Quick reference guide to 2-port networks

Variable Assignment

The following principal convention and variables are used for effort and flow assignment.

	linear Mechanical	Mechanical rotation	electrical
e	velocity v	rotational speed ω	voltage v
f	force f	torque T	current i

$$\begin{bmatrix}e_1\\e_2\end{bmatrix}=Z\begin{bmatrix}f_1\\f_2\end{bmatrix}\qquad\begin{bmatrix}f_1\\f_2\end{bmatrix}=Y\begin{bmatrix}e_1\\e_2\end{bmatrix}\qquad\begin{bmatrix}e_1\\f_2\end{bmatrix}=H\begin{bmatrix}f_1\\e_2\end{bmatrix}\qquad\begin{bmatrix}f_1\\e_2\end{bmatrix}=G\begin{bmatrix}e_1\\f_2\end{bmatrix}\qquad\begin{bmatrix}e_1\\f_1\end{bmatrix}=T\begin{bmatrix}e_2\\-f_2\end{bmatrix}$$

Conversion table

$_{to}\backslash^{from}$	Z	Y	H	G	T
Z	$\begin{bmatrix}z_{11} & z_{12}\\ z_{21} & z_{22}\end{bmatrix}$	$\begin{bmatrix}\frac{y_{22}}{\Delta_y} & \frac{-y_{12}}{\Delta_y}\\ \frac{-y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y}\end{bmatrix}$	$\begin{bmatrix}\frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}}\\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}}\end{bmatrix}$	$\begin{bmatrix}\frac{1}{g_{11}} & \frac{-g_{12}}{g_{11}}\\ \frac{g_{21}}{g_{11}} & \frac{\Delta_g}{g_{11}}\end{bmatrix}$	$\begin{bmatrix}\frac{t_{11}}{t_{21}} & \frac{\Delta_t}{t_{21}}\\ \frac{1}{t_{21}} & \frac{t_{22}}{t_{21}}\end{bmatrix}$
Y	$\begin{bmatrix}\frac{z_{22}}{\Delta_z} & \frac{-z_{12}}{\Delta_z}\\ \frac{-z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z}\end{bmatrix}$	$\begin{bmatrix}y_{11} & y_{12}\\ y_{21} & y_{22}\end{bmatrix}$	$\begin{bmatrix}\frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}}\\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}}\end{bmatrix}$	$\begin{bmatrix}\frac{\Delta_g}{g_{22}} & \frac{g_{12}}{g_{22}}\\ \frac{-g_{21}}{g_{22}} & \frac{1}{g_{22}}\end{bmatrix}$	$\begin{bmatrix}\frac{t_{22}}{t_{12}} & \frac{-\Delta_t}{t_{12}}\\ \frac{-1}{t_{12}} & \frac{t_{11}}{t_{12}}\end{bmatrix}$
H	$\begin{bmatrix}\frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}}\\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}}\end{bmatrix}$	$\begin{bmatrix}\frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}}\\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}}\end{bmatrix}$	$\begin{bmatrix}h_{11} & h_{12}\\ h_{21} & h_{22}\end{bmatrix}$	$\begin{bmatrix}\frac{g_{22}}{\Delta_g} & \frac{-g_{12}}{\Delta_g}\\ \frac{-g_{21}}{\Delta_g} & \frac{g_{11}}{\Delta_g}\end{bmatrix}$	$\begin{bmatrix}\frac{t_{12}}{t_{22}} & \frac{\Delta_t}{t_{22}}\\ \frac{-1}{t_{22}} & \frac{t_{21}}{t_{22}}\end{bmatrix}$
G	$\begin{bmatrix}\frac{1}{z_{11}} & \frac{-z_{12}}{z_{11}}\\ \frac{z_{21}}{z_{11}} & \frac{\Delta_z}{z_{11}}\end{bmatrix}$	$\begin{bmatrix}\frac{\Delta_y}{y_{22}} & \frac{y_{12}}{y_{22}}\\ \frac{-y_{21}}{y_{22}} & \frac{1}{y_{22}}\end{bmatrix}$	$\begin{bmatrix}\frac{h_{2}2}{\Delta_h} & \frac{-h_{12}}{\Delta_h}\\ \frac{-h_{21}}{\Delta_h} & \frac{h_{11}}{\Delta_h}\end{bmatrix}$	$\begin{bmatrix}g_{11} & g_{12}\\ g_{21} & g_{22}\end{bmatrix}$	$\begin{bmatrix}\frac{t_{21}}{t_{11}} & \frac{-\Delta_t}{t_{11}}\\ \frac{1}{t_{11}} & \frac{t_{12}}{t_{11}}\end{bmatrix}$
T	$\begin{bmatrix}\frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}}\\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}}\end{bmatrix}$	$\begin{bmatrix}\frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}}\\ \frac{-\Delta_y}{y_{21}} & \frac{-y_{11}}{y_{21}}\end{bmatrix}$	$\begin{bmatrix}\frac{-\Delta_h}{h_{21}} & \frac{-h_{11}}{h_{21}}\\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}}\end{bmatrix}$	$\begin{bmatrix}\frac{1}{g_{21}} & \frac{g_{22}}{g_{21}}\\ \frac{g_{11}}{g_{21}} & \frac{\Delta_g}{g_{21}}\end{bmatrix}$	$\begin{bmatrix}t_{11} & t_{12}\\ t_{21} & t_{22}\end{bmatrix}$

Combining two-ports

If the two ports are not cascaded then add. Series implies common flow, parallel implies common effort.

Left port	right port	add when in :-
Series	series	Impedance (Z) form
Parallel	parallel	Admittance (Y)form
Series	parallel	Hybrid-H form
Parallel	series	Hybrid-G form

- Series implies that flow is conserved at the connected ports
- parallel implies that effort is conserved at the connected ports

Alternatively and equivalently

$_{port2}\backslash^{port1}$	e	f
e	Y	H
f	G	Z

If the two port networks are in a cascade then they need to be in transmission form T and the combination is then done by matrix multiplication.

Transfer functions (gain table)

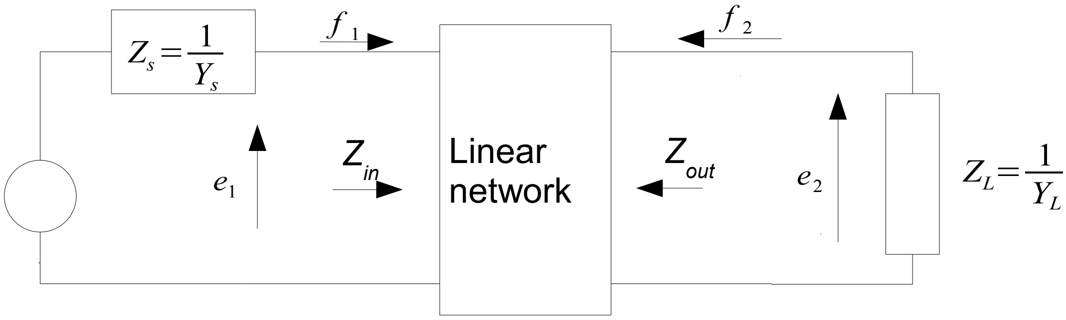

Figure 1. Termination of a generic 2-port network

		Z	Y	G	H	T
Zin	$Z_i = \frac{e_1}{f_1}$	$\frac{\Delta_z + z_{11} Z_L}{z_{22} + Z_L}$	$\frac{y_{22} + Y_L}{\Delta_y + y_{11} Y_L}$	$\frac{g_{22} + Z_L}{\Delta_g + g_{22} Z_L}$	$\frac{\Delta_h + h_{11} Y_L}{h_{22} + Y_L}$	$\frac{t_{11} Z_L + t_{12}}{t_{21} Z_L + t_{22}}$
Zout	$Z_o = \frac{e_2}{f_2}$	$\frac{\Delta_z + z_{22} Z_s}{z_{11} + Z_s}$	$\frac{y_{11} + Y_s}{\Delta_y + y_{22} Y_s}$	$\frac{\Delta_g + g_{22} Y_s}{g_{11} + Y_s}$	$\frac{h_{11} + Z_s}{\Delta_h + h_{22} Z_s}$	$\frac{t_{22} Z_S + t_{12}}{t_{21} Z_S + t_{11}}$
Flow gain	$A_f = -\frac{f_2}{f_1}$	$\frac{z_{21}}{z_{22} + Z_L}$	$\frac{-y_{21} Y_L}{\Delta_y + y_{11} Y_L}$	$\frac{g_{21}}{\Delta_g + g_{11} Z_L}$	$\frac{-h_{21} Y_L}{h_{22} + Y_L}$	$\frac{1}{t_{22} + t_{21} Z_L}$
Effort gain	$A_e = \frac{e_2}{e_1}$	$\frac{z_{21} Z_L}{\Delta_z + z_{11} Z_L}$	$\frac{-y_{21}}{y_{22} + Y_L}$	$\frac{g_{21} Z_L}{g_{22} + Z_L}$	$\frac{-h_{21}}{\Delta_h + h_{11} Y_L}$	$\frac{Z_L}{t_{12} + t_{11} Z_L}$
effort to flow gain	$A_{fe} = \frac{e_2}{f_1}$				$\frac{-h_{21}}{h_{22} + Y_L}$	$\frac{Z_L}{t_{22} + t_{21} Z_L}$
effort to flow gain	$A_{ef} = -\frac{f_2}{e_1}$				$\frac{-h_{21}}{\Delta_h Z_L + h_{11}}$	$\frac{1}{t_{12} + t_{11} Z_L}$

Note that $A_{fe} = \frac{e_2}{f_1} = A_f Z_L$, and $A_{ef} = -\frac{f_2}{e_1} = A_e Y_L$. Also $Z_i = Z_L \frac{A_f}{A_e}$.

W.S.Harwin, University of Reading, School of Systems Engineering (Cybernetics)

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