ON MAXIMUM AVAILABLE FEEDBACK AND PID CONTROL

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Maximum Available Feedback is max loop gain over a specified bandwidth for given stability margins, in a single loop feedback system

Achieved by ensuring phase of (loop of) designed system is flat at key frequencies

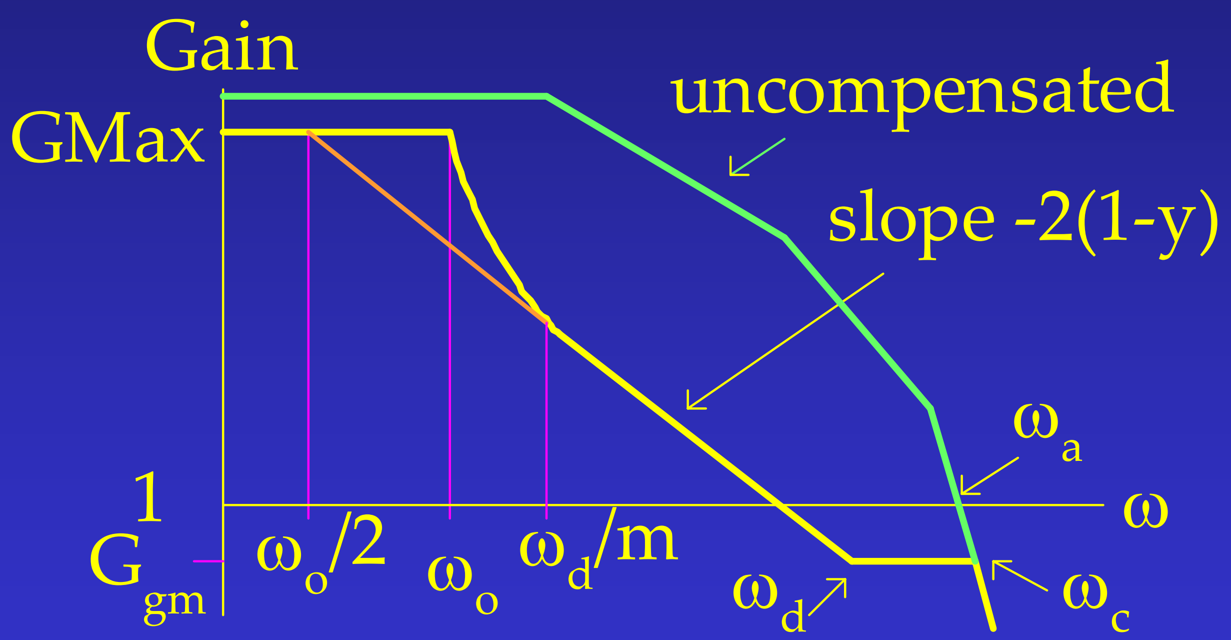
A recent IEEE SMC Paper describes a robust PID controller whose phase is flat at key frequencies

This paper contrasts the two designs.

Frequency Shape for Bode's Design

Uncompensated System: gain = 1 @ ω_a ; slope is –n

Specify ω_o= bw; Margins: x = GM; y = PM/180°

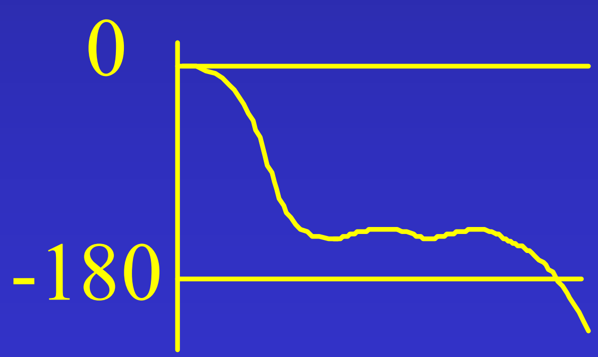


Slope -2(1-y) →
Phase -180°+PM
'Bode Step'
ω_d..ω_c: cancel
phase due to –n
slope + TD
Gain curves up
to double bw

Loop Transfer Function – 3 Parts

Design produces transfer function round loop

- Curved Part : low freq response
  - Third Order Element, corner freq ω_o
  - Lead half way between ω_o and ω_d / m
  - In effect slope -2.5 from ω_o to -2(1-y) slope
- 2+ Lead Lags approximate slope -2(1-y)
  - from ω_d / m to Bode Step (at ω_d)
  - NB Phase not actually flat



- Double Lead for Bode Step at ω_d
- Then n Lags at ω_c

Some Details + Author's Extensions

$$\text{GMax (in dB)} = 40(1 - y)\log_{10}\left(\frac{4(1-y)}{n}10^{\frac{x}{20n}}\frac{\omega_a}{\omega_o}\right) - x$$

Method produced for electronics, adapt for Control:

As gain may equal 1 before system poles, have 'dummy' amplifier which moves ω_a to suitable frequency, then apply method.

Can in fact start with specified Time-to-peak and overshoot to step response, hence estimate y and ω_d and then calculate ω_a and gain of dummy amp

'*m*' fixed so gain = 1 at local minimum of phase lag

Transfer Functions

As comparing with PID, gain slope = -1 up to ω_o.

So low freq achieved by

$$\frac{GMax * \omega_o}{s^2/\omega_o^2 + s/\omega_o + 1} \frac{s/\sqrt{\omega_o \omega_d / m} + 1}{s}$$

P lead-lags

$$\prod_{k=0}^{p-1} \frac{1 + s\, m^{\frac{0.5+y+k}{p}} \omega_d^{-1}}{1 + s\, m^{\frac{0.5-y+k}{p}} \omega_d^{-1}}$$

Bode Step

$$\frac{(1 + s/\omega_d)^2}{(1 + s/\omega_c)^n}$$

New PID Control Method

Y. Chen & K. L. Moore, Relay Feedback Tuning of Robust PID Controllers with Iso-Damping Property, IEEE Trans. SMC B Vol 35, 1, 23-31, 2005

'Modified Ziegler Nichols' PID design moves point on Nyquist locus at particular freq – defines P and I terms; D fixed multiple of I : factor 4.

New method sets D term so phase at this freq is flat: robust as if gain changes, phase & o/s change less.

Similar to Bode's method re PM, but are subtle diffs

More Details

Plant to control defined as $P(s) = \frac{\tilde{P}(s)}{s^m} e^{-s\tau}$

Allows method to work with TDs and integrators

PID controller calculated as

$$C(s) = K_p\left(1 + \frac{1}{sT_i} + sT_d\right)$$

Designed so at 'tangent freq' ω_t, the phase is Φ_m

NB Locus meets sensitivity circle at ω_t: gain $\cos(\Phi_m)$

This < 1 so Φ_m not the phase margin but close

Calculations

Then, if it is defined that $\hat{\Phi} = \Phi_m - \angle P(j\omega_t)$ and

$$s_p(\omega_x)\left\{= \omega_x * \frac{d\angle P(\omega)}{d\omega}\bigg|_{\omega_x}\right\} = \angle\tilde{P}(j\omega_x) + \frac{2}{\pi}\ln\left(\frac{\left|\tilde{P}(j0)\right|}{\left|\tilde{P}(j\omega_x)\right|}\right)$$

and $\Delta = T_i^2\omega_t^2 - 8s_p(\omega_t)\,T_i\omega_t - 4T_i^2\omega_t^2 s_p^2(\omega_t)$

$$K_p = \frac{\left|\cos(\Phi_m)\right|}{\left|P(j\omega_t)\sqrt{1+\tan^2(\hat{\Phi}_m)}\right|} \qquad T_d = \frac{-T_i\omega_t + 2\,s_p(\omega_t) + \sqrt{\Delta}}{2\,s_p(\omega_t)\;\omega_t^2 T_i}$$

$$T_i = \frac{-2}{\omega_t(s_p(\omega_t) + \tan(\hat{\Phi}) + \tan^2(\hat{\Phi})\,s_p(\omega_t))}$$

Example

In paper three plants are tested, presented here is result of third, that for others are similar.

$$P_1(s) = \frac{1}{(1+s)^5} \qquad P_2(s) = \frac{e^{-s}}{(1+s)^3}$$

Here just show $P_3(s) = \dfrac{180}{(s+3)(s+6)(s+10)}$

Do design using this PID controller and modified ZN, using $\omega_t = 7$ rad/s with $\Phi_m = 45^O$

Get o/s ~25% T_{pk} 0.5s.

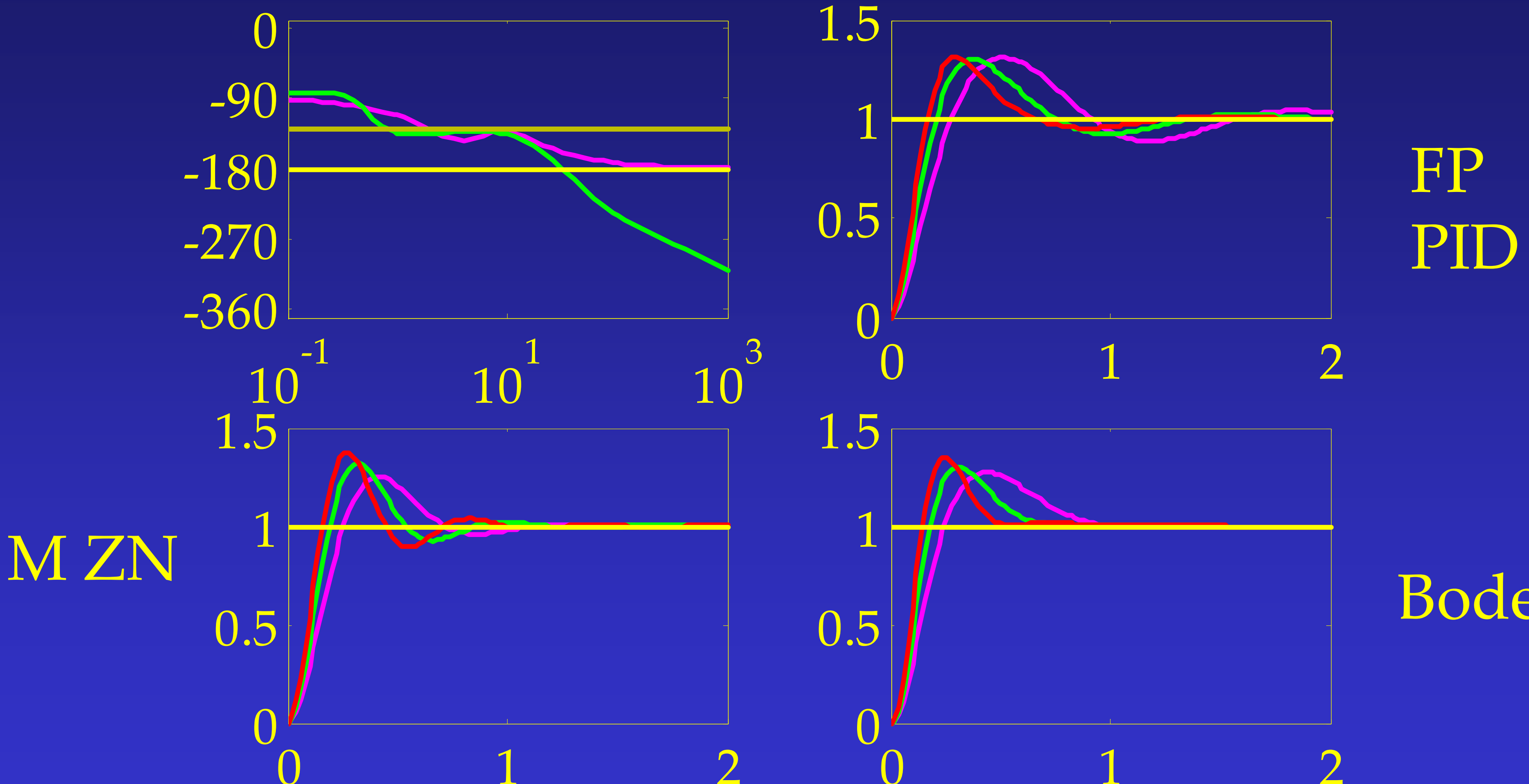
Also do Bode design for same o/s and T_{pk}

Table of Results

Table below shows response of three methods and when P gain changed by factor of 1.5 and of 2, but controllers unchanged.

	Gain * 1			* 1.5		* 2	
	T_{pk}	%os	T_{set}	T_{pk}	%os	T_{pk}	%os
FP PID	0.515	30.9	2.7	0.377	30.1	0.299	31.2
M ZN	0.414	25.0	1.83	0.321	31.9	0.272	37.0
Bode	0.440	27.4	1.22	0.310	30.1	0.249	35.2

Phase of PID + Bode; Step of all 3



M ZN least robust; FP PID more robust but slower

Conclusion

Robust PID Controller is most robust of the three methods re changes in Plant gain

But is slower than Bode design

Both are preferable to Modified Zieger Nichols, which is much less robust.

Bode also better at disturbance rejection (see paper)

Further work needed to look at more examples …

and I have a student whose project is to do just that