Tutorial – Week 6 – Qs 1, 2 and 3

6.1 Find the following differentials,

a) $\frac{d(5 - 5\exp(-4t))}{dt}$ b) $\frac{d\ 0.25*\ln(2x)}{dx}$ c) $\frac{d\cosh(5x)}{dx}$

6.2 Find the following integrals,

a) $\int 10\exp(-0.1x)\,dx$ b) $\int_0^1 t\text{-}5\exp(-2t)\,dt$ c) $\int 3\cosh(4t)\,dt$

6.3 a) Find the equation of the tangent to the graph given by $y = 2x - 5\exp(-3x)$ at $x = 1$.

b) Find the mean of $f(t) = e^{3t}$ from t= -2 to 1;

Tutorial – Week 6 – Qs 4 and 5

6.4 The voltage V in a RC circuit is given by the following when the input voltage is 0.1 t; $V = 0.1\,e^{-t} + 0.1\,t - 0.1$

a) Find $\frac{dV}{dt}$

b) Show that V is a solution of the equation $\frac{dV}{dt} = 0.1t - V$

6.5 The phase shift of an RC circuit is $P = -\tan^{-1}0.5\omega$.

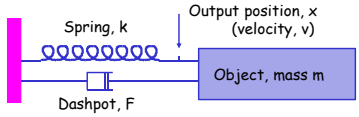
Find $\frac{dP}{d\omega}$.

Tutorial – Week 6 – Q 6

6.6 For the mass spring system below, the accelaration of the mass, $\frac{d^2x}{dt^2}$, is given by $-4\,e^{-t} + 9\,e^{-3t}$. At $t = 0$, its velocity is 1 m/s and its position is 2m.

a) Find an expression for position x.

b) Show that $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = 15$



Tutorial - Week 6 - Hints

6.1 Apply standard rules

6.2 Ditto

6.3 a) m = dy/dx at x = 1; c = y(1) – m

b) Use standard integral for means

6.4 a) straightforward

b) show both sides of equation equal.

6.5 Use inverse function – note diff of tan(x) is $\sec^2(x)$.

6.6 a) Integrate, find constant, integrate, find constant

b) evaluate left hand side of equation – show is 15.

Tutorial – Week 7 – Q1, 2 and 3

7.1 The current i through a diode in terms of voltage V across it is

$i = 0.05(e^{200V} - 1)$

Find the a.c. resistance r which is defined as $r = 1 \Big/ \frac{di}{dV}$.

7.2 In reliability engineering, the distribution function, F,

for a set of components is given by $F = 5 - \frac{30}{6 + 0.25\,t}$

Find the density function = $\frac{dF}{dt}$.

7.3. The gain of a mass-spring, as a function of ω, is

$G = \frac{1}{\sqrt{\omega^4 - 9\omega^2 + 25}}$. Find values of ω where $\frac{dG}{d\omega} = 0$.

Calculate G at all these values to find where G max.

Tutorial – Week 7 – Q4, 5 and 6

7.4 The position, s, of an object moving in a straight line is

$s = \int \frac{8}{\sqrt{40t + 1}}\,dt$.

Find s given that s = 2 when t = 0.

7.5 The sinusoidal output of a system is given by

$O = 5 \sin(4t - 0.2)$

Find its mean value between t = 0 and t = 0.5.

7.6. Find its rms value between t = 0 and t = 0.5.

Tutorial - Week 7 - Hints

7.1 Use Chain rule to find di/dV…

7.2 Again use the chain rule

7.3 Again the chain rule – look for where numerator is 0 – remember can have negative frequencies.

7.4 Use substitution

7.5 and 6

Use same substitution for both mean and rms – remember to change limits – and to use radians.

Tutorial – Week 8 – Q1, 2 and 3

8.1 a) Find the volume of revolution of the solid formed when $y^2 = 4ax$ is rotated about the x-axis from x = 0 to1.

b) A parabolic reflector is formed by rotating $y = 2\sqrt{x}$ about the x-axis, from x = 0 to 1. Find its surface area.

8.2 An aerofoil is defined by $y^2 = 1 - \frac{x^2}{4}$ for $-2 \le x \le 2$

Find the area below this function and above the x-axis.

8.3 The velocity v of a falling object is given by $\frac{dv}{dt} = g - 0.5v$

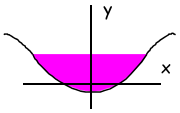
Show that $v = 2g(1-e^{-0.5t})$ if v = 0 at t = 0

Tutorial – Week 8 – Q4 and 5

8.4 The velocity v of a falling object is given by $v = 2g(1-e^{-0.5t})$. Find the distance dropped at time t.

8.5 The figure shows water in a prismatic channel, for which the water height y is defined in terms of horizontal position x by the equation

$$\frac{dy}{dx} = \frac{1}{\sqrt{16x^2 - 1}}$$



Find an expression for y if y = 0 when x = 0.25:

Tutorial – Week 8 – Q6 Extra

8.6 The logistic equation for population P is $\frac{dP}{dt} = P(10 - P)$

a) Show that $t = \int \frac{0.1}{P} + \frac{0.1}{10-P}\, dP$

b) Find P(t) if P = 20 at time t = 0 in the form $P = \frac{A}{1-Be^{-ct}}$

Tutorial - Week 8 - Hints

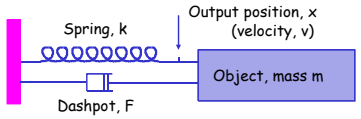
8.1 Use relevant formulae and integrate.
8.2 Is integral of a squareroot – use trig substitution.
8.3 Invert eqn: find t(v) -> v(t). 8.4 Integrate.
8.5 Use relevant hyperbolic substitution.
8.6 Invert eqn, use Partial Fractions and proceed

Tutorial – Week 9 – Q1

9.1 In the mass-spring system below, the position of the mass is given by $x = 2 + e^{-2t}(3\sin(t) + 2\cos(t))$

a) Show that at time t = 0, the velocity v is -1 m/s.

b) Find $\frac{d^2x}{dt^2}$ and hence show that $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 10$



m = 1 kg;
F = 4 Ns/m;
k = 5 N/m

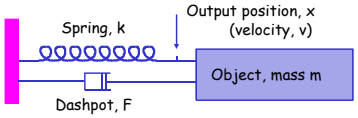
Tutorial – Week 9 – Q2

9.2 For the mass-spring system, v, the mass' velocity, is given by $v = e^{-4t} - 20\,t\,e^{-4t}$

a) Find $\int 20te^{-4t}\,dt$

b) Hence find x, the position of the mass, if at t = 0, x = 3m.

c) Find $\frac{d^2x}{dt^2}$ and so show that $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 16x = 32$



m = 1 kg;
F = 8 Ns/m;
k = 16 N/m

Tutorial – Week 9 – Q3

9.3 Differentiate the following using the quotient rule

a) f(x) = sinc(x) = $\frac{\sin(\pi x)}{\pi x}$ [used in information theory]

b) The displacement of a damped vertical pendulum

$x = \frac{\cos(5t)}{\exp(2t)}$

c) The voltage in an electronic circuit

$V = \frac{t+2}{\exp(0.1t)}$

Tutorial – Week 9 – Q4,5 and 6

9.4 The gain of an electronic circuit, in terms of angular frequency ω is given by $G = \frac{\sqrt{1+\omega^2}}{3+\omega^2}$

Find ω such that G is maximised.

9.5 Find the a_n term for the Fourier Series of a sawtooth.

ie. find $a_n = \frac{2}{\pi}\int_{-\pi/2}^{\pi/2} t\cos(2nt)\,dt$ { recall n is constant }

9.6. Find area A under of Kte^{-at} for t>0 : K and a constant

i.e. solve $A = \int_0^{\infty} K\,t\,e^{-at}dt$; Nb as $t \to \infty$, so $t\,e^{-at} \to 0$

Extra!

Tutorial – Week 9 – Hints

9.1 a) Find dx/dt, and put t = 0. Use diff of product

b) Use diff of product and then show LHS = RHS

9.2 a) Choose u and dv/dt as per examples

b) Straightforward

c) Diff of a product than show LHS = RHS

9.3 Straightforward.

9.4 Use diff of quotient to find dG/dw, find w where this is zero and evaluate G at these values.

9.5 Use Integration by Parts

9.6 Ditto

Tutorial – Week 10 – Q1 and 2

10.1 Small Changes - find change in

a) surface area of a sphere when its radius changes from 10 to 10.1 m. [Surface area = $4\pi r^2$]

b) power in circuit when current changes from 1mA to 0.99 mA, when passing through 10kΩ resistor.

[If I is current going through resistor R, Power = I^2R]

c) gain of RC circuit when angular frequency ω falls by 1% from 0.1 rad/s; if R*C = 10 then Gain $G = \frac{1}{\sqrt{1+\omega^2 100}}$

10.2 For $f = t^3$ estimate its differential at t = 1, for h = 0.1 using the formula $f'(t) \approx \frac{f(t+h) - f(t-h)}{2*h}$. Comment.

Tutorial – Week 10 – Q3 and Q4

10.3 Evaluate $\int_1^5 \sin(0.1t)\,dt$ analytically.

Compare results with Rectangular Trapezoidal and Simpson methods where h is 1.

10.4 Revision

The voltage across the capacitor in a RLC series circuit is

$V = 4 - e^{-t}(\sin(3t) + 3\cos(3t))$

a) Find the current in the circuit, being $I = 0.05\frac{dV}{dt}$

b) Find the voltage across the inductor, being $V_L = 2\frac{dI}{dt}$

c) Verify that $4I + V_L + V = 4$

Tutorial – Week 10 – Q5

10.5 Revision

a) Expand $\frac{d(5Ve^{0.2t})}{dt}$

b) An RC circuit is described by $5\frac{dV}{dt} + V = t$

Show that $V = 0.2e^{-0.2t}\int t\,e^{0.2t}\,dt$

c) Use integration by parts to find $\int te^{0.2t}\,dt$

d) Hence find V given that V = 1 at time t = 0.

Hints Q1, 2 and 3 use methods in this week's notes

Q4 and 5 – look back in previous lectures