

CY3E2 2005 Exam

3. Host-symbiont models reflect the relationship between a host species on which lives a symbiont which often feeds from the host. In the following, H refers to the number of the host species, S the number of symbionts.
- (a) Suppose the host-symbiont model is defined by :

$$\frac{dH}{dt} = H(35 - 6H - H^2 + 10S) \quad \frac{dS}{dt} = S(-8 + 2H - S)$$

Calculate suitable values so as to sketch on the phase plane the zero isoclines for values of S and H in the range 1 to 12. Mark on the sketch the sign of the derivatives of H and S and thus sketch the likely variation of H and S from four different initial values which illustrate what happens with low and high values of H and S . [6]

Answer 3 a)

a) Isoclines calculated by finding where $35-6H-H^2+10S = 0$ and $-8 + 2H - S = 0$.

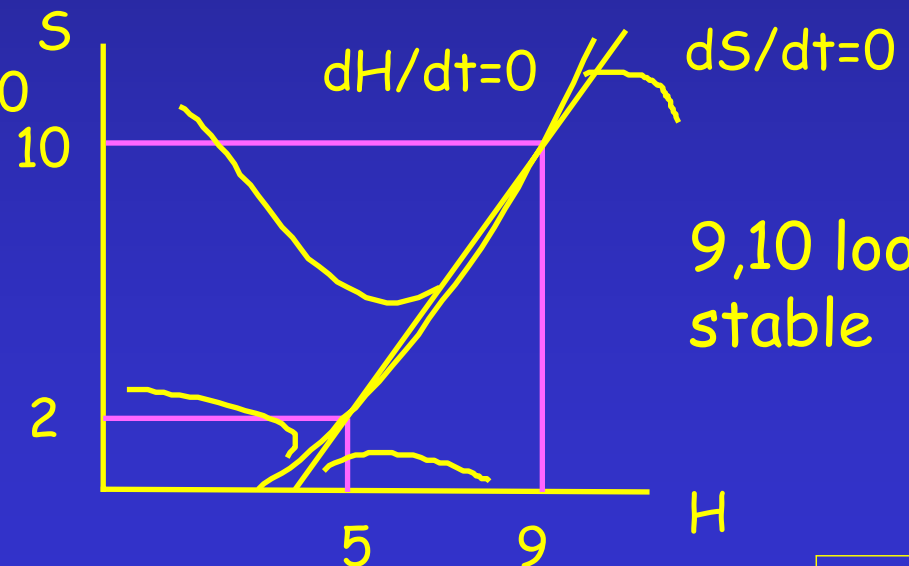
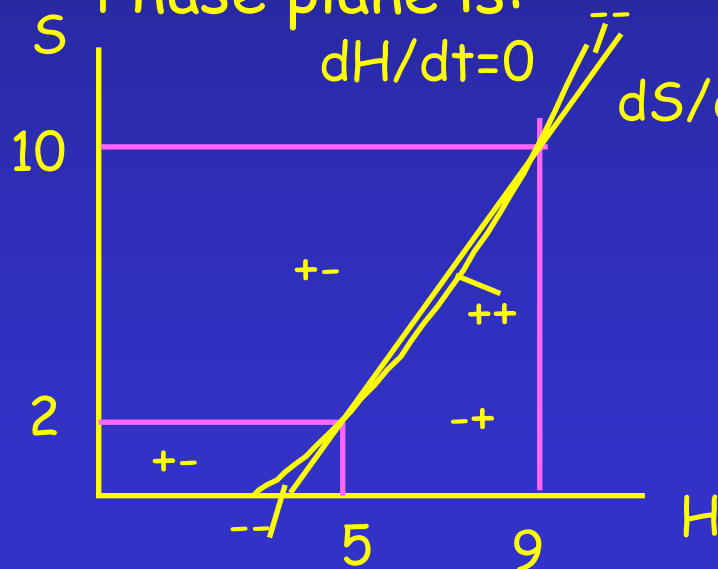
H: For $H = 1:12$; $S = (H^2+6H-35)/10$

= -2.8 -1.9 -0.8 0.5 2.0 3.7 5.6 7.7 10.0 12.5 15.2 18.1

S: For $H = 1:12$;

$S = 2H-8 = -6 -4 -2 0 2 4 6 8 10 12 14 16$.

Phase plane is:



9,10 looks stable

3 a) Continued

An equilibrium point is stable if both the eigenvalues of the associated Jacobean matrix are real and negative. Determine the Jacobean matrix for the two equilibrium points, and use this method to assess the stability of the two equilibrium points. [6]

For formal analysis, the coefficients of the Jacobean matrix can be found as follows, where $F1 = dH/dt$ and $F2 = dS/dt$. For point 5,2

Answer Continued

$$\frac{dH}{dt} = H(35 - 6H - H^2 + 10S) \quad \frac{dS}{dt} = S(-8 + 2H - S)$$

$$\left. \frac{\partial F_1}{\partial H} \right|_{5,2} = H^*(-6 - 2H) \Big|_{5,2} = -80$$

$$\left. \frac{\partial F_1}{\partial S} \right|_{5,2} = H^*10 \Big|_{5,2} = 50$$

$$\left. \frac{\partial F_2}{\partial H} \right|_{5,2} = S^*2 \Big|_{5,2} = 4$$

$$\left. \frac{\partial F_2}{\partial S} \right|_{5,2} = S^*(-1) \Big|_{5,2} = -2$$

So Jacobean is

$$\begin{bmatrix} -80 & 50 \\ 4 & -2 \end{bmatrix}$$

Eigenvalues: $(-80-\lambda)(-2-\lambda)-200=\lambda^2 +82\lambda -40 = 0$;
clearly one is positive and one negative. Saddle

Continued ... 9,10

$$\frac{\partial F_1}{\partial H} \Big|_{9,10} = H^*(-6 - 2H) \Big|_{9,10} = -216$$

$$\frac{\partial F_1}{\partial S} \Big|_{9,10} = H^*10 \Big|_{9,10} = 90$$

$$\frac{\partial F_2}{\partial H} \Big|_{9,10} = S^*2 \Big|_{9,10} = 20$$

$$\frac{\partial F_2}{\partial S} \Big|_{9,10} = S^*-1 \Big|_{9,10} = -10$$

So Jacobean is

$$\begin{bmatrix} -216 & 90 \\ 20 & -10 \end{bmatrix}$$

Eigenvalues: $(-216-\lambda)(-10-\lambda)-1800=\lambda^2 +226 \lambda +360 = 0$;
 $226^2 > 4*360$, so two negative values.

Thus 5,2 is unstable; 9.10 is stable

3 b)

(b) Suppose the host-symbiont model is defined by the following:

$$\frac{dH}{dt} = H(-20 - 4H - H^2 + 16S) \quad \frac{dS}{dt} = S(H - S)$$

Calculate suitable values so as to sketch on the phase plane the zero isoclines for values of S and H in the range 1 to 12. Mark on the sketch the sign of the derivatives of H and S and thus sketch the likely variation of H and S from four different initial values which illustrate what happens with low and high values of H and S .

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Answer

b) Isoclines calculated by finding where

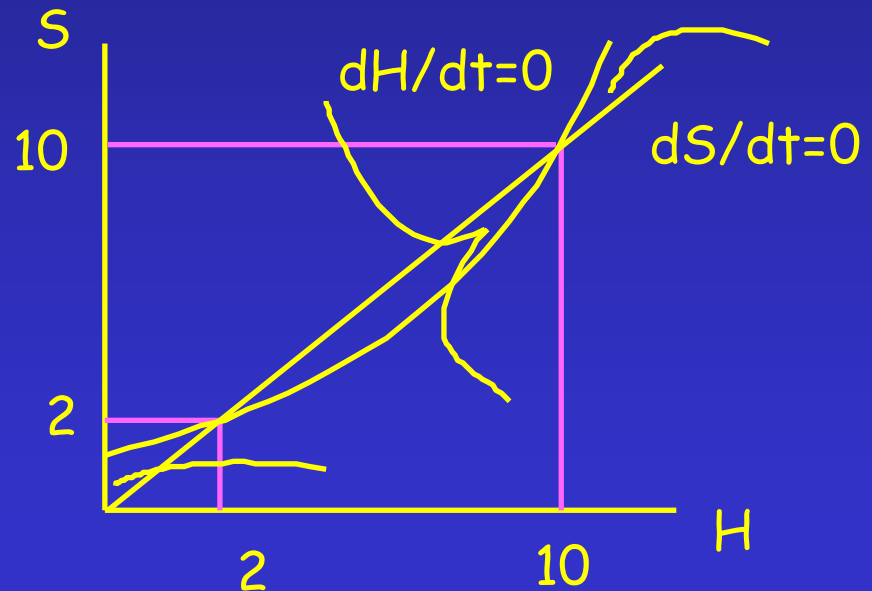
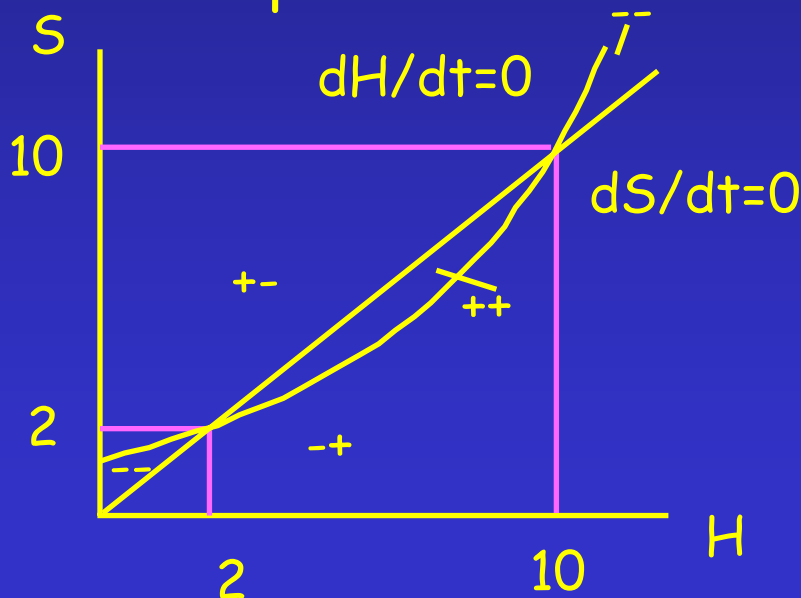
$$-20 - 4H - H^2 + 16S = 0 \text{ and } H - S = 0.$$

H: For $H = 1:12$; $S = (H^2 + 4H + 20)/16$

= 1.6 2.0 2.6 3.3 4.1 5.0 6.1 7.3 8.6 10.0 11.6 13.3

S: For $H = 1:12$; $S = 1:12$

Phase plane:



3 c)

(c) In one of the above models, the host is obligatory. Based on your sketches, state for which model this is true, justifying your answer. [3 Marks]

Answer

The host is obligatory if it cannot exist without the symbiont, and this is true for model b. This is because for model a, when the symbiont dies out, the host population reduces to a non-zero positive value, around 3.6; for model b, when the populations are too low, the trajectory is such that both populations tend to zero.

Question 4

4. You are required to write a short report arguing that the Gaia theory is true, for one of the five levels proposed by Kirchner.

Write the introduction and main body of the report.

[12 Marks]

Write the abstract.

[4 Marks]

Write the conclusion.

[4 Marks]

Answer

Report will need an intro to state the Gaia hypothesis, and Kirchner' spectrum:

Gaia: life an planet producing mutually favourable conditions

Kirchner

Influential

Biota has substantial influence over certain aspects of the world

Status Supported

Co-evolutionary

Biota influences abiotic environment, latter influences evolution of biota

Status Debated

Continued

Homeostatic

Interplay between biota and environment is characterised by stabilising -ve fb loops

Status: Debated

Teleological

Atmosphere kept in homeostasis not just by biosphere but in some sense for biosphere

Status: Daisy-world

Optimising

Biota manipulates its environments to creating favourable conditions for itself

Sceptical

Continued

Answer will then choose one - easiest would be the 'influential' one. Then there would need to be some examples backing this up ... for instance temperature control mention the effect of dark forests, acting like black daisies, to heat the Earth; white clouds over the ocean (the white clouds themselves being influenced by life) acting like white daisies to cool the Earth. Mention also bacterial life controlling evaporation used to remove salt from the sea, etc.

The abstract should be a carefully written paragraph consistent with the first part.

Similarly the conclusion should be brief and to the point, and relevant.