Routh-Hurwitz stability criterion

W.S. Harwin*

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Routh-Hurwitz stability criterion identifies the conditions when the poles of a polynomial cross into the right hand half plane and hence would be considered as unstable in control engineering.

1. TABULAR FORM

For the polynomial

$$D(s) = a_n s^n + a_{n-1} s^{n-1} + ... + a_1 s + a_0$$

the table has n rows and the following structure:

a_n	a_{n-2}	a_{n-4}	...
a_{n-1}	a_{n-3}	a_{n-5}	...
b_1	b_2	b_3	...
c_1	c_2	c_3	...
...	...	...	...

where the elements b_i and c_i can be computed as follows:

$$b_i = \frac{a_{n-1}a_{n-2i} - a_n a_{n-2i-1}}{a_{n-1}}$$

$$c_i = \frac{b_1 a_{n-2i-1} - b_{i+1} a_{n-1}}{b_1}$$

A sign change in column 1 indicates the existence of an unstable pole

In most cases we can assume that all coefficients are already demonstratibly positive.

For the system

$$D(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

this requires

$$b_1 = \frac{a_2 a_1 - a_0 a_3}{a_2} > 0$$

That is for a stable system where a_2 is already tested positive, $a_2 a_1 - a_0 a_3 > 0$

For the system

$$D(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

this requires

$$\frac{a_3 a_2 - a_1 a_4}{a_3} > 0$$

and

$$a_1 - \frac{a_0 a_3^2}{(a_2 a_3 - a_1 a_4)} > 0$$

That is for a stable system where a_3 and the other coefficients have already tested positive, $a_2 a_3 - a_1 a_4 > 0$ and $a_1 a_2 a_3 - a_1^2 a_4 - a_0 a_3^2 > 0$

For the system

$$D(s) = a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

*Digital Notebook, Cybernetics, University of Reading © February 2007 this requires

$$A = \frac{a_3a_4 - a_2a_5}{a_4} > 0$$

$$C = \frac{Aa_2 - Ba_4}{A} > 0$$

$$D = \frac{BC - Aa_0}{C} > 0$$

and

$$a_0 > 0$$

where

$$B = \frac{a_1a_4 - a_0a_5}{a_4}$$