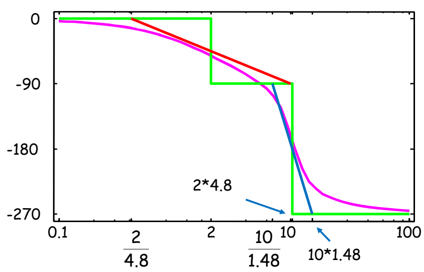
$\zeta = 0.25$

$r = 1.48^2$

Example with single + quadratic poles

$$TF = \frac{400}{(1+s/2)(s^2 + s*0.5*10 + 10^2)}$$

Models 4 : $\frac{8}{j\omega}$: $\frac{800}{j^3\omega^3}$



Extra asyms help to sketch phase.

Shallow 90^O change for pole; Steeper 180^O change for quadratic

Appreciate this for ID

Another Example with Second Order

$$\frac{100(1 + s/4)}{s(s^2 + 5s + 256)}$$

Lead at 4 rad/s;

Second order lag at 16 rad/s

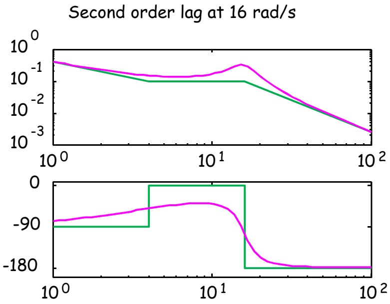
Asym Models

$\omega < 4$ $\frac{100}{j\omega * 256}$

$4 < \omega < 16$ $\frac{25}{256}$

$\omega > 16$ $\frac{25}{j^2\omega^2}$

Phase 90*slope



Plotting Asymptotes GUI

So far, the plotting GUI has required you define asymptote as

$K * (j\omega)^n$ from ω_{start} to ω_{end}

But, particularly for phase of second order elements, it is useful to specify the range around the corner frequencies : e^{π} or $e^{\zeta\pi}$

So GUI also operates in mode where define Corner Freqs

You specify overall ω_{start} to ω_{end} and then at these freqs and CFs:

ω; asymptotic Gain at ω; change in Phase at ω; and Phase Range

On calculating asymptotic gain, either use $K * (j\omega)^n$ at ω_{CF}, or

If gain G at ω_{start}, the slope is n, then at ω_{end}

gain is $G * \left(\frac{\omega_{end}}{\omega_{start}}\right)^n$

Example

$$TF = \frac{400}{(1+s/2)\,(s^2 + s*0.5*10 + 10^2)}$$ G at 2 is 4: at 10 is $4 * \left(\frac{10}{2}\right)^{-1} = 0.8$

Overall 0.1 .. 100

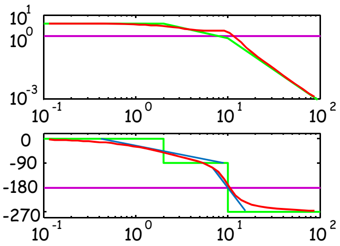
0.1,4,0,0;

2,4,-90,23;

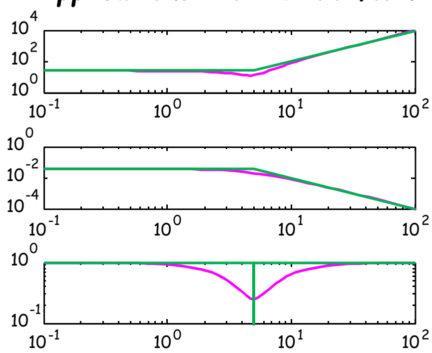
10,0.8,-180,2.2;

100,0.0008,0,0

$0.8*\left(\frac{100}{10}\right)^{-3}$ $e^{0.25\pi}$



Applications: Pole+Zero for Notch Filter



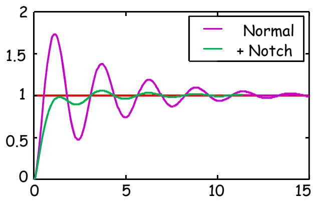
$s^2+2s\omega_{rt}+\omega_{rt}^2$

$\frac{1}{s^2+0.5s\omega_{rt}+\omega_{rt}^2}$

$\frac{s^2+2s\omega_{rt}+\omega_{rt}^2}{s^2+0.5s\omega_{rt}+\omega_{rt}^2}$

Application – Before Loop

Consider system step response of n/d:



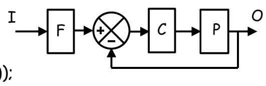
Transient : exp damped sinusoid

ω_{rt} , ang freq of sin, is 2.44 rad/s.

Put a notch filter, at 2.44 rad/s, before loop

$\frac{n}{d} = \frac{CP}{1+CP}$; $F = \frac{pn}{pd}$

```
pn = [1 0.5*wrt wrt^2];
pd = [1 2*wrt wrt^2];
step(conv(pn,n), conv(pn,polyadd(n,d)));
```



Application Second Order Filter

$$F(s) = \frac{2(s^2 + s + 0.25)}{s^2 + 0.01s + 0.5}$$

d.c gain is

high f gain is

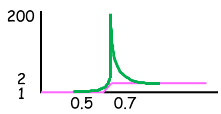
Asymptotes: gain: 1 until numerator corner freq 0.5 rad/s

slope is +2 until denom corner freq 0.707 rad/s, then 2.

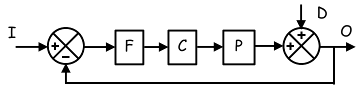
Num overdamped, den very underdamped

Diff of |denom|² wrt ω is 0 at 0.7 rad/s, so |F(jω) |max then : |F(j0.7) | ~ 200

Hence high gain at one freq



If D at that freq, reduce its effect by:



Summary

We have looked more at Bode diagrams and asymptotes, specifically

extra asymptotes for phase – help sketching

second order elements – extra asymptotes helpful

Next week we will move on to

looking at the Bode plot and identifying the system

Recommend look at web page FreqRespSeparate.html

FR plotted, but can include/exclude elements from it

Exercise – Lecture 5

Return to the plotting GUI, but use it in 'corner frequency' mode.

Follow instructions in word doc to plot the specified systems

