Tutorials

Attendance is required at all tutorials BECAUSE the best way to learn maths is to practice, and this you do, with help, in tutorials.

Help is provided – do ask if you are stuck

Complete questions in own time.

Marks contribute to the end of year module mark

Tutorial questions are with lecture notes – look in advance

Bring notes, books + calculator (paper + pen) to tutorials

Write your name clearly at the top of the paper.

Then attempt the questions

Note, there are 'hints' for each Q if you are stuck.

Please feel free to go to Maths Support Centre as well

Tutorial Week 1

1) Suppose O = (I – O) * C * P, derive an expression for O as a function of I, C and P only.

2) RC Circuit: suppose E = 10V; R = 2Ω and C = 0.05F, so T = 0.1s.

Calculate at t = 0, 0.1, 0.2, 0.3, 0.4, 0.5 :

I = E/R * exp(-t/T) and V = E - E *exp(-t/T).

Plot points on graphs of I vs t and V vs t.

Draw smooth curves through these points.

Add tangent to V at t = 0, 0.1 and 0.3 – what is its slope?

Is there a relationship between slope of V and I?

Continued

3) For t = 0, π/6, π/3, π/2, 2π/3, 5π/6 and π:

calculate sin(t) and cos(t) [remember to use radians].

Plot points on graph of sin(t) vs t and cos(t) vs t.

Sketch smooth curve through the points.

Add tangent to sin(t) at t = 0, π/3, π/2, estimate their slopes.

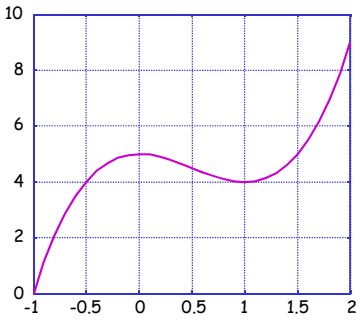
Compare the slopes with cos(t).

4) Repeat for sin(2t) and cos(2t).

Hints. Q1 – see slide on Laplace Operator and RC Circuit.

Q2-4 Calculate values using your calculator, plot some points on graphs, and sketch curves through them. Try to be reasonably accurate, but you don't have to be perfect!

Tutorial – Week 2 – Q1



2.1 $y = 2x^3 - 3x^2 + 5$

a) What is $\frac{dy}{dx}$?

b) Draw tangents at x = -1, 0, 1 and 2 and evaluate their slope.

c) Show answers are consistent.

Tutorial – Week 2 – Q2 and 3

2.2. For the following,

write down the differential of the function

find the equation of the tangent at the given point

a) function: $y = 3x^4$ tangent at x = 2

b) function: $y = 12x^5 - 2x^2 + 5$ tangent at x = 1

c) function: $s = 3t + \frac{1}{2} 7t^2$ tangent at t = 4

2.3 Find the following integrals,

a) $\int 10t^4 \, dt$ b) $\int 5t^3 - 6t \, dt$

c) $\int 2\pi r \, dr$ d) $\int 4\pi r^2 \, dr$ e) $\int 7 + 5t \, dt$

Tutorial – Week 2 – Q4 and 5

2.4 a) An object is accelerating at $3ms^{-2}$ and at time t = 1 s its velocity is $2ms^{-1}$, and at t = 4 s its position is 25m. What are its velocity and position at time t = 10s?

b) Suppose its accelation is 6 t ms^{-2} and at time t = 1 s its velocity is 1 ms^{-1} and its position is 2m. What are its velocity and position at t = 5s?

2.5. The current flowing into a 0.2F capacitor is $i = 2t - 8t^3$

The voltage across it is $\frac{1}{C}\int i \, dt$, and this is 4V at time 1s.

Find an expression for this voltage.

Tutorial – Week 2 – Q6 and Hints

2.6. The voltage across an L = 0.1H inductor is $v = 6t^2 - 3t^4 + 2$

The current through it, $\frac{1}{L}\int v \, dt$, is 3A at time 2s.

Find an expression for this current.

Hints

2.1 : a) use formula, b) draw lines over t = 0.5 touch curve

2.2 : can write down function. See slide on finding tangent

2.3 : Use formulae

2.4 : see slide on 'constants can mean something'

2.5 : Do indefinite integral, use initial value to find const

2.6 : Ditto

Tutorial – Week 3 – Q1, 2 and 3

3.1 Complete the following $\frac{d(t^4)}{dt} = \lim_{\delta t \to 0} \frac{(t+\delta t)^4 - (t)^4}{\delta t}$

3.2. Find the following differentials

a) $\frac{d(t^7)}{dt}$ b) $\frac{d(4t^{-3})}{dt}$ c) $\frac{d(3x^{1/2})}{dx}$ d) $\frac{d(\frac{4}{3}\pi r^3)}{dr}$

3.3. Find the following integrals

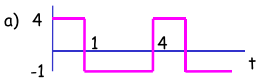
a) $\int t^7 dt$ b) $\int_{-1}^{2} 5t^{-2} dt$ c) $\int 3x^{1/2}\, dx$ d) $\int_{0.1}^{0.5} 4\pi r^2 dr$

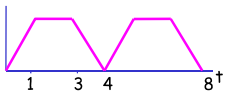
Tutorial – Week 3 – Q4, 5 and 6

3.4. Expand the right hand side of $y = (x-1)^2(x-3)$:
Find the area under the curve between x = 1 and x = 3.

3.5. Find the mean of $f(t) = t^3 - 6t^2 + 12t - 10$ between t = 1 and t = 3.

3.6. Evaluate the mean of the following periodic signals

a) 

b) 

Tutorial Week 3 Hints

3.1	Look at notes for t^3 – Remember 1 4 6 4 1
3.2	Use standard rules
3.3	Ditto
3.4	Expand out; then integrate and evaluate
3.5	Just integrate & evaluate (remember to div by 2)
3.6	Derive f(t) for each line, means will be the summation of integrals of each line.

Tutorial – Week 4 – Q1 and 2

4.1. For $y = 2x^3 + 3x^2 - 72x + 5$;

Find maxima and minima.

Confirm whether each is max/min.

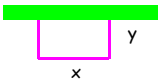
Sketch curve from x = -5..5.

4.2. For $y = 20 + 12x - 3x^2 - 2x^3$, find x such that $\frac{dy}{dx} = 0$.

Evaluate y at these and relevant values to sketch graph for values of x between -3 and +3. Find all roots of y - use Newton Raphson where appropriate.

Tutorial – Week 4 – Q3 and 4

4.3 100m of fence are to be built against a hedge, as shown in the figure.



Derive an expression for the area enclosed as a function of x only, and hence find what x and y should be if the enclosed area is to be maximised.

4.4. Newton-Raphson is to be used on $f(t) = t^3 - 4t^2 - 11t + 30$:

a) What is f'(t)?

b) Why would you not estimate a root starting at t = -1 ?

c) Find a root starting at t = 4 accurate to 4dp

d) Find a root starting at t = -2 accurate to 4dp

Tutorial – Week 4 – Q5 and 6

4.5 Searching sorted data is quicker than unsorted data, but it takes time to sort the data. Suppose a computer sorts n out of 100 items and searches these and the unsorted data separately. The time taken to find data is given by

$$t = 0.0250n^2 + 4.55(100-n) + 9*n^{1/2}$$

Show t is max/min at n = 1 and n = 81 respectively.

4.6 A cantilever of length L bends under its own weight.

At distance x from its base the droop y is given by

$y = k\left(x^4 - 4Lx^3 + 4L^2x^2\right)$ k is a constant

Find the value of x where the droop is maximised.

Tutorial – Week 4 – Hints

4.1 Find max/min using first and second differentials.

Calculate appropriate values and so sketch curve.

4.2 Again find max/min, evaluate and sketch.

One of these gives a repeated root. Your sketch should show a good starting value for the final root.

4.3 Find expression for y, thence one for A with x only.

Then differentiate …

4.4 Find f'(t); find f'(-1); then apply method.

4.5 Find dt/dn and d^2t/dn^2 at these values.

4.6 Find dy/dx and values of x (as function of L) where dy/dx is zero; evaluate d^2y/dx^2 at these values.

Tutorial – Week 5 - Q1 and 2

5.1 Given $\cos^2(x)+\sin^2(x)=1$, and $\cos(2x)=\cos^2(x)-\sin^2(x)$, derive an expression for $\cos(2x)$ not including $\sin^2(x)$.

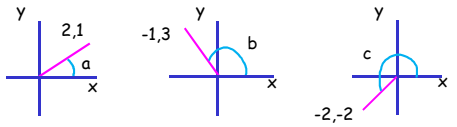
Rearranging your answer, find $\frac{d(\cos^2(x))}{dx}$ and $\int \cos^2(x)\,dx$

5.2 For a pendulum oscillating without friction, if θ is the angle of the string from the vertical, $\theta = -0.01 \sin(10t)$

a) Find $\frac{d^2\theta}{dt^2}$; b) Show that $\frac{d^2\theta}{dt^2} + 100\,\theta = 0$

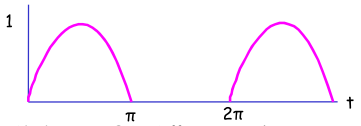
Tutorial – Week 5 – Q3, 4 and 5

5.3 Find angles a, b and c in the following (in degrees):



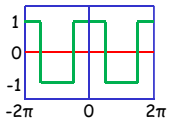
5.4 Express $\sqrt{8}\sin(\omega t) - \sqrt{8}\cos(\omega t)$ in the form $K\sin(\omega t+\phi)$

5.5 Evaluate the mean of the half wave rectified sinusoid:



Tutorial – Week 5 – Q6

5.6 Find the Fourier coefficients a_n and b_n for the square wave.



Tutorial – Week 5 – Hints

5.1 Straightforward; Rearrange to use cos(2a)
5.2 Differentiate twice, evaluate LHS of equation
5.3 Apply the algorithm for each angle
5.4 Find K and then the angle
5.5 and 5.6 Use standard integrals