Artificial Life

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Soft, Hard and Wet (biological/chemical) approaches
Introductions, Theory and Applications
Aims of Module

Aims:

Swarm Intelligence and Artificial Life are two active areas of research in computational optimisation and modelling. This module aims to inspire students into exploring the creative potential of these fields as well as providing insight into the state-of-the-art.

So - will describe work in A-Life:

Overview + Soft, Hard, Wet

Fundamentals & concepts, Progress & achievements

And include latest research presentations - theory and method ; advances and applications
Assessment – 100% coursework

Presentation of academic paper: 30%
  Do in pairs (one group of 3, unless one wants to be on own)
  Find a recent relevant paper (journal/book chapter)
  Read paper and then develop 6 minute presentation on it
  Presentation to be given this Friday afternoon

Web Page: 70%
  For start of next term, develop web page on swarm intelligence and/or artificial life
  Must include novel applet (or video of applet) illustrating work
  Should be eye catching and interesting

Afternoons this week for finding paper, preparing presentation, as well as looking at notes, following up information
History of A Life

Probably the first to actively study and write on related topics was John Von Neumann, mid 20th Century

In “The General and Logical Theory of Automata” he proposed that living organisms are just machines.

He also studied machine self replication, suggesting an organism must contain list of instructions on how to copy itself.

Predating discovery of DNA (Crick, Watson, Franklin, Wilkins)

Also significant, Mathematical Games column in Scientific American, which publicised John Conway’s Cellular Automaton ideas (1960s)

The term ‘artificial life’ was coined by Chris Langton, late 1980s.

He was also responsible for the first specific conference (on Synthesis and Simulation of Living Systems)
What is Life?

“What was life? No one knew. It was undoubtedly aware of itself, so soon as it was life; but it did not know what it was”.

Thomas Mann [1924]

“Life is a dynamic state of matter organized by information”.

Manfred Eigen [1992]

“Life is a complex system for information storage and processing”.

Minoru Kanehisa [2000]

The general condition that distinguishes organisms from inorganic objects and dead organisms, being manifested by growth through metabolism, a means of reproduction, and internal regulation in response to the environment.

Websters Dictionary (other defs also)
What living things have in common

http://www.windows2universe.org/earth/Life/life1.html says biologists have determined that all living things share these:

- Living things need to take in energy
- Living things get rid of waste
- Living things grow and develop
- Living things respond to their environment
- Living things reproduce and pass their traits onto their offspring
- Over time, living things evolve (change slowly) in response to their environment
Also difficult: where did Life come from?

Geogenesis:
Life started on Earth, in a relatively short period of time
Atomic synthesis of C, N, O elements complicated
Exact Conditions required to bootstrap life unknown
Not observed new life being created from elements

Exogenesis:
Life started on an equivalent of Earth
Life (or necessary components) travelled through space
Seeded life then flourished on Earth

Panspermia:
Life (and seeds of life) exists throughout the universe
Life could exist (and may already exist) elsewhere in the universe.

Could A-Life help resolve the uncertainty?
Overview of ALife

Promotion:
Artificial Life, a field that seeks to increase the role of synthesis in the study of biological phenomena, has great potential, both for unlocking the secrets of life and for raising a host of disturbing issues—scientific and technical as well as philosophical and ethical.

Christopher G. Langton

Academic:
Artificial Life ... investigates the scientific, engineering, philosophical, and social issues involved in our rapidly increasing technological ability to synthesize life-like behaviors from scratch in computers, machines, molecules, and other alternative media.

Artificial Life - Journal MIT press

Synthesis:
To make a synthesis of; to put together or combine into a complex whole; to make up by combination of parts or elements.

Oxford English Dictionary
AL views life as a property of the organization of matter, rather than a property of the matter which is so organized.

Whereas biology has largely concerned itself with the material basis of life, AL is concerned with the formal basis of life.

It starts at the bottom, viewing an organism as a large population of simple machines, and works upwards synthetically from there — constructing large aggregates of simple, rule-governed objects which interact with one another nonlinearly in the support of life-like, global dynamics.

The 'key' concept in AL is emergent behavior.”

AL is concerned with tuning the behaviors of such low-level machines that the behavior that emerges at the global level is essentially the same as some behavior exhibited by a natural living system. [...] Artificial Life is concerned with generating lifelike behavior.”
Alife Underview

Alife is “Fact Free Science”  
John Maynard Smith, 1994

Instead of testing a hypothesis on observable data, Alife seeks to synthesise life like behaviour in agents.

[Strong AL vs Weak AL debate as in with AI]

An agent has a set of assigned properties, components or abilities but not globally defined behaviours.

Emergence of global behaviours from local interactions is desired – Alife overlaps with Complex Systems

This 'bottom-up' approach with feedback & environmental interaction has similarities with Cybernetics

Next two slide shows where A Life fits with other disciplines.
How A Life fits into AI

- Genetic Evolutionary Comp.
- Artificial Life
- Fuzzy Logic
- Learning
- Enumeratives
- INTELLIGENT AGENTS
- Knowledge Based
- Expert Systems
- Decision Support
- Case Based Reasoning
- Back-tracking
- Dynamic Programming
- Branch & Bound
- GUIDED
- Las Vegas
- Neuronal Networks
- GUIDED
- Tabu Search
- simulated Annealing
- LCS
- GAs
- Evolution Strategies & Programming
- Genetic Programming
- Kohonen
- Mlp's
- Hopfield
- IMMUNE SYSTEMS
- Ant Colony
- Cellular Automata
Conference Topics - www.alifexi.org/cfp/

Synthesis and origin of life, self-organization, self-replication, artificial chemistries

Evolution and adaptation, evolutionary dynamics, evolutionary games, coevolution, major evolutionary transitions, ecosystems

Development, differentiation, regulation; generative representations

Synthetic biology

Self-organizing technology, self-computing/computational ecosystems

Unconventional and biologically inspired computing

Bio-inspired robots and embodied cognition, autonomous agents, evolutionary robotics

Collective behavior, communication, cooperation

Artificial consciousness; the relationship between life and mind
Continued

Philosophical, ethical, and cultural implications
Mathematical and philosophical foundations of Alife
Evolution in the Brain; Artificial Consciousness: From Alife to Mind
Communication in Embodied Agents
Designing for Self; Amorphous and Soft Robotics
Dynamical Systems Analysis
Trophic Interactions Between Digital Organisms
Autonomous Energy Management for Long Lived Robots
Models for Gaia Theory - including Daisyworld
The Environment and Evolution; Hidden Epistemology
Artificial Life Already?

Soft:
- Cellular Automata,
- Boids,
- Evolutionary algorithms?

Hard:
- Self-replicating machines,
- Self-building robots?

Wet:
- Rat-brained robots,
- DNA cartridges?

Not all criteria for life met.

Especially, adaptability: equilibrium is punctuated & truncated.
Soft A - Life

A Life Components

Soft    Hard    Wet

Lets start with Software and Modelling
flocking,
cellular automata
modelling daisyworld
modelling ‘real life’
attractors and discrete models
fractals and self-similarity

We start by looking at flocking
Flocking

Alife about interacting systems ... So flocking

Collective motion:
  Fish in schools, sheep in herds, birds in flocks, lobsters in lines

Characteristics of animal aggregations:
  Distinctive edges
  Freedom to move within own volume
  Coordinated movement

Benefits of Flocking
  Predator protection
  group foraging
  Social advantages - mating
Craig Reynolds & “boids”

http://cmol.nbi.dk/models/boids/boids.html

3 rules

Separation
Steer to avoid crowding with local flock mates.

Alignment
Steer toward the average heading of local flock mates.

Cohesion
Steer to move toward average position of local flock mates.
Craig Reynolds & “boids”

Each boid has direct access to whole scene’s geometric description.
For Flocking react only to flockmates within its small neighborhood.
Neighbourhood = model of limited perception / region where flockmates influence steering.

distance, measured from the center of the boid

angle, measured from boid’s direction of flight

Flockmates outside local neighborhood ignored

http://www.red3d.com/cwr/boids/

http://dynamicnotions.blogspot.com/2008/12/flocking-boids-c.html
Critique of Flocking

Reynolds' model is hypothetical, gives only appearance of flocking

Flocking is complex - inherent scaling problems

Simple algorithm has asymptotic complexity of $O(n^2)$ each boid assesses each other boid to determine its neighbour

Spatial data structure allows the boids to be kept sorted by their location reduces cost down to nearly $O(n)$

Lack of a quantitative model

When is a flock a flock? Phase transition to become a flock?

When does flock change from cluster to V formation

Heterogeneous vs homogeneous

300° vision cf. 360° vision
“Boids are not birds; they are not even remotely like birds; they have no cohesive physical structure, but rather exist as information structures — processes — within a computer.

But — and this is the critical ‘but’— at the level of behaviors, flocking Boids and flocking birds are two instances of the same phenomenon: flocking.

The ‘artificial’ in Artificial Life refers to the component parts, not the emergent processes. If the component parts are implemented correctly, the processes they support are genuine — every bit as genuine as the natural processes they imitate.

Artificial Life will therefore be genuine life — it will simply be made of different stuff than the life that has evolved on Earth.
Cellular Automata

A regular grid of cells: each in finite states (often 0 or 1).
Commonly, 2D is used for the grid, higher dimensionality possible.
Time is discrete and the state of a cell at time $t$ is a function of the states of a finite number of cells (neighborhood) at time $t-1$.
Every cell has the same rule for updating.
Update based on neighbourhood (consider grid toroidal).

\[t-2\] \[t-1\] \[t\]
Cellular Automata

John Von Neumann working at Los Alamos in the 1940s was interested in self-replicating robots.

Stanislaw Ulam was working on crystal growth at the same time using a mathematical abstraction.

Von Neumann created the first Cellular Automata (CA), but it was complex with 29 states per cell!

1970s John Conway greatly simplified CAs: Game of Life.

Practical uses have included studying crystal growth, casting of metals and biological patterns (e.g. coral).

'Fun' uses include pattern generation, screensavers and PhD studies.

Theoretical uses have shown self replication, infinite growth and computational power.
Conway’s Game of Life

John Conway  (Scientific American, 1970).

http://www.tech.org/~stuart/life/rules.html

Wanted a rule that for certain initial conditions would produce patterns that grow without limit, fade or get stable.

Have grid of cells which are occupied or not … have 8 neighbours

The rules for deriving a generation from the previous one are:

- Occupied cells with 0 or 1 occupied neighbours die of loneliness
- Occupied cells with 4..8 occupied neighbours die of overcrowding
- Otherwise occupied cells survive
- Unoccupied cells with 3 occupied neighbours come to life.

See code at http://blogs.msdn.com/calvin_hsia
**Gliders, guns and spaceships**


Not just pretty patterns

Explores spontaneity, synchronicity and attractors

Pattern Collection

Pattern Viewer

Small Gliders

Map / News / Words
Modern Cellular Automata Rule Notation

Modern CA software accepts multiple forms of rule specification. Rules may be specified in either basic or canonical format. Basic rule notation is based upon traditional "birth /survive". "a" to "e" indicate the number of side neighbours in the rule. "a" corresponds to zero side neighbours ... “e” to four

Here are the meaningful combinations of total and side counts:

0a  1ab  2abc  3abcd
4abcde  5bcde  6cde  7de  8e

Here is full basic rule specification for Game Of Life:
3abcd / 2abc3abcd

Birth / Survive states

Evolutionary Computation for ALife

Aim to find a solution to a particular problem:

1. Create a population of individuals to represent potential solutions
2. Evaluate the individuals
3. Introduce some selective pressure to promote better individuals (or eliminate lesser quality individuals)
4. Apply some variation operators to generate new solutions
5. Repeat
Karl Sims

Evolution of physically realistic agents
Have populations comprising different components ...

http://uk.youtube.com/watch?v=b1rHS3R0llU
Tierra - Tom Ray

Evolution of memory based agents
Useful resources to view here

http://life.ou.edu/pubs/images/
Simulated Hardware

Technically Soft
Alife
as in Karl Sims...

A NN effects motor commands and predicts next state
If agent encounters an unexpected obstacle it learns about itself /
environment
Prof Ralf Der

http://news.bbc.co.uk/go/pr/fr/-/1/hi/technology/7544099.stm
Summary

Have introduced Artificial Life
  Definitions
  Scope
  How relates to other disciplines
  Seen that it divides into Soft/Hard and Wet
We have started on Soft A-Life
  Flocking, Cellular Automata and some Evolutionary Computing
  Tomorrow look at more aspects ...

Consider however the paper mentioned on the next slide ... a good introduction to A-Life
Suggested Introductory Paper

http://people.reed.edu/~mab/publications/papers/BedauTICS03.pdf

Artificial life: organization, adaptation and complexity from the bottom up by Mark A. Bedau

Artificial life attempts to understand the essential general properties of living systems by synthesizing life-like behavior in software, hardware and biochemicals. As many of the essential abstract properties of living systems (e.g. autonomous adaptive and intelligent behavior) are also studied by cognitive science, artificial life and cognitive science have an essential overlap. This review highlights the state of the art in artificial life with respect to dynamical hierarchies, molecular self-organization, evolutionary robotics, the evolution of complexity and language, and other practical applications. It also speculates about future connections between artificial life and cognitive science.
2: More Soft ALife

Today Continue to look at Soft A Life

Daisyworld

Modelling ‘real’ life

Will build on this tomorrow with

Attractors

Discrete Models - and Self Similarity

Fractals
Daisyworld

Andrew J Watson and James E Lovelock; *Biological homeostasis of the global environment: the parable of Daisyworld*, Tellus (1983) 35B

Lovelock's Imaginary world to demonstrate Gaia principle

**Life & Earth work together to mutual advantage**

Grey Planet - black/white daisy seeds in soil

Daisies grow best at 22°C. No grow if < 7°C or > 37°C

Daisyworld’s Sun is heating up: What happens to Daisyworld?
Modelling Life on Daisyworld

Model its temperature: model energy received, absorbed, emitted.

Energy received comes from the sun
Energy absorbed is affected by planet albedo
Planet albedo is affected by the areas of daisies
Areas affected by birth/death rates: affected by temperature

Energy Emitted (Stefan Boltzmann Law) $k \times \text{Temp}^4$

Assume = Energy Absorbed = Energy Received - Energy Reflected
= Solar Luminosity $\times$ Solar Flux Const - Energy Received $\times$ Albedo

For World Temp, solve: Stefan'sConst $\times (\text{WorldTemp} + 273)^4$
= FluxConstant $\times$ Luminosity of Sun $\times (1 - \text{Planet Albedo})$

For any daisy species local temp is different re its albedo

$(\text{Daisy Temp} + 273)^4 = \text{AlbedoToTempConst} \times$
$(\text{Planet Albedo} - \text{Daisy Albedo}) + (\text{WorldTemp} + 273)^4$
Albedos and Areas of Daisies

Suppose have n species of Daisy
Let D_i be area of Daisy Species i, A_i its albedo, T_i its temp
D_0 and A_0 can represent area and albedo of grey soil

\[
\text{Planet Albedo} = \sum_{i=0}^{n} D_i \times A_i
\]

Areas of each daisy, found by solving differential equation
\[
\frac{dD_i}{dt} = D_i \times (\text{Uncolonised Fertile Soil} \times \text{Birth rate} - \text{Death rate})
\]

\[
\text{Uncolonised Fertile Soil} = \text{Prop of Fertile Soil} - \sum_{i=1}^{n} D_i
\]

\[
\text{Birth Rate} = \text{Max } (0, 1 - 0.003265 \times (22.5 - T_i)^2)
\]
\{Parabolic from 5^\circ - 40^\circ, max at 22.5^\circ\}
Algorithm - at each solar time

Initialise areas of daisies

Repeat

Calc Area Grey Soil, \( D_0 = 1 - \sum_{i=1}^{n} D_i \)

Calc Planet Albedo, \( A = \sum_{i=0}^{n} A_i D_i \)

Planet Temp, \( PT = \frac{\text{Flux Constant} \times \text{luminosity} \times (1 - \text{Albedo})}{\text{Stephan's Constant}} \)

For \( i = 1 \) to \( n \) Update \( D_i \)

Until all \( D_i \)'s reached steady value

Update \( D_i \):

\[ T_i = \frac{4}{\Delta} \text{AlbedoToTemp} \times (\text{Albedo} - A_i) + PT^4 - 273 \]

BirthRate = 1.0 - 0.003265 \times \text{Sqr (22.5 - } T_i) \]

\( \Delta = D_i \times (\text{BirthRate} \times \text{AreaFertileSoil} - \text{DeathRate}) \]
Runs with 2, 4 or 8 species

0.2:0.8

no life
temp
all D
DS 1
DS 2
DS 3
DS 4

0 0.2 0.4 0.6 0.8

-20 0 20 40 60 80

no life
temp
all D
DS 1
DS 2
DS 3
DS 4
DS 5
DS 6
DS 7
DS 8

albedo 0.2:0.8

0 0.2 0.4 0.6 0.8

-20 0 20 40 60 80

Runs with 2, 4 or 8 species
Other Approaches / Extensions

Basic model: ‘flat’ earth – for sphere: divide into areas, each receiving different luminosity, and simulating each area.

Can daisies’ albedo can evolve? See [Lenton is Lovelock’s ‘successor’]


At http://www.sussex.ac.uk/Users/jgd20/lisbon2007/

Computational Modelling of the Earth / Life System -
Includes - simulating Daisyworld using Cellular Automata

Also Homeostasis and Rein Control: From Daisyworld to Active Perception, by Inman Harvey, Proc ALife 9 2004

shows also how Rein control can be used for robotics
Consider population models and their analysis
Inc. interacting species: predator-prey, mutualist, competitive

Starting with continuous models
Model by change of population P

\[
\frac{dP}{dt} = (b - d) \times P
\]
b and d are birth and death rates

P constant, rises exponentially or decay to 0

If birth rate \( b - b_2 \times P \); death rate \( d + d_2 \times P \).

\[
\frac{dP}{dt} = (b - d - (b_2 + d_2) \times P) \times P
\]

Pop stabilises at \( \frac{b-d}{b_2 + d_2} \)
**Classic Interacting Species**

Let F be number of foxes and R be number of rabbits.

System model, as follows, where a, b, c, d are constants:

\[
\frac{dR}{dt} = aR - bRF \\
\frac{dF}{dt} = cRF - dF
\]

Stable when 
\[
F = \frac{a}{b} \quad \text{and} \quad R = \frac{d}{c}
\]

For \(a = 20, b = 4, c = 3\) and \(d = 27\): stable at 9,5;

Plot R and F v time but more useful Plot R v F (the phase plane plot)

Initial values set size of ‘egg’
Logistic Rabbit Model

If no Foxes, Rabbits increase exponentially - unrealistic, so

\[
\frac{dR}{dt} = a*R - b*R*F - c*R^2 \\
\frac{dF}{dt} = d*R*F - e*F
\]

Suggests both populations stable at

\[
R = \frac{e}{d} \quad \text{and} \quad F = \frac{a-c*\frac{e}{d}}{b}
\]

where lines \(a-bF-cR=0\) and \(dR-e=0\) meet

\[
\frac{dR}{dt} = R(39 - R - 5F) \\
\frac{dF}{dt} = F(-27 +3R)
\]

With different constants, plot can go straight to equilibrium

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**Mutualist Interaction**

Interacting species which help each other
Also have commensalist (one helps other) systems.
Mutualists - some survive independently, sometimes reliant
Eg Hippo and Bird  
   Clean teeth and food
   Sea-anemone & damsel fish  
   Habitat + protection / food
Plants and Insects  
   Plant pollination insect food
Egret and Cattle  
   Egret eat insects on cattle.

\[
\frac{dx}{dt} = x \left( -13 - 2x^2 + 21y \right) \quad \frac{dy}{dt} = y \left( -13 + 8x - 3y^2 \right)
\]

Analyse on phase plane, noting isoclines (loci where x and y const)
Equilibrium Points, where both x and y are constant
**Plot Zero Isoclines on Phase Plane**

The isoclines for \( \frac{dx}{dt} \) are \( x = 0 \) and \(-13 - 2x^2 + 21y = 0\).

Those for \( \frac{dy}{dt} \) are \( y = 0 \) and \(-12 + 8x - 3y^2 = 0\).

Equilibrium points: where a \( \frac{dx}{dt} \) isocline and a \( \frac{dy}{dt} \) isocline meet.

Main iso's meet at 2,1 and 5,3; \( x = 0 \) and \( y = 0 \) meet at 0,0.

- Show signs of \( x, y \)
- Argue how \( x, y \) change
- Show two points stable
Go further: Phase Plane + Arrows

Arrows show $dx/dt$ and $dy/dt$ at intervals.
Can be used to help sketch $x$ and $y$ values.
See how $x,y$ move from start posns.
To 5,3 or 0,0.
Not to 2,1.

Note Separatrix (thru 2,1): $x,y \to 5,3$ if above this, else $\to 0,0$. 

Specie Y

Specie X

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SE4SI12 Artificial Life - Part A
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Types of Equilibrium Point

Where loci meet are equilibrium point - but different types exist.

NB Also have unstable source and unstable spiral -

How can we categorise a point?
**Jacobian Matrix for Analysis**

Define eq point as \((X_e,Y_e)\), here \(F_1 = \frac{dx}{dt} \& F_2 = \frac{dy}{dt} = 0\).

Model system as being linear around an equilibrium point

\[
\frac{dx}{dt} = a_{11}X + a_{12}Y \\
\frac{dy}{dt} = a_{21}X + a_{22}Y
\]

\[
a_{11} = \frac{\partial F_1}{\partial X} \bigg|_{X_e,Y_e} \quad a_{12} = \frac{\partial F_1}{\partial Y} \bigg|_{X_e,Y_e} \quad a_{21} = \frac{\partial F_2}{\partial X} \bigg|_{X_e,Y_e} \quad a_{22} = \frac{\partial F_2}{\partial Y} \bigg|_{X_e,Y_e}
\]

We then define two matrices \(A\) (the Jacobean) and \(Z\)

\[
A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad Z = \begin{bmatrix} X \\ Y \end{bmatrix}
\]

Then system of equations can be written as \(dZ/dt = A \cdot Z\)
Then Use Eigenvalues of Jacobean

For a 2*2 matrix with eigenvalues $\lambda_1$ and $\lambda_2$:

- If $\lambda_1$ and $\lambda_2$ are both $< 0$, the equilibrium point is stable 'sink'
- If $\lambda_1$ and $\lambda_2$ are both $> 0$, the point is unstable 'source'
- If one $< 0$ and the other $> 0$, have a 'saddle' point
  
  The eigenvector for $\lambda < 0$ is used for the separatrix
  
  If eigenvalues complex have spiral points -
  
  stable (spiral in) if $\text{real}(\lambda_1) < 0$, unstable otherwise
  
  If purely complex, have a 'centre'

For $\lambda^2 + b\lambda + c$:

- Stable sink if $b^2 \geq 4c$; else spiral in
- Source if $b^2 \geq 4c$; else spiral out
- Will be saddle point
Analysis on the Example

\[
\begin{align*}
\frac{dx}{dt} &= F_1 = x(-13 - 2x^2 + 21y) \\
\frac{dy}{dt} &= F_2 = y(-13 + 8x - 3y^2)
\end{align*}
\]

\[
\frac{\partial F_1}{\partial x} = (-13 - 2x^2 + 21y) + x(-4x) \\
\frac{\partial F_1}{\partial y} = 21x \\
\frac{\partial F_2}{\partial x} = 8y \\
\frac{\partial F_2}{\partial y} = (-13 + 8x - 3y^2) - 6y^2
\]

At 0,0, \(A = \begin{bmatrix} -13 & 0 \\ 0 & -13 \end{bmatrix}\) \(\lambda = -13\) and -13 So 0,0 is a stable point

At 5,3, \(A = \begin{bmatrix} -100 & 105 \\ 24 & -54 \end{bmatrix}\) \(\lambda = -132.2178\) and -21.7822 So 5,3 is stable

At 2,1, \(A = \begin{bmatrix} -16 & 42 \\ 8 & -6 \end{bmatrix}\) \(\lambda = -30\) and 8 So 2,1 is a saddle point

Note Eigenvectors are \(\begin{bmatrix} -3 \\ 1 \end{bmatrix}\) and \(\begin{bmatrix} 7 \\ 4 \end{bmatrix}\) for \(\lambda = -30\) and 8
The separatrix can be defined by \( \frac{dy}{dx} \) where:

- Can't solve algebraically, so solve numerically
- Run as ode from a known point on curve (ie saddle point)

One problem: at saddle point, \( \frac{dy}{dx} = 0/0 \); what is \( \frac{dy}{dx} \)?

Answer, use its eigenvector

In example, at point 2,1

\[
\Lambda = \begin{bmatrix} -3 \\ 1 \end{bmatrix}
\]

Slope = \(- \frac{1}{3} \)
Applies to Fox Rabbit

\[
\begin{align*}
\frac{dR}{dt} &= R(20 - 4F); \\
\frac{dF}{dt} &= F(-27 + 3R)
\end{align*}
\]

At 9,5: \( A = \begin{bmatrix} 0 & -36 \\ 15 & 0 \end{bmatrix} \)

\( \lambda = \pm 23.2379j \)

centre

At 0,0: \( A = \begin{bmatrix} 20 & 0 \\ 0 & -27 \end{bmatrix} \)

\( \lambda = 20, -27 \)
saddle
Fox Rabbit (Logistic) Example

\[
\begin{align*}
\frac{dR}{dt} &= R(39 - R - 5F) \\
\frac{dF}{dt} &= F(-27 + 3R)
\end{align*}
\]

At 9,6 \( A = \begin{bmatrix} -9 & -45 \\ 18 & 0 \end{bmatrix} \)

\( \lambda = -4.5000 \pm 28.1025j \)

stable spiral

At 0,0 \( A = \begin{bmatrix} 39 & 0 \\ 0 & -27 \end{bmatrix} \)

saddle
Models of Males and Females (M & F)

\[
\frac{dM}{dt} = r_m F M - d_m M - k_m (M^3 + FM^2)
\]

\[
\frac{dF}{dt} = r_f F M - d_f F - k_f (F^3 + F^2 M)
\]

e.g. \( r_m = 1.3, \quad d_m = 7, \quad k_m = 0.01, \)
\( r_f = 1.1, \quad d_f = 19 \) and \( k_f = 0.01. \)

\[
50, 40: \quad A = \begin{bmatrix} -70 & 40 \\ 28 & -52 \end{bmatrix} \quad \lambda = -95.7 \quad \text{stable}
\]

\[
20, 10: \quad A = \begin{bmatrix} -10 & 22 \\ 10 & -4 \end{bmatrix} \quad \lambda = -22.1 \quad \text{saddle} \quad \Lambda(-ve) = \begin{bmatrix} -0.88 \\ 0.48 \end{bmatrix}
\]

\[
0, 0: \quad A = \begin{bmatrix} -7 & 0 \\ 0 & -19 \end{bmatrix} \quad \lambda = -7 \quad \text{stable}
\]
Lotka-Volterra Mutualism Models

\[ F_1 = \frac{dx}{dt} = x \left( r_x - a_{xx} x + a_{xy} y \right) \]
\[ F_2 = \frac{dy}{dt} = y \left( r_y + a_{yx} x - a_{yy} y \right) \]

Isoclines are lines: eg when \( x = 0 \) or \( r_x - a_{xx} x + a_{xy} y = 0 \), etc.

Assume all ‘a’ parameters > 0. Consider ‘main’ equil. point

First 2 systems stable at main point, 3\textsuperscript{rd} not. First ok with no mutualism; others obligate mutualists - can’t exist on own
Advantage of Mutualism

Note, with no mutualism (ie $a_{xy} = a_{yx} = 0$)

$$F_1 = \frac{dx}{dt} = x \left( r_x - a_{xx} x \right) \quad \text{this is zero when } x = \frac{r_x}{a_{xx}}$$

$$F_2 = \frac{dy}{dt} = y \left( r_y - a_{yy} y \right) \quad \text{this is zero when } y = \frac{r_y}{a_{yy}}$$

But, because of the positive feedback, due to mutualism, stable point is higher than these values.

Note, we will show, if opposite can be true in competitive systems

Next slide shows EQ point stable if gradient of $F_1$ isocline > $F_2$'s

If isoclines are curves, this gradient test also applies …
For Info: Confirm Gradient Test

A matrix is

\[
\begin{bmatrix}
-a_{xx}x_e & a_{xy}x_e \\
a_{yx}y_e & -a_{yy}y_e
\end{bmatrix}
\]

Char Eqn: \((-a_{xx}x_e - \lambda)(-a_{yy}y_e - \lambda) - a_{xy}x_e a_{yx}y_e = 0\)

\[\lambda^2 + \lambda(a_{xx}x_e + a_{yy}y_e) + a_{xx}x_e a_{yy}y_e - a_{xy}x_e a_{yx}y_e = 0\]

To be stable, need two negative eigenvalues, ie

\[a_{xx}x_e a_{yy}y_e > a_{xy}x_e a_{yx}y_e\]

ie \(a_{xx} a_{yy} > a_{xy} a_{yx}\) or \(\frac{a_{xx}}{a_{xy}} > \frac{a_{yx}}{a_{yy}}\)

ie gradient of \(F_1\) isocline > that of \(F_2\) isocline
Lotka–Volterra Example

\[
F_1 = \frac{dx}{dt} = x(-7x + 2y + 20)
\]

\[
F_2 = \frac{dy}{dt} = y(x - 3y + 8)
\]

At 4,4: \( A = \begin{bmatrix} -28 & 8 \\ 4 & -12 \end{bmatrix} \)

\( \lambda = -29.8 \ -10.2 \) stable

At 0,8/3: \( A = \begin{bmatrix} 25.33 & 0 \\ 2.667 & -8 \end{bmatrix} \)

\( \lambda = -8 \ 25.333 \) saddle

At \( \frac{20}{7} \),0: \( A = \begin{bmatrix} -20 & 5.714 \\ 0 & 10.86 \end{bmatrix} \)

\( \lambda = -20 \ 10.86 \) saddle

At 0,0: \( A = \begin{bmatrix} 20 & 0 \\ 0 & 8 \end{bmatrix} \)

\( \lambda = 20 \ 8 \) unstable
**Competitive Species**

\[
\frac{dx}{dt} = x(8 - 0.5x - 2y) \quad \frac{dy}{dt} = y(10 - x - 2y)
\]

\[
\frac{\partial F_1}{\partial x} = 8-x-2y \quad \frac{\partial F_1}{\partial y} = -2x \quad \frac{\partial F_2}{\partial x} = -y \quad \frac{\partial F_2}{\partial y} = 10-x-4y
\]

At 4,3, \( A = \begin{bmatrix} -2 & -8 \\ -3 & -6 \end{bmatrix} \), \( \lambda = 1.29 \) and \(-9.29\) saddle

At 16,0, \( A = \begin{bmatrix} -8 & -32 \\ 0 & -6 \end{bmatrix} \), \( \lambda = -8, -6 \) stable

At 0,5, \( A = \begin{bmatrix} -2 & 0 \\ -5 & -10 \end{bmatrix} \), \( \lambda = -10, -2 \) stable

At 0,0, \( A = \begin{bmatrix} 8 & 0 \\ 0 & 10 \end{bmatrix} \), \( \lambda = 8,10 \) unstable
Result – only one species survives

At 4,3
Saddle
A = \[
\begin{pmatrix}
-2 & -8 \\
-3 & -6
\end{pmatrix}
\]

At 16,0
Stable
A = \[
\begin{pmatrix}
-8 & -32 \\
0 & -6
\end{pmatrix}
\]

At 0,5
Stable
A = \[
\begin{pmatrix}
-2 & 0 \\
-5 & -10
\end{pmatrix}
\]

At 0,0
Unstable
A = \[
\begin{pmatrix}
8 & 0 \\
0 & 10
\end{pmatrix}
\]

Separatrix thru 4,3 – whether go to 16,0 or 0,5
**Competitive But Stable EQ - swap lines**

\[
\frac{dx}{dt} = x(10 - x - 2y); \quad \frac{dy}{dt} = y(8 - 0.5x - 2y)
\]

At 4,3 \[ A = \begin{bmatrix} -4 & -8 \\ -1.5 & -6 \end{bmatrix} \]

\[ \lambda = -1.394 \text{ & } -8.606 \text{ stable} \]

At 10,0 \[ A = \begin{bmatrix} -10 & -20 \\ 0 & 3 \end{bmatrix} \]

\[ \lambda = -10 \text{ and } 3 \text{ saddle} \]

At 0,4 \[ A = \begin{bmatrix} 2 & -20 \\ -2 & -8 \end{bmatrix} \]

\[ \lambda = -8 \text{ and } 2 \text{ saddle} \]

At 0,0 \[ A = \begin{bmatrix} 10 & 0 \\ 0 & 8 \end{bmatrix} \text{ u/s} \]

No competition, \( x = 10, y = 4; \) with, stable at \( x = 4, y = 3 \)

comp bad!
Summary

We have seen simple populations of models.

Single species,

Interacting mutualists, predator-prey and competitors

Isoclines, where one pop stable, interact at equilibrium points.

These can be stable sink, unstable source, saddle, stable spiral, unstable spiral or centre

And that the state can be found from the eigenvalues of the Jacobean matrix for each point.

Mutualists - stable point larger pop than if no interaction

Competitors - if stable smaller pop than if no interaction.

This leads to considerations of attractors, which leads to fractals
3: More Modelling

In this lecture we build on population modelling
Looking at attractors,
Discrete models - with recurrence
See some effects of self similarity
This then leads to fractals for artificial life
Emergence

We wish for soft artificial life to emerge in a computer – so create a system that changes states over time:

A *phase space* (cf mathematics) is a space in which all possible states of a system are represented. Each possible state of the system corresponds to one unique point in the phase space.

A *state space* representation (cf control engineering) is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations.

Can solve these to look at transient behaviour

But for A-Life more interested in steady state

This is Long-term behaviour: characterised by *Attractors*.

This relates to the equilibria we saw last time.
An attractor is a 'set', 'curve', or 'space' that a dynamical system irreversibly evolves to if left undisturbed.

May be known as a 'limit set'

Not necessarily trivial
Dynamics of Attractors

Activation of each system unit is associated with direction in a multidimensional space (configuration space).

Every point in the space represents a possible state of the system (state vector).

Motion of this vector represents its evolution in time (described as a dynamic system).

http://www.scholarpedia.org/article/Attractor_network

There are three types of attractors:
- point attractors,
- periodic attractors
- strange attractors
**Why use Attractors?**

No need to know the start time or duration or to observe entire evolution of system to get result

Only need to know the attractor, a given initial condition involves a path towards an attractor and an indication that the system has reached the attractor

They allow control of timing of system’s response

Robust to small changes in system parameters
   - dilution (removal of system connections)
   - asymmetry
   - clipping/quantization of parameter values

NB system could be ‘computer’ based (e.g. networks), biological (e.g. heart) or mechanical (e.g. pendulum).
Using Attractors

These systems are typically defined as series of ODEs

\[
\begin{align*}
\frac{dx}{dt} &= 10(y-x) \\
\frac{dy}{dt} &= -xz + 28x - y \\
\frac{dz}{dt} &= xy - \frac{8}{3} y
\end{align*}
\]

http://www.edc.ncl.ac.uk/highlight/rhnovember2006g02.php/

Computation performed by mapping from an initial condition to a particular attractor

Dynamics partition the configuration space into basins of attraction around the attractors. Let’s see some.
Fixed Point Attractor

The state vector comes to rest

Results are computed as different input data settle into different fixed points

The region of initial states that settle into a single fixed point is called its basin of attraction

Most networks are fixed point networks

C.f. Stable equilibrium point in population examples

Example with three variables
The state vector settles into a periodic cycle.

The attractor is a limit cycle.

- Shown in red
- Transient in purple
Strange Attractor - chaotic

Two copies of the system that initially have nearly identical states will grow dissimilar as they evolve. Divergence is restricted so that in many directions the state vectors are growing closer.
Higher Dimensions

chaotic attractors (also encounter repellors)

\[
\begin{align*}
\frac{dy}{dt} &= x + ay \\
\frac{dz}{dt} &= b + z(x - c) \\
\frac{dx}{dt} &= -y - z
\end{align*}
\]

Rössler studied chaotic attractor \(a = 0.2, b = 0.2, c = 5.7\)

NB \(a = 0.1, b = 0.1,\) and \(c = 14\) more commonly used since
Lorenz attractor

\[
\frac{dx}{dt} = \sigma(y - x) \\
\frac{dy}{dt} = x(\rho - z) - y \\
\frac{dz}{dt} = xy - \beta z
\]

\(\sigma\) is called the Prandtl number, 
\(\rho\) is called the Rayleigh number. 
All \(\sigma, \rho, \beta > 0\), but usually 
\(\sigma = 10, \beta = 8/3\) and \(\rho\) is varied

Simple model of convection in atmosphere.
Sensitive to initial conditions.
Some Discrete Models

In the above, the models were continuous
They can be of single species, or multiple.
We now move to considering discrete models
Here $x_n$ is the 'population' at 'time' $n$
The change in $x_n$ is set by a recurrence relation of the form

$$x_{n+1} = r \times x_n (1 - x_n)$$

These are of interest as the value of $r$ affects what happens
Different steady states are found
These show self similarity, which leads nicely to fractals.
Logistic Map – discrete model

\[ x_{n+1} = r \times_n (1 - \times_n) \]

\[ \times_0 = 0.2; \quad r = 2 \]

<table>
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<tr>
<th>n</th>
<th>\times</th>
</tr>
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</tr>
<tr>
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</tr>
<tr>
<td>2</td>
<td>0.4352</td>
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<tr>
<td>3</td>
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<td>0.5000</td>
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<td>8</td>
<td>0.5000</td>
</tr>
<tr>
<td>9</td>
<td>0.5000</td>
</tr>
<tr>
<td>10</td>
<td>0.5000</td>
</tr>
</tbody>
</table>
But if $r$ changed to 3.1

$$x_{n+1} = r \cdot x_n (1 - x_n)$$

$x_0 = 0.2; \ r = 3.1$

<table>
<thead>
<tr>
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<th>$x$</th>
</tr>
</thead>
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<td>0.7646</td>
</tr>
<tr>
<td>24</td>
<td>0.5579</td>
</tr>
</tbody>
</table>

Finally - oscillates between 2 values
But if \( r \) is 3.5 ...

\[ x_{n+1} = r \times x_n (1 - x_n) \]

\[
\begin{array}{l|l}
 n & x \\
 0 & 0.2000 \\
 1 & 0.5600 \\
 2 & 0.8624 \\
 3 & 0.4153 \\
 4 & 0.8499 \\
 \vdots & \vdots \\
 27 & 0.8750 \\
 28 & 0.3828 \\
 29 & 0.8269 \\
 30 & 0.5009 \\
 31 & 0.8750 \\
\end{array}
\]

\( x_0 = 0.2; \ r = 3.5 \)

Oscillates between 4 values
'Final' values for diff $r$

\[ x_{n+1} = r x_n (1 - x_n) \]

Self-similarity

Compare prev with when zoom in ..
Orbits

The Hénon map is a discrete-time dynamical system - exhibiting chaotic behavior. The Hénon map takes a point \((x, y)\) in the plane and maps it to a new point

\[
x_{n+1} = y_n + 1 - ax_n^2
\]

\[
y_{n+1} = bx_n
\]

Here \(a = 1.3\) \(b = 0.4\)

Known to be chaotic

Orbit - plots of \(x, y\)

For other values of \(a\) and \(b\) the map may be chaotic, intermittent, or converge to a periodic orbit
The self similarity observed earlier, leads to Fractals. These have been used in various ways re modelling life. Examples include trees, Plants, Clouds, Mountains. D'arcy Wentworth Thompson, On Growth and Form, 1917. Laid foundations for biomathematics; found equations to describe static forms of organisms; saw transformations by changing paras.
Fractals

Complex objects defined by systematically and recursively replacing parts of a simple start object with another, using a simple rule.

Simplest: Have initiator and generator, both many lines.

Replace each line in the initiator with the generator shape.

Makes more lines, so replace all these lines with generator.
Koch, SnowFlake, Forest Examples
Sierpinski : Space Filling Curve

four shapes \_/ (+ rotations) A B C D, joined by four corners.
A(n) is A(n-1) \ B(n-1) _ D(n-1) / A(n-1) \{n > 0\}
A(0) is nowt.
Similar for B(n), C(n) and D(n)
Also

Sierpinski Gasket: triangle from which smaller triangles are cut

Can also get 'natural' fractals..

More sophisticated replication methods...
Lindenmayer System (L-Systems)

Mathematical formalism proposed by biologist Aristid Lindenmayer in 1968: foundation for axiomatic theory of biological development.

A Lindenmayer system is a variant of a formal grammar (a set of rules and symbols), acting as a parallel rewriting system.

It models the growth processes of plants, organisms and self-similar fractals - due to the recursive nature of the rules.

Useful: http://algorithmicbotany.org/papers/#abop
L-systems are defined as a tuple

\[ G = \{ V, S, \omega, P \} \]

where

- \( V \) (the alphabet) is a set of symbols containing elements that can be replaced (variables)
- \( S \) is a set of symbols containing elements that remain fixed (constants)
- \( \omega \) (start, axiom or initiator) is a string of symbols from \( V \) defining the initial state of the system
- \( P \) is a set of production rules defining the way variables can be replaced with combinations of constants and other variables.
An L-system is an ordered triplet

\[ G = \langle V, w, P \rangle \]

- \( V \) = alphabet of the symbols in the system; \( V = \{F, B\} \)
- \( w \) = nonempty word, the axiom: \( B \)
- \( P \) = finite set of production rules (productions)

- \( B := F[-B][+B] \)
- \( F := FF \)
Production Rules for Artificial Plants

Add branching symbols [ ]

simple example

Main trunk shoots off one side branch

• Angle 45
• Axiom: F
• Seed Cell
• Rule: F=F[+F]F
• Angle

Gen 1

Gen 2

Gen 3

Gen 8
Some Examples

V = \{F, X\} the alphabet

the axiom: X

P = finite set of production rules

X := F[+X][-X]FX

F := FF

Probabilistic production rules

A := B C (P = 0.3)
A := F A (P = 0.5)
A := A B (P = 0.2)

http://coco.ccu.uniovi.es/malva/sketchbook/
More Example L - Systems

Colin McRae Dirt: pre-generated and preloaded!
Making Life Realistic

Fractals etc can help make realistic looking images
But 'life' tends to move and interact
So want artificial life to also behave realistically
Need to define appropriate behaviour, dependent on surroundings
Of interest is having situations with multiple entities

Applications
- Film and TV
- Games
- Simulations for engineering, architecture and transport

Premier system is MASSIVE ...
MASSIVE

Software package from Stephen Regelous for visual effects
Key feature: can create 1000s ...1000000s of agents
Fuzzy logic used so each agent react individually to surroundings
Used to control prerecorded animation clips
(say from motion capture or hand animation)
Creates characters that move, act and react realistically
Developed initially for Lord of the Rings ...
Used in Avatar, King Kong, Narnia, I Robot, Doctor Who, WallE, ...

http://www.massivesoftware.com/
Some Images

new car agent

Engineering Simulation

Television & Games

© Dr Richard Mitchell 2014
Summary

We have looked at more modelling of systems

Some differential equations, and the associated attractors which define their steady state

We have considered discrete models - recurrence relations, and seen the different states, and the associated self similarity

This lead to fractal systems, including Lindenmayer systems, which have been used to produce computer generated images

More sophisticated examples also exist, of 'agents' interacting with others, determining their actions/ movements.

Next time, we move to hardware systems and how they learn.