Mechatronics SE3ME11 2012 Assignment

Due date Wednesday 14th March 2012 (Treat like an exam question, so please write your answers by hand and spend approx 90-120 minutes on the solution). The assignment will be returned on the 23rd April 2012.

Part 1 (Spring)

1. A linear spring can be considered as a 'grounded' or 'ungrounded' component as indicated schematically in the figure.

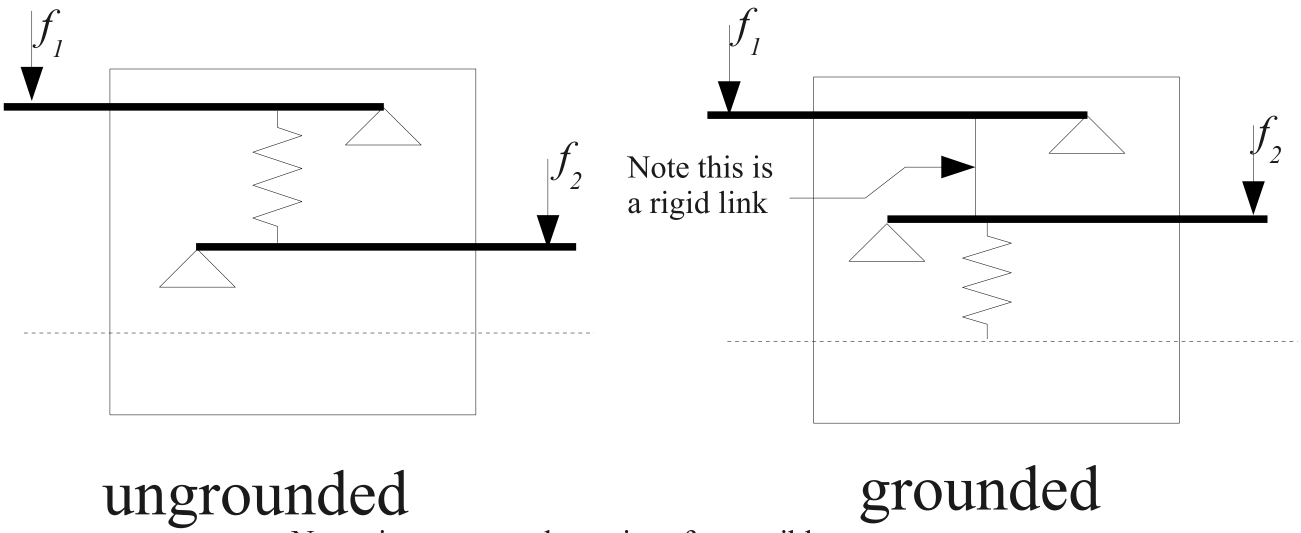
a) Show that the equations for an ungrounded spring

$f_1+f_2=0, f_2=\frac{K_1}{s}(v_2-v_1)$ can be write this as a transmission two-port.

b) Show that the equations for a grounded spring

$v_1=v_2, f_1+f_2=\frac{K_1}{s}v_2$

can also be written as a transmission two-port.



Note pictures are schematics of a possible arrangement

2. Show that you can put two ungrounded springs in series by cascading two ungrounded spring two-ports and hence calculate the equivalent stiffness.

3. Show that you can put two ungrounded springs in parallel and hence calculate the equivalent stiffness. (note, the efforts (velocities) at ports 1 and 2 must be conserved hence you will need to convert to admittance form)

4. Show that you can put two grounded springs in parallel by cascading two grounded spring two-ports.

5. Derive a simple mass, spring, damper equation by cascading a grounded spring two-port with a grounded damper two-port and adding a mass load. Give the gain that would indicate the position response to a force input (e_2/f_1).

6. Check your results using the Routh-Hurwitz stability criteria (ie for this simple system make sure all denominator coefficients are positive)

Part 2 (Motor)

A motor is represented in H-form as $H_{motor}=\begin{bmatrix} r+sL & K_t \\ -K_t & sJ_{arm} \end{bmatrix}$ where r is resistance, L is inductance, Kt is the torque/generator constant and Jarm is the armature inertia. The motor is connected to a spring shown in Transmission form as

$$T_{spring}=\begin{bmatrix} 1 & 0 \\ K_s/s & 1 \end{bmatrix}$$

through a gear $T_{gear}=\begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix}$ where Ks is the spring constant, and n is the gear ratio.

1. Give the transmission form for the motor, gear and spring.
2. Give the input impedance if the assembly is connected to an impedance ZL

Part 3 (Phase planes)

A control system can be modelled as a mass *m* sliding on a linear bearing with constant coulomb friction F_c. The mass has an instantaneous velocity of *v*. The actuator can apply three possible forces to the actuator $-F_a$, 0, F_a. The control scheme is to apply force $+F_a$ when the position sensor reading (measuring *x*) is less than zero, $-F_a$ when the reading is greater than zero and turn the actuator off when the sensor reads 0.

(a) assuming there is no friction, use Newton's equation in the form $F=m\frac{dv}{dt}$ to show the time independent form of equation that can be plotted on a position-velocity phase plane.
(b) Use the values $m=1$, $x_0=-1.5$, to sketch a single cycle, showing the velocities at the transition points.

(c) If coulomb friction is now included and the mass starts at the position $x=-x_0$, with zero velocity, calculate the theoretical velocity v_1 when the mass passes zero (the point where the actuator reverses direction).

(d) Calculate the new point along the *x* axis at which the velocity changes from positive to negative (i.e. friction reverses direction (at x_2)), and hence the velocity where the mass passes zero with a negative velocity and once again the actuator reverses direction (at v_3).

(e) Sketch this cycle on the phase plane.

W.S. Harwin