EC933-G-AU – Lecture 6

23 November 2005

Aggregate (Ad-Hoc) Monetary Models of Exchange Rates under Rational Expectations

Alexander Mihailov

University of Essex

Plan of talk

- **introduction**

1. the **rational expectations** revolution
2. **flexible-price** models under *perfect foresight*
3. **sticky-price** models under *perfect foresight*: **Dornbusch (1976)**
    - *motivation* for the model
    - key elements and *assumptions*
    - model *equilibrium* and *transition paths*
    - key result: model versions *with* and *without* **exchange-rate overshooting**

- **wrap-up**

Aim and learning outcomes

- **aim:** understand the mechanics of *exchange-rate overshooting* in one of the most influential models in international macroeconomics
- **learning outcomes**
  - distinguish and discuss
    - *rational* expectations and *perfect* foresight
    - *flexible*-price vs *sticky*-price monetary models of exchange rate dynamics
  - derive and interpret
    - Dornbusch (1976) model *equilibrium* and *transition paths*
    - *effects of monetary expansion* in Dornbusch (1976) model
      - when NER overshooting *always occurs*
      - when NER overshooting *does not necessarily occur*
  - analyse the basic set-up of *ad-hoc dynamic monetary* OEMs

The rational expectations revolution

- **Muth (1961) on rational expectations**
  - "Rational Expectations and the Theory of Price Movements", *Econometrica* 29 (3, July)
  - introduces (to the economics profession)
    - the *concept* of **rational expectations (RE)**
    - and the (minimum) necessary **mathematics to implement** it
  - **RE** imply that
    - agents *forecast* in a way that is **internally consistent with the model** *generating the variable(s)* whose future behaviour they try to predict
    - agents are **rational** in the sense that they
      - incorporate **all available *relevant* information** (in their *information set*)
      - and **do not make *systematic* errors** *in* their *predictions*
- the **"rational expectations revolution"** (in economics)

Flexible-price models under perfect foresight

- **Cagan (1956) on hyperinflation**
  - "The Monetary Dynamics of Hyperinflation", in Friedman, Milton (ed.), *Studies in the Quantity Theory of Money*, Chicago University Press
  - **empirical** *closed*-economy **model** to study hyperinflation in 7 countries
  - argued that during a hyperinflation **expected inflation** *swamps* all other influences on *money demand* => specified the MDF in a *simple* way
    - real money balances depend *only* on expected inflation
    - and *not* on real income and interest rates as in the *conventional* (IS-LM) MDF
  - **adaptive forecasting**: expected future inflation depends on *lagged* inflation
- the (*flexible* NER) **monetary model** of lecture 3 ⇔ Cagan model:
  - extended to *open* economy
  - with the *conventional* MDF
  - assuming *perfect foresight* (as an *extreme* form of rational expectations)
  - and underpinned with some *basic* (but *ad-hoc*) macro-*theory*
- to read **more:** Obstfeld and Rogoff (1996), chapter 8

Sticky-price models under perfect foresight

- **Dornbusch (1976):** perhaps the *best known OEM example*
  - "Expectations and Exchange Rate Dynamics", *Journal of Political Economy* 84 (December)
  - extension of the (Keynesian) *static* Mundell-Fleming model we studied in lecture 2 to a *dynamic* setting under *perfect foresight*
  - very influential model => Mundell-Fleming-Dornbusch tradition/paradigm
    - among *academic circles* involved with open-economy macro
    - among *policy makers* at national and international institutions
  - summary of technical *approach*
    - *ad-hoc* (not optimising!) model, *assuming* explicit functional forms (not deriving them from microfoundations!)
    - written in *continuous* time (but can be "translated" to a *discrete* version: see Obstfeld-Rogoff textbook, section 9.2)
    - all variables (except interest rates) in *logarithms*: $z \equiv \ln Z$ for any Z

Dornbusch (1976): *motivation* for model

- *stylised facts* to explain
  - observed **large exchange rate volatility** after the *demise* of Bretton-Woods
    - much higher than that of *underlying fundamentals*
    - needed to be *consistent with rational expectations formation*
  - **effects of a monetary expansion**
    - in the *short run*, an immediate domestic currency **depreciation** => monetary expansion seems to account for *fluctuations* in the exchange rate and ToT
    - during the *adjustment* process, rising prices may be accompanied by an **appreciation** => the *trend* behaviour of exchange rates and the *cyclical* behaviour of exchange rates and prices stand potentially in contrast
    - during the *adjustment* process, there is also a direct effect of the exchange rate on domestic **inflation** => the exchange rate as a critical channel for the *transmission* of monetary policy to AD for domestic output
  - **differential speed of adjustment** of *asset* vs *goods* markets

    which was becoming an interesting *novel* topic for research

Dornbusch (1976): *general* assumptions

1. **SOE** =>
   – the world (nominal) *interest rate* is given in the world *asset* market ι^*
   – the world *price of* SOE's *imports* is given in the world *goods* market p^*
2. *domestic output* is an ***imperfect* substitute** for *imports*

   => demand for H (SOE) and F (RoW) output will be affected by their relative price, $s + p - p^*$ in logs (from PPP, SP/P^*, in levels)
3. (NB!) ***differential* adjustment speed of markets**
   – *goods* markets adjust *slow relative to*
   – *asset* (including *exchange rate*) markets, which adjust *instantaneously*
4. (NB!) ***consistent* expectations (formation)**
   – formation of (rational) expectations is *consistent*
   – with *perfect foresight*, i.e., the strongest, *extreme* form of RE

Dornbusch (1976): assumptions on *capital mobility* and *NER expectations formation*

1. ***perfect* capital mobility**
   - assets denominated in domestic and foreign currency are *perfect* substitutes given a proper *premium/discount* to offset anticipated NER changes
   - equivalently, this is (a form of) UIP: $\imath = \imath^* + x$
2. **exchange-rate *expectations* formation**
   - *expected* depreciation, $x > 0$, of current spot NER, s, is assumed proportional (via a coefficient θ) to its discrepancy w.r.t. long-run NER, $\overline{s} = const$ (for now taken as *known*, but later an expression which determines it will be developed): $x = \theta(\overline{s} - s)$
   - Dornbusch (1976) makes the point that
     - while expectations formation, as assumed, *may appear ad hoc*
     - it will be *consistent with perfect foresight* (to be shown further down)

Dornbusch (1976): assumptions on *money market* structure and equilibrium

1. ***conventional* money demand** function ⇔ a *monetary* model
2. **money supply** /stock/ ***exogenous***(ly given, by SOE's CnBk)
3. (NB!) real income (or **output**, or AS) is ***fixed* at** its **full-employment level**, y (initially ↔ *variable* output later)
4. **money market *equilibrium*:** $m^s = m^d = m$

   => *money demand* can be written as $m - p = -\lambda\iota + \phi y \Leftrightarrow \iota = -\frac{1}{\lambda}(m - p) + \frac{\phi}{\lambda} y$

   => *relationship* linking spot and LR NER with price level

$$p - m = -\phi y + \lambda\iota^* + \lambda\theta(\overline{s} - s)$$

5. ***stationary* money supply process:** *long-run* equilibrium would imply $s = \overline{s}$ and hence, through *UIP* $\iota = \iota^*$ => an expression for the *long-run* equilibrium price level $\overline{p} = m - \phi y + \lambda\iota^*$

Dornbusch (1976): key equation on the relationship b/n the NER and the price level

- $$s = \overline{s} - \frac{1}{\lambda\theta}(p - \overline{p})$$
- solving it for p determines ***asset* market equilibrium**, to be *graphically* represented later as the *QQ* schedule:

$$p = \overline{p} - \lambda\theta(s - \overline{s}) = \overline{p} - \lambda\theta s + \lambda\theta\overline{s}$$

- **interpretation** of key equation: for *given* LR values $\overline{s}$ and $\overline{p}$, the spot NER s can be determined as a f-n of the current price level p
  - *given* the price level p, we have a domestic interest rate ι from MDF and an interest rate differential $\iota - \iota^*$ from UIP determined endogenously
  - *given also* the LR NER $\overline{s}$, from expected depreciation eq. there is a unique level of the spot NER s such that expected depreciation x matches the interest differential $\iota - \iota^*$
  - so $p\uparrow$ => (from MDF) $\iota\uparrow$ => an incipient capital *inflow* => (from UIP and expected depreciation eq.) $s\downarrow$ (appreciation) **to the point where** the *anticipated depreciation x offsets exactly* the increase in ι

Dornbusch (1976): assumptions on *goods market* structure and equilibrium

1. the *foreign* price level *normalised* $p^* \equiv \ln P^* = \ln 1 = 0$ so that the **relative price** of domestic goods $s + p^* - p$ becomes $s - p$
2. a special, *ad-hoc* ***demand*** **f-n for domestic output**

$$\ln D = u + \delta(s-p) + \gamma y - \sigma \iota$$

3. a special, *ad-hoc* **f-n for the *rate of increase* of the *price* of domestic goods**: *proportional* to an ***excess*** **demand** measure

$$\dot{p} = \pi \ln \tfrac{D}{Y} = \pi \left(\ln D - \underbrace{\ln Y}_{\equiv y} \right) = \pi[u + \delta(s-p) + (\gamma - 1)y - \sigma \iota]$$

4. with the relevant **LR** assumptions $\dot{p} = 0$ and $\iota = \iota^*$ and further substituting for $\iota = \iota^*$ above from MDF, *solving* for s yields the **LR equilibrium NER:** $\overline{s} = \overline{p} + \frac{1}{\delta}[\sigma \iota^* + (1-\gamma)y - u]$

Dornbusch (1976): price level and NER *dynamics*

- now the earlier price adjustment eq. can be simplified using the LR NER to substitute in it as well as UIP and expected depreciation eq. to express $x = \iota - \iota^* = \theta(\overline{s} - s)$ =>

$$\dot{p} = -\pi\left(\frac{\delta+\sigma\theta}{\theta\lambda} + \delta\right)(p - \overline{p}) = -\nu(p - \overline{p}) \qquad \nu \equiv \pi\left(\frac{\delta+\sigma\theta}{\theta\lambda} + \delta\right)$$

- recall that a 1st-order linear homogeneous differential eq. of some f-n of time *z(t)*, $\underbrace{\frac{dz}{dt}}_{\equiv \dot{z}} + cz = 0$, has a (definite) s-n of the form $z(t) = z(0)e^{-ct}$
- solving, by analogy, the price eq. above yields $p(t) = \overline{p} + (p_0 - \overline{p})e^{-\nu t}$
- substituting in key NER eq.: $s(t) = \overline{s} - \frac{1}{\lambda\theta}(p_0 - \overline{p})e^{-\nu t} = \overline{s} + (s_0 - \overline{s})e^{-\nu t}$
- **interpretation**
  - the spot NER *will converge* to its LR level
  - NER will *appreciate* if prices are *initially below* their LR level, and conversely

Dornbusch (1976): graphical analysis of model equilibrium and transition paths

- **Figure 1, p. 1166**, in the original paper to be discussed in class
- the **QQ schedule** describes **asset** (here also *money*) market *equilibrium* and has a *negative* slope (as can be checked earlier), so it must intersect the 45^0-line
- the $\dot{p} = 0$ **schedule** shows all combinations of price levels and exchange rates for which the **goods** market **and** the **money** market *clear*: can be derived from the $\dot{p}$ eq. with $\dot{p} = 0$ and the MDF:

$$\underbrace{0}_{=\dot{p}} = \pi \left[u + \delta(s - p) + (\gamma - 1)y - \sigma \underbrace{\frac{m - p - \phi y}{-\lambda}}_{=\iota,\ \text{from MDF}} \right]$$

- solving for p, $p = \frac{\lambda}{\lambda\delta+\sigma}u + \frac{\lambda\delta}{\lambda\delta+\sigma}s + \frac{\sigma}{\lambda\delta+\sigma}m + \frac{\lambda(\gamma-1)-\phi\sigma}{\lambda\delta+\sigma}y$
- the *slope* is $0 < \frac{\lambda\delta}{\lambda\delta+\sigma} < 1$, hence $\dot{p} = 0$ must intersect the 45^0-line too => **model equilibrium => model transition paths** (to be analysed on Figure 1, p. 1166)

Dornbusch (1976): *consistent expectations*

- if the expectations formation process in expected depreciation eq., driven by θ, must *correctly* predict – in compliance with **perfect foresight** – the *actual* path of exchange rates, determined by v, it must be true that $\theta = v$
- hence, $\theta = v \equiv \pi\left(\frac{\delta+\sigma\theta}{\theta\lambda} + \delta\right)$
- the **consistent expectations coefficient**, $\tilde{\theta}$, is obtained as the (positive and stable) solution to the quadratic equation implied by the above expression:

$$\tilde{\theta}(\delta, \lambda, \pi, \sigma) = \pi \frac{\frac{\sigma}{\lambda}+\delta}{2} + \sqrt{\left(\pi \frac{\frac{\sigma}{\lambda}+\delta}{2}\right)^2 + \frac{\pi\delta}{\lambda}}$$

- gives the **rate** at which Dornbusch (1976) economy will **converge** to long-run equilibrium along the perfect foresight path: of interest because this assumption
  - is not arbitrary
  - does not involve persistent prediction errors
- for any given *price adjustment* parameter π, convergence will be **faster**
  - the *lower* the interest-rate response of money demand, λ
  - and the *higher*
    - the interest-rate response of goods demand, σ
    - and the price elasticity of demand for domestic output, δ

Dornbusch (1976): graphical and analytical account of *NER overshooting*

- **Figure 2, p. 1169**, in the original paper to be discussed in class
  - a *two-stage adjustment* which implies **exchange rate overshooting**: a phenomenon whereby the spot exchange rate temporarily exceeds its long-run value, illustrated by the short-run equilibrium at point *B* in **Figure 2**
  - to better understand the key model result, let us also derive it **analytically**
    - totally differentiate MDF, noting that p is instantaneously fixed and y is always fixed $\frac{d\imath}{dm} = -\frac{1}{\lambda} < 0$
    - in the *LR*, an ↑ in money causes an *equiproportionate* ↑ in prices and the exchange rate: $d\overline{s} = d\overline{p} = dm$
    - totally differentiate UIP eq. while holding $\imath^*$ constant and use $d\overline{s} = dm$ in expected depreciation eq. to obtain $d\imath = \theta\left(\underbrace{dm}_{=d\overline{s}} - ds\right)$
    - use this to eliminate $d\imath$ in *liquidity effect* eq. above and solve for ds

$$ds = \left(1 + \frac{1}{\lambda\theta}\right)\underbrace{dm}_{=d\overline{s}} \qquad 1 + \frac{1}{\lambda\theta} > 1 \qquad ds > d\overline{s}$$

Dornbusch (1976): version *without* NER overshooting (I)

- **output** is now assumed **variable** => an *output gap* is thus defined
- the ad-hoc *f-n of the rate of increase of the price of domestic goods* is *replaced* by **two new equations**
  - an **equilibrium cond. in the domestic goods market** (written in two ways)

$$y = \ln D \equiv u + \delta(s - p) + \gamma y - \sigma \iota$$

$$y = \mu[u + \delta(s - p) - \sigma \iota], \qquad \mu \equiv \tfrac{1}{1-\gamma} > 0$$

  - a **price adjustment eq. related to the output gap**: ad-hoc, again, but motivated by *Phillips curve* (*wage inflation* – unemployment: *negative*) + *Okun's law* (unemployment – *output gap*: *positive*): $\dot{p} = \pi(y - \overline{y})$, where $\overline{y}$ denotes the *full-employment* level of output
- **LR** equilibrium implies $y = \overline{y}$ and $\iota = \iota^*$ so the output eq. becomes

$$\overline{y} = \mu[u + \delta(\overline{s} - \overline{p}) - \sigma \iota^*]$$

Dornbusch (1976): version *without* NER overshooting (II)

- *subtracting* the LR from the SR equilibrium **output** eq.

$$y - \overline{y} = \mu(\delta + \sigma\theta)(s - \overline{s}) + \mu\delta(\overline{p} - p)$$

- doing the *same* (i.e., rewriting in terms of *deviations* from LR equilibrium) with **money** market equilibrium

$$\phi(y - \overline{y}) + (p - \overline{p}) = \lambda\theta(\overline{s} - s)$$

- *solving* the above simultaneous system for the spot NER and the level of output as a f-n of the existing price level, one obtains

$$y - \overline{y} = -\omega(p - \overline{p}) \qquad \omega \equiv \frac{\mu(\delta+\theta\sigma)+\mu\delta\theta\lambda}{\Delta}, \qquad \Delta \equiv \phi\mu(\delta + \theta\sigma) + \lambda\theta$$

$$s - \overline{s} = -\frac{1-\phi\mu\delta}{\Delta}(p - \overline{p})$$

Dornbusch (1976): version *without* NER overshooting (III)

- substituting the **solution for output** into price adjustment yields

$$\dot{p} = -\pi\omega(p - \overline{p})$$

- **consistent expectations** here *require* $\theta = \pi\omega$ (which can be solved for $\widetilde{\theta}$)
- in the LR, $d\overline{s} = d\overline{p} = dm$, and considering from the **NER solution**

$$\frac{ds}{dm} = 1 + \frac{1-\phi\mu\delta}{\Delta} > 0$$

- thus, whether NER increases *more* **or** *less* proportionately than the nominal quantity of money depends on the condition

$$\frac{1-\phi\mu\delta}{\Delta} \gtrless 0$$

- which determines *too* whether the domestic (nominal) interest rate declines (*liquidity effect* of monetary expansion) **or** increases
- to sum-up, **two assumptions** are *crucial* to producing **NER overshooting**
  1. output (AS) *y* is *fixed* => does not respond to changes in AD (some induced by *dm*)
  2. income elasticity of real money demand, ϕ , is *low enough*, so that even if output (AS) *y* is *variable* and responds to AD (and *dm*), the change in output is not sufficient, from MDF, to prevent the other channel of adjustment, via the interest rate, ι , from working

Concluding wrap-up

- **What have we learnt?**
  - distinguish and discuss
    - *rational* expectations and *perfect* foresight
    - *flexible*-price vs *sticky*-price models of exchange rate *dynamics*
  - derive and understand Dornbusch (1976) model
    - *equilibrium* and *transition paths*
    - *effects of monetary expansion*
      - when and why NER overshooting *occurs*
      - when and why NER overshooting *needs not necessarily occur*
  - analyse the basic set-up of *ad-hoc* dynamic monetary OEMs
- **Where we go next**: to the *microfounded* DSGEM of exchange rates by Lucas (1982)