ACHIEVING THE STABILITY MARGINS IN BODE'S METHOD

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Overview

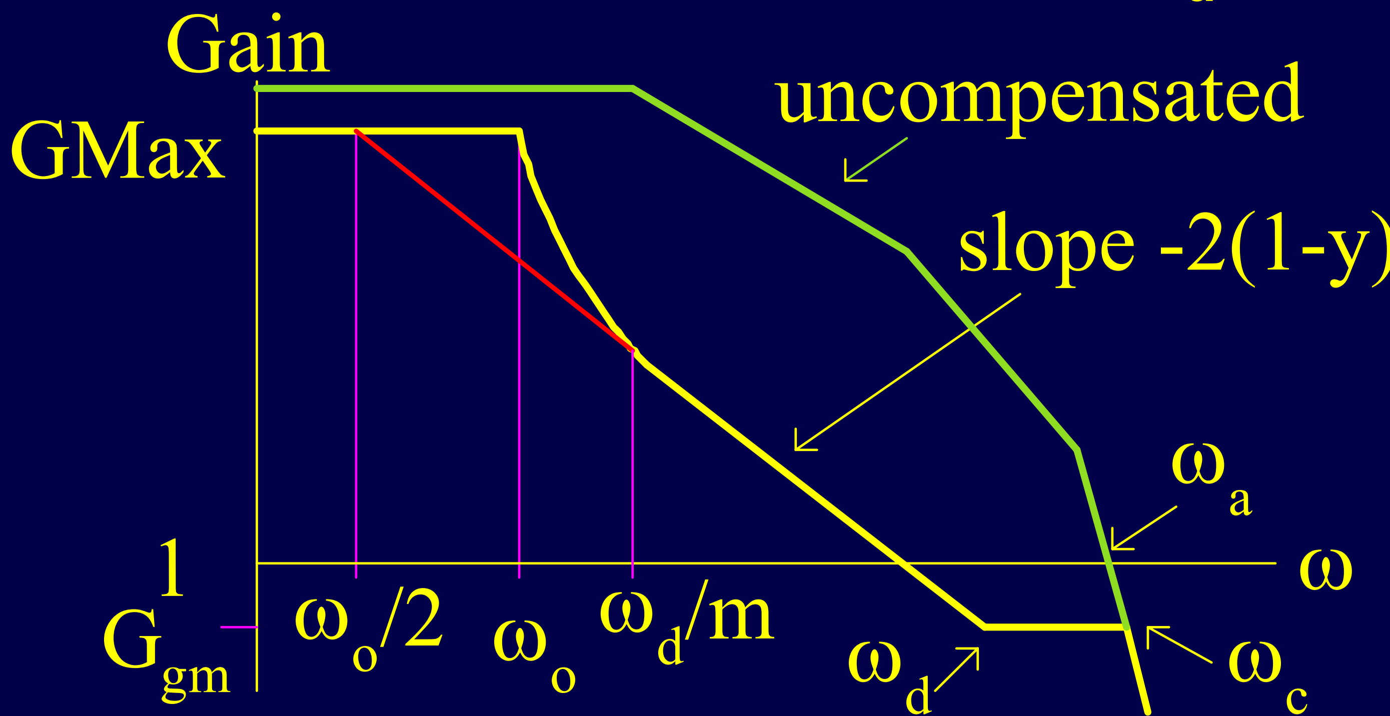
- Bode's fundamental work uses asymptotes to allow a system to be stabilised having max possible gain over a given bandwidth with suitable gain and phase margin

  (It's a method of placing poles/zeros)

- But, as uses asymptotes, actual margins can be very different from specified
- A solution is presented, in which margins are preprocessed before being applied

Frequency Shape for Bode's Design

Uncompensated: gain = 1 at ω_a when slope -n



Design
ω_o= bw
x = Gain Margin
y = Rel Phase Margin PM/180

Slope -2(1-y) → Phase = -180 + PM;
'Bode Step' $\omega_d..\omega_c$: cancel phase due to –n slope

Loop Transfer Function

$$\frac{GMax}{s^2/\omega_o^2 + s/\omega_o + 1} * \frac{1 + s/\omega_1}{1 + s/\omega_2} * \frac{(1 + s/\omega_d)^2}{(1 + s/\omega_c)^n}$$

Second order element for low freq response.

(easier for students to understand than Bode's irrational element)

Lead Lag to approximate slope -2(1-y)

Can be better to have multiple lead lags

But actual GM and PM differ from specified …

So Iterate Design for GM and PM

As actual margins differ from specified, do design, note errors and redesign

```
gms = gm; pms = pm;      % initialise specified gm/pm
DoBodeDesign;            % and calc gma and pma
while num of iterations < 8 & ...
        abs(gm – gma) + abs (pm – pma) > 0  do
    gms = gms - gma + gm;  % new gm to specify
    pms = pms - pma + pm;
    DoBodeDesign
end    % limit iterations in case algorithm cycles
```

Typical Result

(ω_a = 100 rad/s, n = 4)

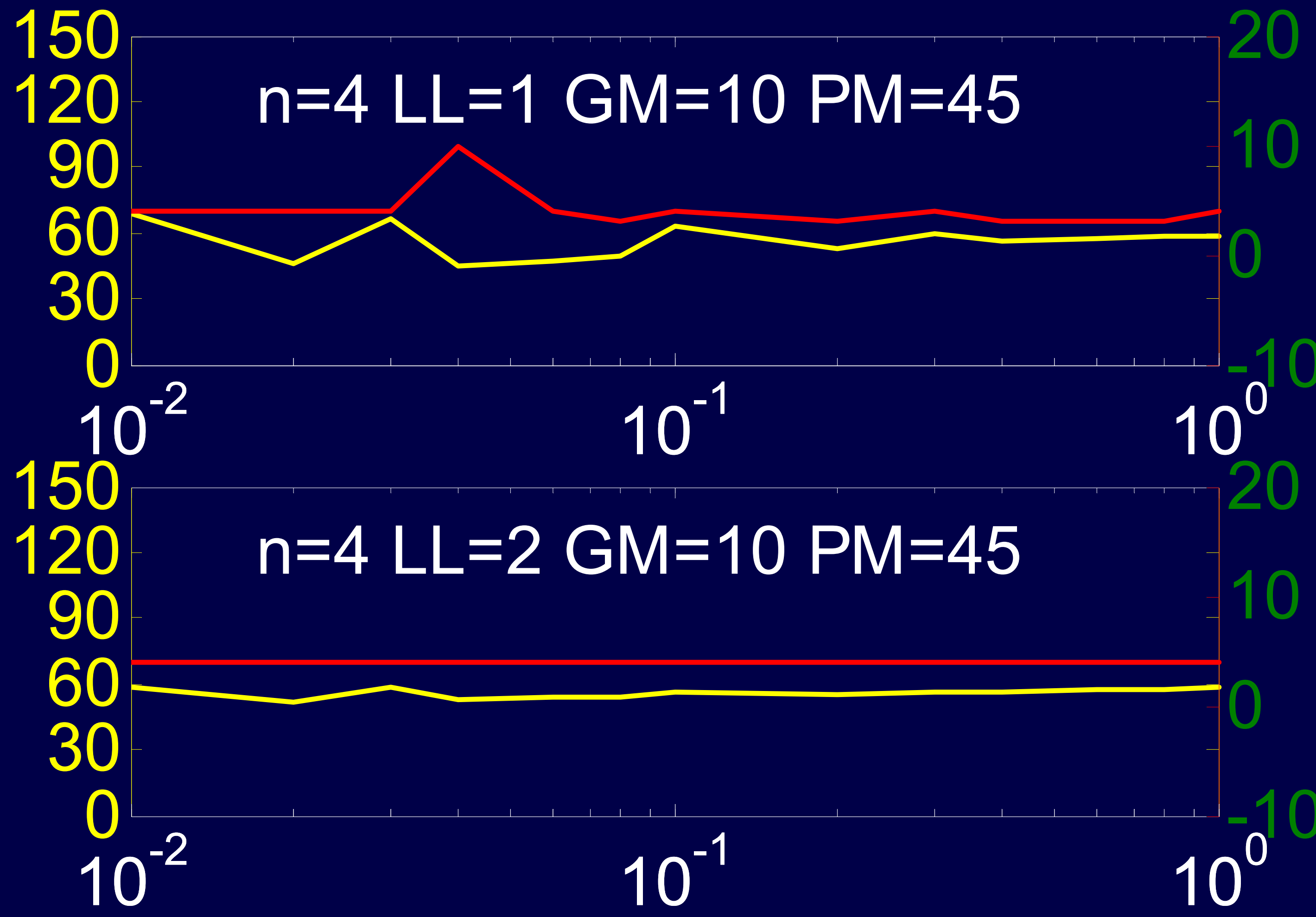
ω_o	LLag	GM	PM	GMax	GMact	PMact
0.03	1	15	45	42478	18	39
0.03	2	15	45	42478	19	48

Then do iteration, results of which

GMs	PMs	GMax	GMact	PMact
11	56	18629	15	45
11	45	56645	15	45

Run tests for different n, ω_o, GM and PM

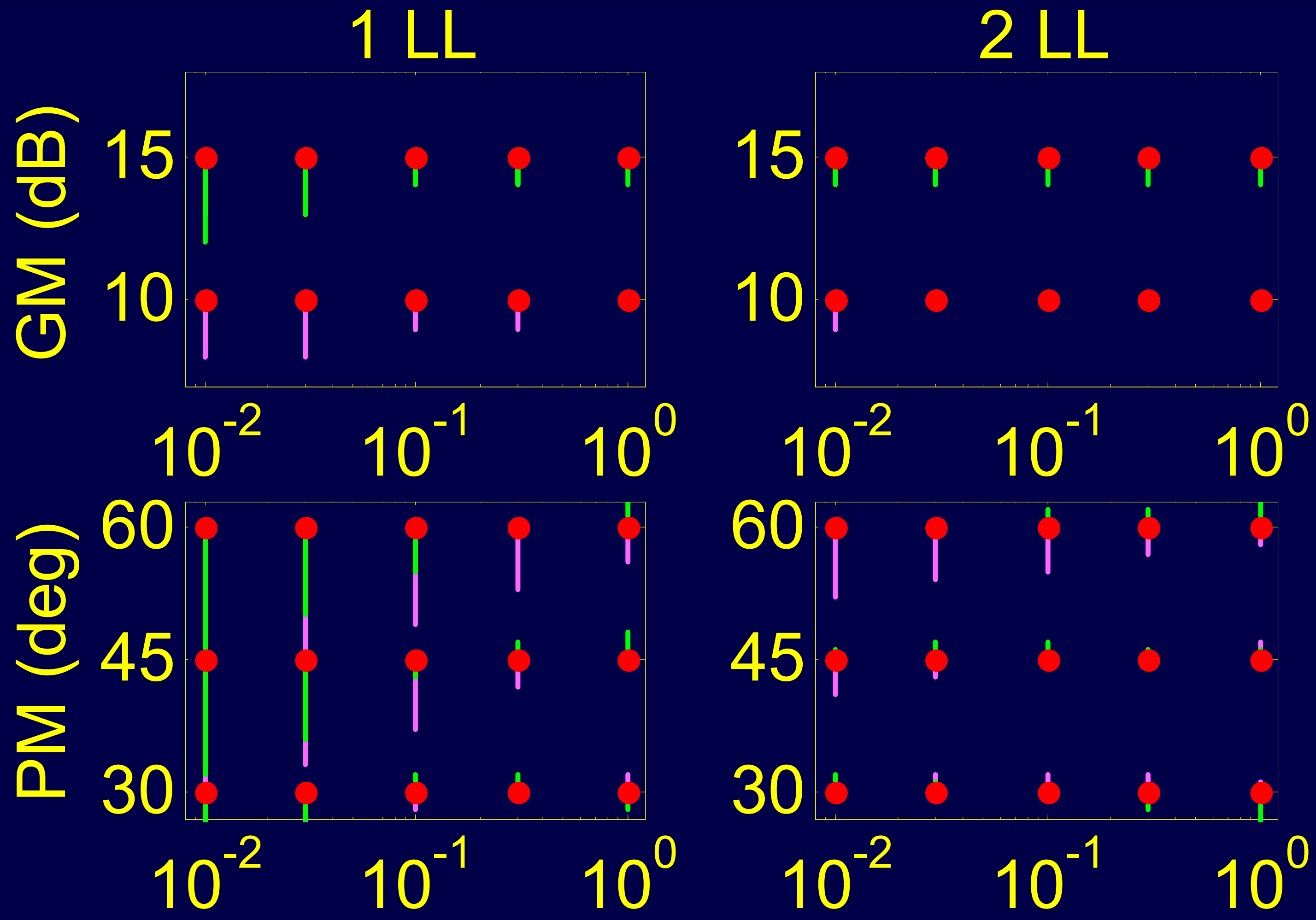
Graphs: GMs and PMs vs ω_o



Models

- Graphs show ~ linear relationship between ω_o and GMs and PMs, for different values of n, LLag, GM and PM; partic if use 2 LLag
- So produce separate models for each
-     GMs = Goff + Gfac * ω_o
-     PMs = Poff + Pfac * ω_o
- But use many models, so for each n, LLag:
-     GMs = G0 + $G\omega * \omega_o$ + Ggm*GM + Gpm*PM
-     PMs = P0 + $P\omega * \omega_o$ + Pgm*GM + Ppm*PM
- Both work, second nicer as fewer models

Results Actual GM/PM (for n = 4)



2LL better

For GM=15 dB, PM good, worse for 10dB

Conclusion

- Bode’s Maximum Available Feedback Method gives approximate response: often actual stability margins in error.
- However, particularly if have 2 lead-lags for ‘fractional slope’, a linear model can be used to pre-process the margins, the results being fed into Bode’s method, so that the desired margins are achieved.