EC933-G-AU – Lecture 7

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Microfounded (Optimising) Models of Exchange Rates under Flexible Prices

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Plan of talk

- **introduction**
1. Lucas (1976) critique
2. Lucas (1982) DSGEM of exchange rates
    1. *barter* two-country economy
    2. *single*-currency two-country ("two-sector closed") *monetary* economy
    3. *two*-currency two-country ("world") *monetary* economy
        1. under perfectly *flexible* exchange rate regime
        2. under perfectly *fixed* exchange rate regime
3. RBC models in closed economy
4. I(R)BC models
- **wrap-up**

Aim and learning outcomes

- **aim**: provide microfoundations to exchange rate models
- **learning outcomes**
  - motivate *microfoundations* from the perspective of the Lucas (1976) critique
  - derive Lucas (1982) model in its *three* versions/stages
  - distinguish analytically and interpret the role of *money* and of the *exchange rate* in Lucas (1982)
  - discuss the welfare implications of alternative *exchange rate regimes* under *flexible* prices and *complete* asset markets
  - summarise/critically asses the methodology of RBC/I(R)BC

Lucas (1976) critique

- Lucas, Jr., Robert (1976)
  - “Econometric Policy Evaluation: A Critique”
  - in Brunner, Karl and Meltzer, Allan (eds.), *The Phillips Curve and Labour Markets*, Amsterdam: North Holland
- main point
  - **econometric estimates** rely on *coefficients* assumed *stable*
  - but this is so only **given** certain economic **policy**, already *incorporated* in the rational expectations of agents about the future
  - hence, once there is a **change in policy**, mechanistic econometric estimates based on *past* behaviour will not truly reflect realities
  - therefore, the importance of **deriving models from first principles**
    - any time a policy changes, the *underlying analytical model* is rederived
    - and its empirical implications reformulated: i.e., the *empirical model* (or *test equation*) is accordingly respecified to take account of the policy change

Lucas (1982): summary

- a theoretical study of the determination of prices, interest rates and exchange rates in an infinitely-lived two-country world
  - subject both to stochastic endowment shocks: real ("output") shocks
  - and to monetary instability: nominal ("money supply") shocks
- "highly ambitious" in some respects and "very modest" in others
  - ambitious in integrating monetary theory with financial economics, by "replicating all of the classical results of monetary theory as well as the main formulas for securities pricing"
  - modest because "many, perhaps most, of the central substantive questions of monetary economics are left unanswered"
- consists of three models (stages), building upon one another
  - *barter* two-country economy, essentially a variation on Lucas (1978)
  - *single*-currency two-country ("two-sector closed") *monetary* economy
  - *two*-currency two-country ("world") *monetary* economy
- main concern are "alternative monetary arrangements": *float-peg*

Lucas (1982): key assumptions

- **general environment**
  - complete information
  - rational expectations
  - no market imperfections
  - no nominal rigidities
- *2* countries, *H* and *F*, populated by a large number of individuals
  - identical utility functions
  - identical wealth
  - constant population normalised to *1*

  ⇔ **representative agent** model: the *simplest* way to "aggregate"
- **why study** such a perfect world?
  - as a *benchmark*, against which to measure progress (in research)
  - confront its implications with *data* => extensions and refinements of theory

Lucas (1982): "firms" (or "fruit trees")

- pure endowment streams generate a
  - homogeneous
  - non-storable
  - country-specific $i_t \equiv (1+g_t)i_{t-1}$ and $i_t^* \equiv (1+g_t^*)i_{t-1}^*$

  good, using no labour or capital inputs
- number of "firms" in each country also normalised to *1*
- each "firm"
  - issues one *perfectly divisible* share of common stock, e_t and e_t^*, which is traded at a competitive stock market
  - pays out all revenue from sales of output as *dividends* to shareholders
- in the *barter* model
  - dividends form the *sole* source of income for individuals
  - i_t is the *numéraire* good and q_t is the (relative) price of i_t^* in terms of i_t at t

H consolidated period budget constraint

- H *initial* current-period wealth (*brought* into period t)

$$\mathcal{W}_t = \omega_{it-1} \underbrace{(i_t + e_t)}_{\textit{with}\text{-dividend value of } H \text{ firm}} + \omega_{i^*t-1} \underbrace{(q_t i_t^* + e_t^*)}_{\textit{with}\text{-dividend value of } F \text{ firm}}$$

- H current-period *allocation* of wealth (in period t)

$$\mathcal{W}_t \geq \underbrace{\omega_{it} e_t + \omega_{i^*t} e_t^*}_{\text{saving: purchases of new shares}} + \underbrace{c_{it} + q_t c_{i^*t}}_{\text{consumption}}$$

- H *consolidated* current-period BC: equating the above
- H and F current-period utility: $u(c_{it}, c_{i^*t})$ and $u(c_{it}^*, c_{i^*t}^*)$

H-agent optimisation problem

- choose *sequences* of consumption and stock purchases

$$\left\{c_{it+k}, c_{i^*t+k}, \omega_{it+k}, \omega_{i^*t+k}\right\}_{k=0}^{\infty}$$

- to maximise expected lifetime utility

$$\underbrace{E_t \underbrace{\left[\sum_{k=0}^{\infty} \beta^k \underbrace{u(c_{it+k}, c_{i^*t+k})}_{H\ period\ t+k\ \text{utility}}\right]}_{H\ lifetime\ \text{utility}}}_{H\ expected\ \text{lifetime utility}}$$

- subject to the *consolidated* budget constraint

H unconstrained problem and FONCs

$$u\left[\underbrace{\omega_{it-1}(i_t + e_t) + \omega_{i^*t-1}(q_t i_t^* + e_t^*) - \omega_{it} e_t - \omega_{i^*t} e_t^* - q_t c_{i^*t}}_{c_{it}}, c_{i^*t}\right] +$$

$$+E_t \beta u\left[\underbrace{\omega_{it}(i_{t+1} + e_{t+1}) + \omega_{i^*t}(q_{t+1} i_{t+1}^* + e_{t+1}^*) - \omega_{it+1} e_{t+1} - \omega_{i^*t+1} e_{t+1}^* - q_{t+1} c_{i^*t+1}}_{c_{it+1}}, c_{i^*t+1}\right] +$$

$$+E_t \beta^2 u\left[\underbrace{\omega_{it+1}(i_{t+2} + e_{t+2}) + \omega_{i^*t+1}(q_{t+2} i_{t+2}^* + e_{t+2}^*) - \omega_{it+2} e_{t+2} - \omega_{i^*t+2} e_{t+2}^* - q_{t+2} c_{i^*t+2}}_{c_{it+2}}, c_{i^*t+2}\right] + \ldots$$

$$c_{i^*t} : \quad q_t u_1(c_{it}, c_{i^*t}) = u_2(c_{it}, c_{i^*t}) \Leftrightarrow \frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})} = q_t$$

$$\omega_{it} : \quad e_t u_1(c_{it}, c_{i^*t}) = \beta E_t[u_1(c_{it+1}, c_{i^*t+1})(i_{t+1} + e_{t+1})]$$

$$\omega_{i^*t} : \quad e_t^* u_1(c_{it}, c_{i^*t}) = \beta E_t[u_1(c_{it+1}, c_{i^*t+1})(q_{t+1} i_{t+1}^* + e_{t+1}^*)]$$

Market clearing and stationary distribution

- adding-up constraints (accounting identities)
  - on *outstanding* **equity shares** $\omega_{it} + \omega^*_{it} = 1 \qquad \omega_{i^*t} + \omega^*_{i^*t} = 1$
  - on *exhaustion* of **output** $c_{it} + c^*_{it} = i_t \qquad c_{i^*t} + c^*_{i^*t} = i^*_t$
- assumptions on the stochastic processes
  - Mark's (2001) textbook: **2 possible realisations of output:** *high* and *low*
    - at each *date*
    - in each *economy*, hence
    - **4 states of nature:** $s_1 \equiv (i_h, i^*_h)$, $s_2 \equiv (i_h, i^*_l)$, $s_3 \equiv (i_l, i^*_h)$ and $s_4 \equiv (i_l, i^*_l)$
    - with the *set* of all possible states being the **same:** $(s_1, s_2, s_3, s_4) \equiv \mathcal{S}$
  - Lucas (1982) paper: the *more general* case of a **stationary distribution**
  - possible to **reformulate** the **DS**GE economy into **static competitive** GE (Arrow-Debreu) model, the properties of which have been well studied
    - if *complete asset* markets (assumed)
    - if *competitive* setting (assumed)
  - in such a way, *all* possible future **outcomes** and the **corresponding** *unique* **Arrow-Debreu goods** are *completely* spelled out

Arrow-Debreu planner's problem: centralised and decentralised optimum (I)

- it is known from **static** GE analysis that
  - the **solution** to the *social planner's problem*
  - is a **Pareto optimal** *allocation*
  - moreover, from the **fundamental theorems of welfare economics** it follows that the Pareto optimum *supports* (i.e., *can be replicated by*) a *competitive equilibrium*
  - hence, the *centralised* social optimum solution can also be **decentralised** into a *competitive market economy* equilibrium
- s.t. resource constraints, we first solve the planner's problem, by maximising

$$\underbrace{E_t \left\{ \sum_{k=0}^{\infty} \beta^k \underbrace{\left[\phi \underbrace{u(c_{it+k}, c_{i^*t+k})}_{H period\ t+k\ \text{utility}} + (1-\phi) \underbrace{u(c^*_{it+k}, c^*_{i^*t+k})}_{F period\ t+k\ \text{utility}} \right]}_{global\ \text{period}\ t+k\ \text{welfare} \equiv \text{weighted}\ national\ \text{period}\ t+k\ \text{utility}} \right\}}_{\text{global social welfare} \equiv \text{global}\ lifetime\ \text{utility}}$$

Arrow-Debreu planner's problem: centralised and decentralised optimum (II)

- goods are *non-storable* => problem reduces to a *timeless* one of maximising

$$\underbrace{\phi u(c_{it}, c_{i^*t}) + (1-\phi)u(c^*_{it}, c^*_{i^*t})}_{\text{global period } t \text{ welfare} \equiv \text{weighted national period } t \text{ utility}}$$

- s.t. resource constraints => Euler eqs (FONCs) give the **optimal /efficient/ risk sharing**: consumption is allocated so that *MUC* of *H* agent w.r.t. both goods is *proportional*, therefore *perfectly correlated*, to *MUC* of *F* agent

$$\underbrace{u_1(c_{it}, c_{i^*t})}_{\text{MUC of } H \text{ agent w.r.t. } H \text{ good}} = \frac{1-\phi}{\phi} \underbrace{u_1(c^*_{it}, c^*_{i^*t})}_{\text{MUC of } F \text{ agent w.r.t. } H \text{ good}}$$

$$\underbrace{u_2(c_{it}, c_{i^*t})}_{\text{MUC of } H \text{ agent w.r.t. } F \text{ good}} = \frac{1-\phi}{\phi} \underbrace{u_2(c^*_{it}, c^*_{i^*t})}_{\text{MUC of } F \text{ agent w.r.t. } F \text{ good}}$$

- the *Pareto optimal* allocation is to **split** the available output **equally**, by holding a *perfectly diversified portfolio* of assets ⇔ **pooling equilibrium**

$c_{it} = c^*_{it} = \frac{i_t}{2}$ and $c_{i^*t} = c^*_{i^*t} = \frac{i^*_t}{2}$ $\qquad \omega_{it} = \omega^*_{it} = \omega_{i^*t} = \omega^*_{i^*t} = \frac{1}{2}$

Arrow-Debreu planner's problem: explicit solution under CRRA utility

- if CRRA utility defined over a **Cobb-Douglas** (real) *consumption* **index**

$$u(c_{it}, c_{i^*t}) \equiv \frac{c_t^{1-\gamma}}{1-\gamma}, \qquad c_t \equiv c_{it}^{\theta} c_{i^*t}^{1-\theta}$$

- then

$$u_1(c_{it}, c_{i^*t}) = \theta \frac{c_t^{1-\gamma}}{c_{it}}, \qquad u_2(c_{it}, c_{i^*t}) = (1-\theta) \frac{c_t^{1-\gamma}}{c_{i^*t}}$$

- and **Euler eqs** become

$$q_t = \frac{1-\theta}{\theta} \frac{i_t}{i_t^*}, \quad \frac{e_t}{i_t} = \beta E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{1-\gamma} \left(1 + \frac{e_{t+1}}{i_{t+1}}\right) \right], \quad \frac{e_t^*}{q_t i_t^*} = \beta E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{1-\gamma} \left(1 + \frac{e_{t+1}^*}{q_{t+1} i_{t+1}^*}\right) \right]$$

- RER q_t is determined by relative output levels
- the other two FONCs are stochastic difference eqs in **the "dividend/price" (inverted) ratios** and can be solved forward *if* an assumption is made about the *stochastic process* governing output
- *no actual asset trading* in the Lucas (1982) model: agents *hold* their *investments forever* and *never rebalance* their portfolios ⇔ *asset prices* are thus **shadow prices**: these must be respected in order for agents to *willingly* hold the outstanding equity shares *according to* the risk pooling equilibrium

Single-currency monetary economy: money via CiA => timing of events

- ***single*-currency** two-country *open* economy ⇔ ***two*-sector** *closed* economy
- imposing **cash-in-advance (CiA) constraint**: Clower (1967) on *money*
- with *CiA* and *uncertainty*, **timing of events** becomes important

1. *endowment* shock realisations revealed: $i_t \equiv (1+g_t)i_{t-1}$ and $i_t^* \equiv (1+g_t^*)i_{t-1}^*$

2. *money* supply shock realisation revealed: $M_t \equiv (1+\mu_t)M_{t-1}$

- economy-wide increment $\Delta M_t \equiv M_t - M_{t-1} \equiv (1+\mu_t)M_{t-1} - M_{t-1} = \mu_t M_{t-1}$
- distributed evenly to all individuals, so representative agent gets $\frac{\Delta M_t}{2} = \frac{\mu_t M_{t-1}}{2}$

3. a *centralised* **securities market** opens: agents allocate their wealth b/n stock purchases (risk-sharing) and cash to buy goods (consumption)

4. *decentralised* **goods trading** takes place in the "shopping mall": each **household splits** into "worker-seller" and "shopper"

- **shopper** takes the cash from the securities market trading and buys goods from other "stores" in the mall (shoppers are not allowed to buy from their own stores)
- the *H*-country **worker-seller** collects the *i*-good endowment and offers it for sale at an *i*-good store in the "mall", and similarly does the *F*-country worker-seller

5. cash value of goods sales distributed to stockholders as dividends => stock-holders carry these nominal dividend payments into next period

Single-currency monetary economy: why extra cash not carried and CiA binding

- now the state of the world is summarised by a triplet, $s \equiv (g_t, g_t^*, \mu_t)$, and is *revealed* **before** *trading* => representative household can **precisely** determine the amount of money it needs to finance its current-period consumption plan
- if the **(shadow) *nominal* interest rate** is *always positive* (as we assume), it is *optimal* for households to use up *all* their cash intended for consumption
- initial current-period wealth (brought in *t*)

$$\mathcal{W}_t = \underbrace{\frac{P_{t-1}}{P_t}(\omega_{it-1} i_{t-1} + \omega_{i^*t-1} q_{t-1} i^*_{t-1})}_{\text{dividends}} + \underbrace{\omega_{it-1} e_t + \omega_{i^*t-1} e^*_t}_{\text{ex-dividend share values}} + \underbrace{\frac{\Delta M_t}{2P_t}}_{\text{money transfer}}$$

- allocation of current-period wealth (in *t*)

$$\mathcal{W}_t \geq \underbrace{\frac{M_{Ht}}{P_t}}_{\text{cash to buy consumption}} + \underbrace{\omega_{it} e_t + \omega_{i^*t} e^*_t}_{\text{insurance: purchases of new shares}}$$

Single-currency monetary economy: difference with barter in FONCs and clearing

- with *positive* nominal interest rate **CiA binds**: $\frac{M_{Ht}}{P_t} = \underbrace{c_{it} + q_t c_{i^*t}}_{\text{real consumption}}$
- substituting this into last eq on previous slide
- results in a simpler expression for the **consolidated budget constraint (CBC)**:

$$\frac{P_{t-1}}{P_t}(\omega_{it-1} i_{t-1} + \omega_{i^*t-1} q_{t-1} i^*_{t-1}) + \omega_{it-1} e_t + \omega_{i^*t-1} e^*_t + \frac{\Delta M_t}{2P_t} = \omega_{it} e_t + \omega_{i^*t} e^*_t + c_{it} + q_t c_{i^*t}$$

- maximising the *same* objective as under barter but using *CBC above* gives the same *consumption* Euler eq but **modifies *equity* pricing FONCs** by the *inverse* of the **gross inflation rate**: *inflation premium* due to *nominal* dividends carried

$$\omega_{it}: \quad e_t u_1(c_{it}, c_{i^*t}) = \beta E_t\left[u_1(c_{it+1}, c_{i^*t+1})\left(\frac{P_t}{P_{t+1}} i_t + e_{t+1}\right)\right]$$

$$\omega_{i^*t}: \quad e^*_t u_1(c_{it}, c_{i^*t}) = \beta E_t\left[u_1(c_{it+1}, c_{i^*t+1})\left(\frac{P_t}{P_{t+1}} q_t i^*_t + e_{t+1}\right)\right]$$

- a **5th market clearing cond** (now, with money) complements the 4 other (barter)

$$M_t \equiv M_{Ht} + M_{Ft}$$

- aggregating for *H* and *F* and using the other market clearing conds yields QTM

$$M_{Ht} = P_t(c_{it} + q_t c_{i^*t}), \qquad M_{Ft} = P_t(c^*_{it} + q_t c^*_{i^*t}), \qquad M_t = P_t(i_t + q_t i^*_t)$$

Single-currency monetary economy: difference with barter in CRRA solution

- under CRRA utility, RER is the same as under barter => substituting it into QTM eq produces an expression for the inflation premium: $\frac{P_t}{P_{t+1}} = \frac{M_t}{M_{t+1}} \frac{i_{t+1}}{i_t}$
- it can be used, with Euler eqs under CRRA utility, to re-write equity prices:

$$\frac{e_t}{i_t} = \beta E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{1-\gamma} \left(\frac{M_t}{M_{t+1}} + \frac{e_{t+1}}{i_{t+1}} \right) \right], \qquad \frac{e_t^*}{q_t i_t^*} = \beta E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{1-\gamma} \left(\frac{M_t}{M_{t+1}} + \frac{e_{t+1}^*}{q_{t+1} i_{t+1}^*} \right) \right]$$

- **to price *nominal* bonds**, one looks for the *shadow price* of a *hypothetical* nominal bond such that the public willingly keeps it in ***zero* net supply**
  - let B_t be the **nominal price** of a *zero-coupon discount bond* that pays (with certainty) *1 unit of currency* at the end of the period
  - the **utility cost** of *buying* the nominal bond is $u_1(c_{it}, c_{i^*t}) \frac{Bt}{P_t}$
  - in equilibrium, this is offset by the **discounted expected marginal utility of the payoff** of *1 monetary unit* $\beta E_t\left[u_1(c_{it+1}, c_{i^*t+1}) \frac{1}{P_{t+1}} \right]$
  - hence, under the CRRA utility

$$B_t = \beta E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{1-\gamma} \frac{M_t}{M_{t+1}} \right] \qquad B_t = \frac{1}{1+\iota_t}$$

Two-currency monetary economy under float: additional assumptions

- 2nd national currency: *traditional* assumption on invoicing (pricing: PCP)

$$\Delta M_t^* \equiv M_t^* - M_{t-1}^* \equiv (1+\mu_t^*)M_{t-1}^* - M_{t-1}^* = \mu_t^* M_{t-1}^*$$

- a *new* country-specific risk thus introduced: foreign purchasing power risk
- agents can *acquire* the *foreign currency* needed for consumption or saving
  - from foreign dividends
  - during securities market trading
- *complete markets* paradigm
  - allows markets to develop whenever there is a demand for a product
  - the product that individuals desire in the present context are *claims* to future *H*-currency and *F*-currency *transfers*: one *perfectly divisible claim* outstanding for *each* of these two monetary transfer streams is assumed => denoted r_t and r_t^*
  - *initially*, the *H*-agent is endowed with claims *only* to his *national* currency, and similarly the *F*-agent; *onwards*, they are allowed to freely trade these claims

$$\psi_M = 1, \psi_{M^*} = 0 \qquad\qquad \psi_M^* = 0, \psi_{M^*}^* = 1$$

Two-currency monetary economy under float: initial wealth and allocation of wealth

$$\mathcal{W}_t = \underbrace{\frac{P_{t-1}}{P_t}\omega_{it-1} i_t + \frac{S_t P^*_{t-1}}{P_t}\omega_{i^*t-1} i^*_t}_{\text{dividends}} + \underbrace{\frac{\psi_{Mt-1}\Delta M_t}{P_t} + \frac{\psi_{M^*t-1}\Delta M^*_t}{P_t}}_{\text{monetary transfers}} +$$

$$+ \underbrace{\underbrace{\omega_{it-1} e_t + \omega_{i^*t-1} e^*_t}_{\text{market value of ex-dividend shares}} + \underbrace{\psi_{Mt-1} r_t + \psi_{M^*t-1} r^*_t}_{\text{market value of monetary transfer claims}}}_{\text{market value of securities}}$$

$$\mathcal{W}_t \geq \underbrace{\frac{M_{Ht}}{P_t} + \frac{S_t M^*_{Ht}}{P_t}}_{\text{cash to buy consumption}} + \underbrace{\omega_{it} e_t + \omega_{i^*t} e^*_t}_{\textit{output}\text{ insurance: purchases of new shares}} + \underbrace{\psi_{Mt} r_t + \psi_{M^*t} r^*_t}_{\textit{money}\text{ insurance: purchases of new claims}}$$

Two-currency monetary economy under float: *H* consolidated budget constraint

- using binding CiA constraints $M_{Ht} = P_t c_{it}$ and $M^*_{Ht} = P^*_t c_{i^*t}$ to eliminate money held by the *H*-agent into the last eq above, we can rewrite it as:

$$\mathcal{W}_t = \underbrace{c_{it} + \frac{S_t P^*_t}{P_t} c_{i^*t}}_{\text{consumption: goods}} + \underbrace{\omega_{it} e_t + \omega_{i^*t} e^*_t}_{\text{saving: new equity}} + \underbrace{\psi_{Mt} r_t + \psi_{M^*t} r^*_t}_{\text{insurance: new monetary transfer claims}}$$

- the CBC of the *H*-agent now becomes

$$\frac{P_{t-1}}{P_t}\omega_{it-1} i_{t-1} + \frac{S_t P^*_{t-1}}{P_t}\omega_{i^*t-1} i^*_{t-1} +$$
$$+\omega_{it-1} e_t + \omega_{i^*t-1} e^*_t + \psi_{Mt-1} r_t + \psi_{M^*t-1} r^*_t +$$
$$+\frac{\psi_{Mt-1}\Delta M_t}{P_t} + \frac{\psi_{M^*t-1} S_t \Delta M^*_t}{P_t} =$$
$$= \omega_{it} e_t + \omega_{i^*t} e^*_t + \psi_{Mt} r_t + \psi_{M^*t} r^*_t + c_{it} + \frac{S_t P^*_{t-1}}{P_t} c_{i^*t}$$

Two-currency monetary economy under float: *H* Euler equations

- maximising the same objective but s.t. the CBC above, Euler eqs now are:

$$c_{i^*t}: \quad \frac{S_t P_t^*}{P_t} u_1(c_{it}, c_{i^*t}) = u_2(c_{it}, c_{i^*t}) \Leftrightarrow \frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})} = \frac{S_t P_t^*}{P_t} \equiv RER_t$$

$$\omega_{it}: \quad e_t u_1(c_{i^*t}, c_{i^*t}) = \beta E_t\left[u_1(c_{it+1}, c_{i^*t+1})\left(\frac{P_t}{P_{t+1}} i_t + e_{t+1}\right)\right]$$

$$\omega_{i^*t}: \quad e_t^* u_1(c_{i^*t}, c_{i^*t}) = \beta E_t\left[u_1(c_{it+1}, c_{i^*t+1})\left(\frac{S_t P_t^*}{P_{t+1}} i_t^* + e_{t+1}\right)\right]$$

$$\psi_{Mt}: \quad r_t u_1(c_{it}, c_{i^*t}) = \beta E_t\left[u_1(c_{it+1}, c_{i^*t+1})\left(\frac{\Delta M_{t+1}}{P_{t+1}} i_t + r_{t+1}\right)\right]$$

$$\psi_{M^*t}: \quad r_t^* u_1(c_{it}, c_{i^*t}) = \beta E_t\left[u_1(c_{it+1}, c_{i^*t+1})\left(\frac{S_{t+1} \Delta M_{t+1}^*}{P_{t+1}} i_t + r_{t+1}^*\right)\right]$$

Two-currency monetary economy under float: perfect risk-pooling equilibrium

- imposing the market clearing conditions allows to write $M_t^* \equiv M_{Ht}^* + M_{Ft}^*$

$$M_t = P_t i_t, \quad M_t^* = P_t^* i_t^*$$

- which can be used to eliminate endogenous nominal price levels from Euler eqs
- the equilibrium with **perfect risk-pooling** of *country-specific risks* is given by

$$\omega_{it} = \omega_{it}^* = \omega_{i^*t} = \omega_{i^*t}^* = \psi_{Mt} = \psi_{Mt}^* = \psi_{M^*t} = \psi_{M^*t}^* = \frac{1}{2}$$

- in this equilibrium, both the *H* and *F* representative household own:
  - half of the domestic endowment (output) stream
  - half of the foreign endowment (output) stream
  - half of all future domestic monetary transfers
  - half of all future foreign monetary transfers
  - in short, the world resources are **split equally** between the *H* and *F* representative agents, subjected to country-specific *endowment (output)* and *monetary* risks (uncertainty): the **pooling equilibrium** thus supports the symmetric allocation

$$c_{it} = c_{it}^* = \frac{i_t}{2} \text{ and } c_{i^*t} = c_{i^*t}^* = \frac{i_t^*}{2}$$

Two-currency monetary economy under float: equilibrium NER and CRRA utility solution

- to solve for NER we use RER eq and QTM eqs to get

$$\frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})} = \frac{S_t P^*_{t-1}}{P_t}, \qquad \frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})} = \frac{S_t M^*_t i_t}{M_t i^*_t}, \qquad S_t = \frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})} \frac{M_t}{M^*_t} \frac{i^*_t}{i_t}$$

- under CRRA utility
  - equilibrium NER becomes $S_t = \frac{1-\theta}{\theta} \frac{M_t}{M^*_t}$
  - in addition to Euler eqs, two new ones relate to each of the currencies in circulation

$$\frac{r_t}{i_t} = \beta E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{1-\gamma} \left(\frac{\Delta M_{t+1}}{M_{t+1}} + \frac{r_{t+1}}{i_{t+1}} \right) \right], \quad \frac{r^*_t}{i_t} = \beta E_t\left[\frac{1-\theta}{\theta} \left(\frac{c_{t+1}}{c_t}\right)^{1-\gamma} \left(\frac{\Delta M^*_{t+1}}{M^*_{t+1}} + \frac{r^*_{t+1}}{i_{t+1}} \right) \right]$$

  - and a *Foreign* bond price equation adds to the earlier *Home* bond price equation

$$B^*_t = \beta E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{1-\gamma} \frac{M^*_t}{M^*_{t+1}} \right] \qquad B^*_t = \frac{1}{1+\imath^*_t}$$

Two-currency monetary economy under peg

- if NER is to be **fixed**, some **agency** has to ensure and implement this fixity: this role is assigned to a single, central authority, holding **reserves of both currencies**
- Lucas (1982) points out that to analyse such a regime under **RE**, it is necessary
  - **either** to assume that behaviour of this central authority, in combination with monetary and real shock processes in the countries, is consistent with permanent maintenance of the pegged level
  - **or** to incorporate into the analysis the possibility of deviations and of consecutive speculative activity: approach avoided by Lucas (1982)
- reserves held by the central authority **before** and **after** trading: $R_0 = R + \overline{S} R^*$
- under *positive* interest rates, QTM eqs give the NER: $\overline{S} = \frac{M-R}{M^*-R^*} \frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})} \frac{i_t^*}{i_t}$
- viability of the peg requires, $R > 0$ and $R^* > 0$, for all *s*, which is the same as

$$R_0 > \overline{S} M^* - M \frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})} \frac{i_t^*}{i_t} \text{ and } R_0 > M - \frac{\overline{S} M^*}{\frac{u_2\left(c_{it}, c_{i^*t}\right)}{u_1\left(c_{it}, c_{i^*t}\right)} \frac{i_t^*}{i_t}} \quad \text{with } 0 < \frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})} \frac{i_t^*}{i_t} < \infty$$

- maintenance of **fixed** exchange rate requires sufficient **reserves** and **coordination**
  - the *rest* of the analysis is *precisely the same* as in the single-currency world economy =>
  - **peg versus float** debate **does not matter** for the equilibrium allocation of real consumption with *complete* asset markets and *flexible* prices

RBC research in closed economy

- Kydland and Prescott (1982), *Econometrica*, **started** this literature
  - **analytical model** ⇔ microfoundations => optimising behaviour
  - **calibration**
    - *functional form(s)*
    - *parameters*: long-run (stable) relations “extracted” from the *data*
  - **simulation**
  - **model vs data: comparison of moments** => mean, SD, corrs (comovement)
- **textbook** treatment
  - sections 4.4, 4.5 and 5.1 in Mark (2001) provide a compact introduction to the RBC approach
  - sections 7.4.3.1 – 7.4.3.4 in Obstfeld and Rogoff (1996) offer another
- King, Plosser and Rebelo (1988 a, b), *JME*
  - discuss in greater detail the **technical aspects** of RBC research
  - itself viewed as **extending** the *basic neoclassical growth model*

I(R)BC models

- **extension** of RBC research to *open* economies
- Backus, Kehoe and Kydland (1992), *JPE*, **started** this literature
- essential **features and techniques** of I(R)BC research
  - *textbook* treatment
    - section 5.2 in Mark (2001)
    - section 7.4.3.5 in Obstfeld and Rogoff (1996)
  - Baxter (1995), *NBER* WP, provides *another* insightful account
- Baxter and Crucini (1995), *IER*, focus on the **solution algorithm** of these models

Concluding wrap-up

- **What have we learnt?**
  - justify the need for *microfounded* models following Lucas (1976) critique
  - derive and discuss Lucas (1982) model in its *three* versions/stages
  - distinguish analytically and interpret the role of *money*, *exchange rates* and the *nominal exchange rate regime* in Lucas (1982)
  - summarise Lucas (1982) conclusion on the welfare implications of *peg vs float* under flexible prices and complete asset markets
  - describe and critically asses the methodology of *RBC/I(R)BC research*
- **Where we go next**: to Obstfeld-Rogoff (1995) "redux" model, the 1st microfounded DGE of exchange rates under *sticky* prices