Titles and Abstracts

Stability of the Nonlinear Filter for Random Expanding Maps

Jochen Broecker

A ubiquitous problem in science and engineering is to reconstruct the state of a hidden Markov process (the ``signal process'') from noise corrupted observations. The optimal or nonlinear filter is given by the conditional probability of the current state of the signal process given present and past observations. This probability obeys a dynamical equation which is the subject of this talk. An important question is whether these equations are stable with respect to the choice of the initial condition, i.e. the unconditional probability distribution of the initial state of the signal process. This stability problem has attracted quite some interest recently. In this talk, we will show that the filter is stable for Markov processes which arise through iterations of randomly chosen expanding mappings. We show that the filter dynamics form a random contraction on certain cones of positive densities. The approach is novel in the sense that we have to work with a random set of cones, rather than only a single cone, as previous studies did.

Kusuoka-Stroock bounds for the solution of the stochastic filtering problem with applications to particle filters

Dan Crisan

Abstract: In the first part of the talk, I introduce sharp gradient bounds for the perturbed diffusion semigroup. In contrast with existing results, the perturbation studied here is random and the bounds obtained are pathwise. The approach builds on the classical work of Kusuoka and Stroock. It extends their program developed for the heat semi-group to solutions of stochastic partial differential equations. The work is motivated by and applied to nonlinear filtering. The analysis allows us to derive pathwise gradient bounds for the un-normalized conditional distribution of a partially observed signal. by a random process. It uses a pathwise representation of the perturbed semigroup in the spirit of the one introduced by Ocone. The estimates we derive have sharp short time asymptotics.

In the second part of the talk, I introduce a class of particle approximations (particle filters) to the (normalized) perturbed diffusion semigroup. The method combines the Kusuoka–Lyons–Victoir (KLV) cubature method on Wiener space to approximate the law of the signal with a minimal variance ‘thinning’ method, called the tree-based branching algorithm (TBBA) to keep the size of the cubature tree constant in time. The novelty of the approach resides in the adaptation of the TBBA algorithm to *simultaneously* control the computational effort and incorporate the observation data into the system. We provide the rate of convergence of the approximating particle filter in terms of the computational effort (number of particles) and the time discretization grid mesh.

This is joint work with C. Litterer (Imperial), S. Ortiz-Latorre (Oslo) and T. Lyons (Oxford).

Two non-self-adjoint specral problems

E Brian Davies

Abstract: We describe the asymptotic spectral behaviour of two large non- self-adjoint matrices. The first concerns a non-self-adjoint Anderson model, whose spectral behaviour is contrasted with some very recent work done jointly with Michael Levitin. This concerns a matrix analogue of an indefinite self-adjoint linear pencil that concerns a Dirac operator with an indefinite potential. In some sense it is the simplest matrix example of its type, but its analysis is still far more complex than one might expect.

Sequential Monte Carlo for Markov random fields

Richard Everitt

In a range of applications, including population genetics, epidemic modelling and social network analysis, the data from which we wish to estimate parameters of interest consists of noisy or incomplete observations of an unobserved process. Bayesian statistics offers a framework in which to tackle this problem, accurately accounting for the uncertainty present due to the missing data. However, standard Markov chain Monte Carlo (MCMC) methods that are used to implement the Bayesian approach can perform poorly in this situation. In this talk we describe two alternatives to standard MCMC approaches: approximate Bayesian computation (ABC) and particle MCMC. Both methods are applied to parameter estimation of a hidden Markov random field, and are compared to the standard data augmentation approach.

Adaptive and Interacting Markov chain Monte Carlo samplers

Gersende Fort

Abstract : Markov chain Monte Carlo (MCMC) algorithms are designed to produce paths of a Markov chain with a given distribution as unique stationary distribution. These algorithms depend on many design parameters which play a crucial role on the efficiency of these samplers. Therefore new generations of MCMC samplers provide adaptive and interacting methods which are able to learn on the fly better (ideally, the best) values for the design parameters. Nevertheless, the stochastic process described by these adaptive and interactive MCMC samplers is no more a Markov chain. Thus new tools have been proposed in order to address the asymptotic behavior of these algorithms. In the approach presented in this talk, adaptive and interacting MCMC samplers are seen as controlled Markov chains.

We will first illustrate how adaptive and interacting MCMC samplers can be seen as controlled Markov chains: the adaptive Metropolis algorithm and the Wang-Landau algorithm (resp. the Equi-Energy sampler) which are adaptive (resp. interacting) MCMC algorithms, will be used as examples.
Then, the talk will be devoted to the asymptotic behavior of controlled Markov chains: sufficient conditions for the convergence in distribution, for a law of large numbers and for a Central Limit Theorem will be given.

This talk relies on joint works with Prof. Eric Moulines (Telecom ParisTech, France) and Prof. Pierre Priouret (Univ. Paris 6, France).

On Extreme Value Distributions of Chaotic Dynamical Systems

Mark Holland

Abstract: For chaotic dynamical systems we consider the time series of maxima along typical orbits. We formulate a set of checkable conditions the dynamical system must satisfy in order for the time series (under suitable normalization) to convergence to an extreme value distribution. These conditions will be based on quantitative Poincare recurrence time statistics. For a range of applications, including Lorenz maps and Henon maps, we verify that these conditions hold. We also formulate and check corresponding conditions for suspension flows. The conditions we formulate are also suitable for numerical implementation, especially for applications were analytic results are not readily available. We also study quantitative recurrence statistics of certain quasi-periodic systems. Work is joint with Pau Rabassa and Alel Sterk.

Particle systems with non-local boundary conditions

Martin Kolb

Abstract: In the talk I will outline the notion of a quasistationary distribution and introduce an interacting particle systems which can be associated to it in some natural way. The main aim of the talk is to present the basic open problem in this area and to explain the difficulty involved in proving it.

Piecewise Approximate Bayesian Computation: fast inference for discretely observed Markov models.

Theodore Kypraios

Abstract: Many modern statistical applications involve inference for complicated stochastic models for which the likelihood function is difficult or even impossible to calculate, and hence conventional likelihood-based inferential techniques

cannot be used. In such settings, Bayesian inference can be performed using a simulation based approach: Approximate Bayesian Computation(ABC). However, in spite of many recent developments to ABC methodology, in many applications the computational cost of ABC necessitates the choice of summary statistics and tolerances that can potentially severely bias the estimate of the posterior distribution.

We propose a new “piecewise” simulation-based approach suitable for discretely observed Markov models that involves writing the posterior density of the parameters as a product of factors, each a function of only a subset of the data, and then using ABC within each factor. The approach has the advantage of side-stepping the need to choose a summary statistic and it enables a stringent tolerance to be set, making the posterior “less approximate”. We investigate two methods for estimating the posterior density based on ABC samples for each of the factors: the first is to use a Gaussian approximation for each factor, and the second is to use a kernel density estimate. Both methods have their merits. The Gaussian approximation is simple, fast, and probably adequate for many applications. On the other hand, using instead a kernel density estimate has the benefit of consistently estimating the true piecewise ABC posterior as the number of ABC samples tends to infinity. We illustrate the piecewise ABC approach with four examples; in each case, the approach offers fast and accurate inference.

Hilbert’s metrics and positive operators

Bas Lemmens

Abstract: In this talk I will survey some applications of Hilbert's metric in the analysis of positive operators. In particular, I will discuss Birkhoff's contraction ratio theorem, which can be used to prove the existence and uniqueness of positive eigenvectors for certain positive operators using a contraction mapping argument. This idea will be illustrated by looking at so-called Perron-Frobenius operators (transfer operators).

Spectral representations for affine processes

Ronnie Loeffen

Abstract: In this talk we look at discrete spectral representations of a certain subclass of Markov processes, called affine processes. We start by considering one-dimensional, positive affine processes for which, under certain conditions and for large times, a spectral representation exists for its transition semigroup acting on functions which vanish at infinity fast enough. Then we give, again under various conditions, a spectral representation for the transition semigroup of multi-dimensional affine processes but now acting only on exponential

functions. In other words, we give a spectral representation of the Laplace transform of the process. We discuss why these results can be very useful for pricing in mathematical finance with affine models.

Invariant measures for random expanding maps

Gianluigi Del Magno

It is well known that expanding maps admit a unique absolute continuous invariant measure. In this talk, we address the problem of the existence of absolute continuous invariant measures for random expanding maps. To construct these measures, we study a random transfer operator, and prove a random version of the Ruelle-Perron-Frobenius Theorem. The invariant measures obtained this way are ergodic, and enjoy fiberwise exponential decay of correlations. As a method of proof, we construct a family of cones of positive functions for which the transfer operator is a strict contraction.

Mirror and Synchronous Couplings of Geometric Brownian Motions

Alexander Mijatovic

Do the classical mirror and synchronous couplings of two Brownian motions minimise and maximise, respectively, the coupling time of the corresponding geometric Brownian motions? We establish a characterisation of the optimality of the two couplings over any finite time horizon and show that, unlike in the case of Brownian motion, the optimality fails in general even if the geometric Brownian motions are martingales. On the other hand, we prove that in the cases of the ergodic average and the infinite time horizon criteria, the mirror coupling and the synchronous coupling are always optimal for general (possibly non-martingale) geometric Brownian motions. We show that the two couplings are efficient if and only if they are optimal over a finite time horizon and give a conjectural answer for the efficient couplings when they are suboptimal. This is joint work with Saul Jacka and Dejan Siraj.

The spectral structure of PDEs

Beatrice Pelloni

Abstract: In this talk, I will discuss a point of view that extends the connection between integral transforms (such as the Fourier transform) widely used to solve PDEs, and the spectral analysis of differential operators. This point of views originates in the so–called Inverse Scattering Transform approach, a sort of non-linear Fourier transform method, and the resulting approach provides a unified treatment of initial and boundary value problems for linear and integrable nonlinear PDEs in two independent variables.

Some spectral properties for a class of non-self-adjoint operators related to Markov processes

Mladen Savov

Abstract: In this talk we consider the spectral properties of the semigroup of the positive self- similar Markov processes. When the process jumps downwards only we describe the point spectrum, the eigen- and coeigenfunctions and derive spectral expansions for the semigroup.

Random matrix models for population ecology

David Steinsaltz

Abstract: Population growth rates are related in sometimes obscure ways to an organism's demographic rates, the age-specific pattern of mortality and fertility. Since the work of Leslie in the 1940s, matrix models have been the primary tool used by demographers, ecologists, and evolutionary biologists to study population dynamics. Since the 1980s environmental variability has come to be seen as a crucial factor, leading to random matrix models.

This talk will outline the key Monte Carlo algorithms that are used to analyse population growth rates in random environments, and some new theoretical results on the evolution of diapause and migration.