EC933-G-AU INTERNATIONAL FINANCE – LECTURE 9

FOREIGN EXCHANGE MARKET EFFICIENCY AND MICROSTRUCTURE: MODELS WITH NOISE TRADERS

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Abstract. Until now, all exchange rate determination models considered in this course assumed *rational* expectations. This lecture weakens the latter assumption by allowing for certain *departures* from rationality, based on the *microstructure* of the forex market. Microstructure is, essentially, about the *heterogeneity* of the various sets of "players" in the foreign exchange market, such as traders and investors, part of whom may be rational but another part not, as well as the monetary authorities or governments at large, which intervene in this market pursuing policy objectives under economic and political constraints. We start, in section 1, by summarising the *efficient market hypothesis*, as it relates to the *foreign exchange* market, and its tests, mostly based on the covered (CIP) and uncovered (UIP) interest parity conditions outlined in lecture 1. Section 2 then sketches the major lines of research and the key outcomes within the literature on official forex market intervention. Having thus reviewed the principal arbitrage conditions in the forex market and the behaviour of the most typical agents in it, and, in particular, the main reasons for the failure of UIP, we focus on some of the possible explanations of this failure in the recent research on forex market microstructure briefly introduced in section 3. We then illustrate this approach in detail by analysing, in section 4, the model of exchange rate determination and regime evaluation with noise traders proposed by Jeanne and Rose (2002).

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This set of lecture notes is preliminary and incomplete. It is based on parts of the four textbooks suggested as essential and supplementary reading for my graduate course in international finance at Essex as well as on the related literature (see the course outline and reading list at http://courses/essex.ac.uk/ec/ec933/). The notes are intended to be of some help to the students attending the course and, in this sense, many aspects of them will be clarified during lectures. The present second draft may be developed and completed in future revisions. The responsibility for any errors and misinterpretations is, of course, only mine. Comments are welcome, preferably by e-mail at mihailov@essex.ac.uk and/or a_mihailov@hotmail.com.

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1. Forex Market Efficiency

The notion of foreign exchange (forex) market efficiency is closely related to the *efficient market hypothesis* (EMH) in finance. In its *simplest* form, EMH is, in fact, a *joint* hypothesis that, in an aggregate sense, the participants in the (forex) market:

(1) hold *rational* expectations;
(2) are risk-*neutral*.

EMH can also be modified to account for risk-*averse* market participants: its second part in this case becomes a model of equilibrium returns allowing for risk *premia*.

Following Fama (1970), *three* forms of market efficiency are usually distinguished:

(1) *weak* form: prices reflect all the information contained in *past* prices;
(2) *semi-strong* form: prices reflect all *publicly* available information, including that contained in past prices; this form of market efficiency seems closest to the rational expectations hypothesis;
(3) *strong* form: prices reflect *all* information that can possibly be known; this form of market efficiency is not likely to hold, even in theory, mainly because of *secret non-random intervention* in forex markets by central banks (a topic we briefly summarise in section 2).

If a financial – or, in particular, the foreign exchange – market is efficient, then:

- prices should *fully* reflect information available to market participants; and
- it should be *impossible* for a trader to earn excess return by speculation.

Academic interest in forex market efficiency, as Sarno and Taylor (2002), p. 5, point out, has usually been related to:

- the *information content* of financial market prices; and
- the implications for *social efficiency*.

Tests of forex market efficiency have traditionally involved tests of the interest parity conditions we introduced in lecture 1 of this course, CIP and UIP.

1.1. **Testing CIP.**

Recall that, algebraically, covered interest parity (CIP) is expressed (*ignoring* transaction costs) as:

$$\frac{F_{t,k}}{S_t} = \frac{1+\iota_{t,k}}{1+\iota^*_{t,k}}, \tag{1.1}$$

where S_t is the spot exchange rate (i.e., the current, date t, domestic price of foreign currency), $F_{t,k}$ is the current k-period forward rate (i.e., the rate agreed now, at date t, for an exchange of currencies k periods ahead), and $\iota_{t,k}$ and $\iota^*_{t,k}$ are the net interest rates on otherwise *identical* securities (with k periods to maturity) but denominated in *different* currencies.

A *logarithmic* approximation to (1.1) is:

$$f_{t,k} - s_t = \iota_{t,k} - \iota^*_{t,k}. \tag{1.2}$$

Two approaches have dominated empirical research on the validity of CIP.

The first one relies on computing the actual deviations from CIP, to check if these "significantly" differ from zero. This significance of departures is often defined w.r.t. a "neutral band", determined by transaction costs. Frenkel and Levich (1975, 1977) started this literature, based on the idea that transactions costs create a neutral band within which prices of spot and forward exchange rates and domestic and foreign interest rates of identical securities which differ only in their currency of denomination can fluctuate without any profit opportunities. Then the question they pose is: how frequent are there observations that lie *outside* the bands? They find that around 80% of apparent profit opportunities lie *within* the neutral band for Treasury bills and almost 100% for Eurocurrency rates.

A second approach for testing the validity of CIP is based on regression analysis. It reduces to estimation of (1.2), in the form of

$$f_{t,k} - s_t = \alpha + \beta \left(\iota_{t,k} - \iota^*_{t,k} \right) + \epsilon_t \tag{1.3}$$

and (again) abstracting from transaction costs. CIP is considered to hold if estimates of α and β are not significantly different from zero and unity, respectively, given also that the error term ϵ_t is not autocorrelated. Sarno and Taylor (2002), p. 8, sum up the results from such CIP regressions in the following way:

> "The main conclusion to be drawn from this line of research is that, broadly speaking, CIP is supported in that although there are significant deviations of α from zero (reflecting perhaps nonzero transactions costs) the estimates of β differ insignificantly from unity in the majority of cases."

Thus, as we said in lecture 1, CIP is largely confirmed by the empirical evidence. Yet all these early tests of CIP suffer from a number of methodological problems, as has later been recognised.

1.2. **Testing UIP.** The basic relationship which has been used to asses forex market efficiency is uncovered interest parity (UIP). Recall (from lecture 1) that UIP is defined by a similar condition to CIP, with the only difference being that, instead of the forward rate, it is now the *expected* future (at date $t+k$) exchange rate, *conditional* on all available information at date t, that appears in the definition:

$$E_t \left[s_{t+k} \right] - s_t = \iota_{t,k} - \iota^*_{t,k}, \tag{1.4}$$

where the exchange rate variables are in *logarithms*.

The first checks for forex market efficiency, understood as validity of UIP, have tested for *randomness* in exchange rate *changes*. But – as Sarno and Taylor (2002), p. 11, put it – only if (i) the nominal interest rate differential is identically equal to a *constant*, $\iota_{t,k} - \iota^*_{t,k} \equiv const$, and (ii) expectations are *rational* does (1.4) imply a *random walk* in the exchange rate (with *drift* if the constant is non-zero): $E_t \left[s_{t+k} \right] = const + s_t$. An extension of such analysis is Cumby and Obstfeld (1981). They tested for, and rejected, the randomness of deviations from UIP.

Most often, tests of UIP have applied regression analysis to the spot and forward exchange rates. Assuming CIP, UIP implies that the forward *premium* (or *discount*) – i.e., the *percentage* deviation of the current *forward* rate from the current *spot* rate (recall this definition from lecture 1) – should be equal to the *market expectation* (at the relevant maturity) of the exchange rate *depreciation* (or, inversely, *appreciation*). To see this point clearly, just compare (1.2) and (1.4): these equations imply

$$E_t \left[s_{t+k} \right] - s_t = f_{t,k} - s_t,$$

$$E_t \left[s_{t+k} \right] = f_{t,k}.$$

This means that, under rational expectations (RE), the expected change in the exchange rate should differ from the actual change only by a *RE forecast error.* Hence, assuming CIP, the UIP condition (1.4) can be tested by estimating a regression of the form:

$$s_{t+k} - s_t = \alpha + \beta \left[f_{t,k} - s_t \right] + \epsilon_{t+k}, \tag{1.5}$$

where ϵ_{t+k} is a disturbance term, with $E_t \left[\epsilon_{t+k} \right] = 0$. If agents are risk-neutral and have rational expectations, the econometrician should expect the slope parameter $\beta = 1$.

Sarno and Taylor (2002), p. 12, write:

> "Empirical studies based on the estimation of (1.5), for a large variety of currencies and time periods, generally report results which are unfavourable to the efficient market hypothesis under risk neutrality (e.g., Frankel, 1980, Fama, 1984, Bekaert and Hodrick, 1993)."

Early empirical research on UIP was impaired by the nonstationarity of many of the series included in such regressions, which was understood later and attempts to correct for the problems were made. As the sophistication of econometric techniques has been increasing, stronger evidence has been generated that the simple, no-risk premium forex market efficiency hypothesis tested via UIP does not hold in the data. Therefore, four major explanations of the failure of UIP, and corresponding improvements in modelling and estimating it, have emerged. We only enumerate them below. For more detail, the books by Sarno and Taylor (2002), chapter 2, and Mark (2001), chapter 6, could be a good starting point.

1.3. **Reasons Why UIP Does Not Hold in the Data.**

(1) risk-*averse* agents;
(2) a *failure*, in some sense, of the RE component of the joint hypothesis involved in the formulation of EMH:
   (a) *rational bubbles*: may occur because of multiple RE equilibria in addition to the "fundamentals" solution (recall the monetary model in lecture 3);
   (b) regime shifts and *rational learning* in the forex market: Lewis (1989 a,b);
   (c) the *peso problem*: Rogoff (1979);
   (d) *inefficiencies in information processing* (i.e., non-RE) revealed from survey data studies on exchange rate expectations: Bilson (1981).

To sum up, data have persistently rejected the simple risk-neutral forex market efficiency hypothesis tested via UIP. Recent work based on *survey data* of exchange rate expectations tends to suggest that this is due to problems with *both* assumptions underlying the joint hypothesis of efficient markets, namely risk *neutrality* and *rational* expectations.

2. Official Intervention in the Forex Market

We shall have time to only briefly summarise this topic. For further reading, one may start by Sarno and Taylor (2002), chapter 7, on which we base our summary in the present section.

Official intervention in the foreign exchange market occurs when the authorities buy or sell foreign currency, normally against the national currency, with the objective to affect the (current) exchange rate. *Exchange-rate management* under *float*, mostly implemented through official interventions, is of crucial policy importance, which has given rise to a huge theoretical and empirical literature. However, questions of interest such as (i) whether, (ii) by what means, (iii) by how much and (iv) for how long the authorities can affect the exchange rate through intervening in the forex market remain open, and are subject to a substantial and ongoing controversy.

2.1. **Rationale.**

- *"wrong rate" argument*: under float, an inefficient forex market may tend to generate the "wrong" exchange rate, which implies *ex ante* abnormal returns, rather than the "correct" rate, defined as corresponding to economic fundamentals;
- *information set mismatch*: some information available to, and used by, market participants may be inaccurate or misleading in comparison to the information set of the authorities;
- *offsetting temporary disturbances*: e.g., exchange rate overshooting or cross-country policy interdependence; this point is eloquently made by Pilbeam (1991), as interpreted in Sarno and Taylor (2002), p. 211:

  "Clearly, in a perfect world with no frictions of any sort, where market clearing occurs instantaneously in every market, information is perfect and agents form rational expectations, exchange rate overshooting and cross-country policy interdependence do not necessarily provide a case for official intervention in the foreign exchange market. In a "second best" world, however, with rigidities and imperfections of various sorts in both labour and goods markets, the textbook model may be misleading if taken too seriously and it is a well-known result of the theory of the second best that the introduction of a further distortion in the presence of existing distortions does *not necessarily* lower welfare (Pilbeam, 1991).";
- *adjustment-smoothing argument*: smoothing the adjustment process of exchange rates from short run values to long run values.

2.2. **Types.**

(1) non-sterilised (as defined thus far) vs *sterilised* forex intervention: when the authorities – simultaneously or with a very short lag – take action to offset, or "sterilise", the effects of a change in official foreign asset holdings on the domestic monetary base (recall the balance sheet of the central bank we introduced in lecture 1 and the mechanics of a forex intervention reflected in it);
(2) public (announced) vs *secret* forex intervention;
(3) internationally coordinated (*concerted*) vs non-coordinated forex intervention.

2.3. **Profitability and Data.**

- *profitability*: empirical evidence on profitability of intervention in the 1980s – surveyed by Edison (1993) and Sweeney (1997) – suggests that profits from intervention (made at the expense of forex speculators) vary significantly according to the sample period considered but, in general and in the long run, *central banks do make profits* from intervening in the foreign exchange market;
- *data*: no matter that monetary statistics is regularly (monthly) published, it has been very difficult until recently to collect data on official intervention *at reasonable frequencies.*

2.4. **Channels of Influencing the NER by Sterilised Intervention.**

(1) *portfolio balance (adjustment) channel*: can be analysed within the framework of the portfolio balance model (PBM) of exchange rate determination (recall lecture 3), in which investors continuously rebalance their portfolio among assets of various currencies and countries on the basis of their relative expected returns;
(2) *signalling (policy intentions) channel* (Mussa, 1981): assumes that intervention affects exchange rates by providing the market with new relevant information, under the implicit assumptions that
   (a) the authorities have *superior* information to other market participants;
   (b) they are *willing to reveal* this information through their actions in the forex market;
   - more precisely, the effect of sterilised intervention occurs because private agents change their exchange rate expectations
     - either because they change their view of the *likely future actions* of the central bank;
     - or because they change their view of the *impact* of certain actions of the central bank.

2.5. **Effectiveness.**

- a strong consensus exists that *non-sterilised* foreign exchange intervention acts like monetary expansion or contraction, and that it is rather *effective* in inducing changes in the stock of the monetary base, hence in the broader monetary aggregates and interest rates, and ultimately in market expectations and the exchange rate;
- the effectiveness of *sterilised* intervention is, on the contrary, very controversial, evidence is quite *mixed*, and – accordingly – the core of the debate on effectiveness of official forex interventions largely concerns sterilised intervention;
- overall, the more recent literature suggests a significant *signalling effect* of official intervention on both the level and the change of exchange rates.

2.6. **Central Bank Reaction Functions.** A whole literature has emerged, as a positive approach to official intervention in particular, but more generally on monetary policy analysis, describing the objectives and constraints of central banks both on a theoretical and applied level. Empirical research has focused on estimating monetary policy (or central bank) reaction functions (or feedback rules). It is not of direct interest for our course to go into its details. For a compact review, see Sarno and Taylor (2002), section 7.3, and Mihailov (2005), section 2, among others.

3. Forex Market Microstructure

The recent literature on foreign exchange market microstructure arose in an attempt to understand the substantial and persistent deviations of exchange rates from economic fundamentals documented by numerous studies. This literature is at the same time concerned with other, related issues such as the transmission of information among market participants, the behaviour of agents and – last but not least – the heterogeneity of agents' expectations and the implications of such heterogeneity for trading volume and exchange rate volatility.

It is exactly by this latter aspect that we shall illustrate in the next section in more detail some of the assumptions and approaches that are representative for the forex market microstructure literature. In addition, the model we are going to analyse at the end of this course will be important in another sense: until now, we have only studied models embodying the strong assumption of rational expectations; yet the Jeanne and Rose (2002) model with noise traders will relax this assumption, and will show how irrational behaviour can also be incorporated in a usual exchange rate determination set-up we know from previous lectures.

4. Jeanne-Rose (2002) Model with Noise Traders

The model is exposed in section "II. A Microstructural Theory of Exchange Rate Regimes" of the original article.[1] As the authors admit, it consists of two components, or blocks, each taken from a disparate part of economic theory:

- the macroeconomic theory of *exchange rate determination* ⇔ *macro*economic fundamentals (section II.1 in the original article): the *conventional* monetary model of the exchange rate with flexible prices, *augmented* by portfolio considerations, is used;
- the *noise trading* approach to asset price volatility ⇔ *micro*structure: trading behaviour (section II.2 in the original article): the well-known model of noise trading developed by De Long, Shleifer, Summers and Waldmann (1987, 1990).

  "As in chemistry, we make the experiment illuminating by combining two components that are as pure (uncontaminated by tangential complications) as possible." (JR02, pp. 541-542)

4.1. **Macro-block.** A simple money market equilibrium is *posited* in both countries (foreign variables are denoted by an asterisk), linking the (natural logarithm of the) money stock m, deflated by the (log of the) price level p, to the interest rate ι (but – observe – not income y, as is standard in such money demand related ad-hoc models) at a point in time t. Prices are assumed *perfectly flexible* and purchasing power parity (PPP) is satisfied *on average*, so that the log of the nominal exchange rate is the difference of the logs of the price levels plus an independently and identically distributed (*i.i.d.*) *normal* shock ϵ. Thus

$$m_t - p_t = -\alpha\iota_t, \tag{4.1}$$

$$m_t^* - p_t^* = -\alpha\iota_t^*, \tag{4.2}$$

$$s_t = p_t - p_t^* + \epsilon_t \tag{4.3}$$

so that

$$s_t = \overbrace{p_t}^{m_t+\alpha\iota_t} - \overbrace{p_t^*}^{m_t^*+\alpha\iota_t^*} + \epsilon_t = (m_t - m_t^*) + \alpha(\iota_t - \iota_t^*) + \epsilon_t. \tag{4.4}$$

To better focus on the impact of policy changes in the domestic country, the authors assume that the *foreign* country is in a *steady state* with *constant* money supply, price level, and interest rate. Accordingly, the time index is dropped for these foreign variables. The log of the foreign

[1] We generally keep the Jeanne-Rose (2002) notation, except on the following three occasions: as is consistent with our course, (i) the nominal interest rate is denoted by ι (iota) instead of i, (ii) the (logarithm of the) nominal exchange rate by s instead of e and (iii) the constant real level of bonds by $\overline{b}$ instead of $\overline{B}$.

price level is then *normalised* to zero ($p_t^* = 0$). *Initially*, the *domestic* money supply is assumed to be an *exogenous* policy variable and to follow a stochastic *i.i.d. normal* process centered on $\overline{m}$, as would be appropriate if the exchange rate *floats freely.*[2]

The (domestic-country) interest rate is determined by equilibrium in the international bonds market. The authors assume that investors in that market care about the return of their portfolio measured in real terms (or equivalently in terms of the foreign currency, since the foreign price level is constant). Investors are risk *averse* and require a risk *premium* to hold bonds denominated in the *domestic* currency if the exchange rate is stochastic (as it is, according to (4.4)).

> "One may think of the foreign country as the center of the international financial system, and of the domestic country as a small open economy at the periphery. For the sake of brevity and *couleur locale*, we shall sometimes call the domestic currency "peso" and the foreign currency "dollar" (although we do not wish to imply that our model is meant to work especially well for developing countries)." (JR02, p. 543)

The quantity of *domestic (Home) external liabilities* results, in equilibrium, from the current account (CA) *and* the balance of payments (BoP). These external liabilities can take the form of *bonds* denominated in either currency. The supply of bonds denominated *in peso* is the result from actions of the domestic monetary and fiscal authorities, in particular from the respective *shares* of peso and dollar bonds on the *asset* side of the central bank's balance sheet (B/S). Some assumption is now required to *endogenise* the *currency composition* of the domestic country's *external debt.* An assumption to this end, whose major benefit is analytical convenience (i.e., keeping the model simple), is that the domestic authorities maintain the supply of peso bonds to international investors at a *constant real* level $\overline{b}$.

4.2. **Micro-block.** Foreign exchange traders are modelled as *overlapping generations of investors* who live for two periods and *allocate their portfolio* between peso- and dollar-denominated one-period nominal bonds in the *first* period of their life. Traders have the same endowments and tastes, but differ in their ability to trade in the peso bonds market: *"informed" traders* are able to form *accurate (i.e., rational)* expectations on risk and returns *costlessly*, while *noise traders* have *noisy (irrational)* expectations and have to pay an *entry cost* to invest in *peso* bonds.

At each period a generation of N traders $j = 1, ..., N$ is born. Each individual trader j receives a *real* endowment of $\mathcal{W}$, which can be invested in dollar bonds at no cost. Traders decide whether or not to enter the peso bond market. The *entry* decision of trader j at time t is characterised by a *dummy* variable δ_t^j equal to 1 if trader j enters and zero if she does not. Of course, traders enter the peso-bond market only if such a choice increases their *expected utility*, as trader j's entry decision is taken on the basis of *(conditional on) information* available at $t-1$ and *before* the time t shocks are revealed:

$$\forall j, t \qquad \delta_t^j = 1 \Leftarrow E_{t-1}^j \left[U_t^j \mid \delta_t^j = 1 \right] \geq E_{t-1}^j \left[U_t^j \mid \delta_t^j = 0 \right]. \tag{4.5}$$

The expectation operator bears the traders's index to allow for *heterogeneity*; whenever it is written *without* index, this denotes the *rational* expectation.

Once having entered the peso-bond market (i.e., once $\delta_t^j = 1$), the trader invests b_t^j in peso bonds so as to maximise the expected utility of her *end-of-life* (period when *old*) wealth. Jeanne and Rose (2002) assume the trader j's *portfolio allocation problem* at time t (the period when the same trader is *young*) to be

$$\max_{b_t^j} \quad U_j^j = E_j^j \left[-\exp\left(-a\mathcal{W}_{t+1}^j \right) \right], \tag{4.6}$$

where a is the coefficient of absolute risk aversion (CARA) and $\mathcal{W}_{t+1}^j$, the end-of-life wealth of trader j, is given by

[2] Note as well that the Jeanne-Rose (2002) macro-block implicitly assumes the *same sensitivity* of the real money balances to changes in the interest rate in both countries ($\alpha \equiv \alpha^*$), which generally would not be the case.

$$\mathcal{W}_{t+1}^{j} = (1+\iota^*)\,\mathcal{W} + \delta_t^j \left(b_t^j \rho_{t+1} - c^j \right). \tag{4.7}$$

Trader j's end-of-life wealth is equal to her initial endowment times the (gross) yield on dollar bonds plus, if j enters, the excess return on peso bonds minus a fixed cost of entry. The log-linearised excess return on peso bonds between t and $t+1$ is

$$\rho_{t+1} \equiv \iota_t - (s_{t+1} - s_t) - \iota^*. \tag{4.8}$$

Note that the right-hand side above is, if equal to zero, the *ex-post* uncovered interest parity (UIP) condition (with the NER in logs). The excess return is thus (implicitly) defined here, as is standard, by the deviation (ex-post) $\rho_{t+1} \neq 0$ from UIP.

The entry cost – much discussed in the literature – may include informational problems, tax issues, etc. and is not small given the size of the "home market effect" in Lewis (1995) survey.

The informed traders are N_i in number, with $j = 1, ..., N_i$. The remaining N_n traders, with $j = N_i + 1, ..., N$, are noise traders, and clearly $N \equiv N_i + N_n$.

Thus, for $j \leq N_i$ who hold rational expectations one can write

$$E_t^j \left[\rho_{t+1}\right] = E_t \left[\rho_{t+1}\right], \qquad \forall j \leq N_i,$$

$$var_t^j \left[\rho_{t+1}\right] = var_t \left[\rho_{t+1}\right], \qquad \forall j \leq N_i.$$

Jeanne and Rose (2002) follow the (standard) assumption that noise traders perceive the second moment of returns correctly, but allow their perception of first moments to be affected by noise that is unrelated to economic fundamentals. In other words, noise traders have irrationally volatile *expectations*. The noise is *common* across traders; there is no private information. This assumption is needed because, as the authors stress, the impact of noise on the exchange rate should not cancel out in aggregate, which is the case only if the noise has a component that is common across all noise traders. Furthermore, the authors assume, for simplicity, that *only* this component is present, claiming that the addition of another, idiosyncratic component within the noise term would not change the essence of their results. Formally, these assumptions are summarised as follows:

$$E_t^j \left[\rho_{t+1}\right] = \overline{\rho} + \nu_t, \qquad \forall j > N_i,$$

$$var_t^j \left[\rho_{t+1}\right] = var_t \left[\rho_{t+1}\right], \qquad \forall j > N_i,$$

where $\overline{\rho}$ is the *unconditional* mean of the excess return (or average risk premium) and the *noise* term ν_t is a stochastic *i.i.d. normal* shock *common* across $j > N_i$ and uncorrelated with m_t and ϵ_t. Jeanne and Rose (2002) interpret the noise term as a fad that is widespread but nonfundamental. Unlike De Long, Shleifer, Summers and Waldmann (1990) on which they build, the former two coauthors assume that noise traders do not make systematic errors in their prediction of excess returns. They link the size of noise trader errors to economic (or fundamental) uncertainty by assuming that the variance of the noise is proportional to the true *unconditional* variance of the exchange rate:

$$var\left[\nu\right] = \lambda var\left[s\right], \tag{4.9}$$

where λ is a positive coefficient.

At this stage in the model, *individual heterogeneity across noise traders* is introduced through the *cost of their entry into the peso-bond market.* Jeanne and Rose (2002) mention that such a heterogeneity can be rationalised in a number of ways. It may, for example, reflect the fact that some traders inherit a larger stock of knowledge on the domestic economy and so can afford to invest less in the acquisition of information. Without loss of generality, the authors *order* noise traders by *increasing* entry cost:

$$c^j = 0 \quad \text{for } j \leq N_i,$$

$$c^j \geq 0 \quad \text{increasing with } j \text{ for } j > N_i.$$

Another assumption made here is that the entry cost of noise traders is *not too small*:

$$\forall j > N_i, \quad c^j > \frac{1}{2a} \log(1+\lambda).$$

4.3. **Equilibrium.** An equilibrium in this model consists of *stochastic processes* for

- the exchange rate $\{s_t\}$,
- the risk premium $\{\rho_t\}$,
- and individual traders' decision rules $\left\{\delta_t^j\right\}$ and $\left\{b_t^j\right\}$

such that at each period t,

- δ_t^j satisfies the entry condition (4.5),
- b_t^j is the solution to the optimal portfolio allocation problem (4.6),
- and the market for domestic (peso) bonds is in equilibrium:

$$\overline{b} = \sum_{j=1,\ldots,N} \delta_t^j b_t^j.$$

This equilibrium is difficult to determine since it involves entry decisions by a set of heterogeneous agents in a stochastic environment. Jeanne and Rose (2002) exploit the assumption that the driving shocks are i.i.d., which suggests that the set of equilibrium individual decisions take a simple form.

They thus solve the model with a "guess-and-verify" technique, first postulating its properties, then checking that they are satisfied. They conjecture that

- the fluctuations of the exchange rate are *i.i.d. around an average* level $\overline{s}$;
- *all* informed traders and a *constant* number of noise traders, n, enter the peso bond market at each period.

Jeanne and Rose (2002) characterise the equilibrium in two steps.

(1) they determine the equilibrium exchange rate, taking the number of noise traders in the domestic market as given;
(2) they then endogenise the number of noise traders, using the entry condition.

4.4. **Exogenous Number of Noise Traders.** In equilibrium the domestic interest rate and the risk premium are i.i.d. around average values denoted $\overline{\rho}$ and $\overline{\iota}$, respectively. Hence, the average risk premium is

$$\overline{\rho} = \overline{\iota} - \iota^*, \tag{4.10}$$

which, taking the expectation of equation (4.4), implies that

$$\overline{s} = \overline{m} - m^* + \alpha\overline{\rho}. \tag{4.11}$$

A rise in $\overline{s}$ corresponds to a depreciation of the domestic currency (the peso). Equation (4.11) says that a higher average interest rate differential, by decreasing the demand for domestic money relative to foreign money in (4.1) and (4.2) leads to domestic currency depreciation.

The risk premium is determined in equilibrium in the market for peso bonds. If the excess return on these bonds is *normally distributed* (in addition to being *i.i.d.*, as already assumed), which is shown to be true in equilibrium below, it is well-known – recall lecture 1 – that maximising (4.6) is equivalent to maximizing the mean-variance objective function

$$E_t^j\left[\mathcal{W}_{t+1}\right] - \frac{a}{2} var_t^j\left[\mathcal{W}_{t+1}\right],$$

and that the individual trader j's demand for peso bonds is given by

$$b_t^j = \frac{E_t^j\left[\rho_{t+1}\right]}{a \quad var_t^j\left[\rho_{t+1}\right]}.$$

The equality of demand and supply in the peso bond market implies that

$$N_i \frac{E_t\left[\rho_{t+1}\right]}{a \quad var_t\left[\rho_{t+1}\right]} + n \frac{\overline{\rho} + \nu_t}{a \quad var_t\left[\rho_{t+1}\right]} = \overline{b},$$

$$\frac{N_i E_t\left[\rho_{t+1}\right] + n\left(\overline{\rho} + \nu_t\right)}{a \quad var\left[s\right]} = \overline{b},$$

or

$$N_i E_t\left[\rho_{t+1}\right] + n\left(\overline{\rho} + \nu_t\right) = \overline{b} a \quad \underbrace{var\left[s\right]}_{\text{known constant}} \tag{4.12}$$

Now taking the expectation of the above expression at $t-1$ gives the average risk premium:

$$E_{t-1}\left\{N_i E_t\left[\rho_{t+1}\right]\right\} + E_{t-1}\left\{n\left(\overline{\rho} + \nu_t\right)\right\} = E_{t-1}\left\{a\overline{b} var\left[s\right]\right\},$$

$$N_i E_{t-1}\left[\rho_{t+1}\right] + n\left(\overline{\rho} + E_{t-1}\left[\nu_t\right]\right) = a\overline{b} var\left[s\right],$$

$$N_i \overline{\rho} + n\left(\overline{\rho} + 0\right) = a\overline{b} var\left[s\right],$$

$$\left(N_i + n\right)\overline{\rho} = a\overline{b} var\left[s\right],$$

$$\overline{\rho} = a \frac{\overline{b}}{N_i + n} var\left[s\right]. \tag{4.13}$$

We can now rewrite equation (4.4) as

$$s_t - \overline{s} = m_t - \overline{m} + \alpha\left(\iota_t - \overline{\iota}\right) + \epsilon_t.$$

It follows from (4.12) and (4.13) that

$$E_t\left[\rho_{t+1}\right] = \overline{\rho} - \frac{n}{N_i}\nu_t.$$

Taking expectation of (4.8) gives

$$E_t\left[\rho_{t+1}\right] = \overline{\rho} + \left(\iota_t - \overline{\iota}\right) + \left(s_t - \overline{s}\right),$$

so that

$$\iota_t - \overline{\iota} = -\left(s_t - \overline{s}\right) - \frac{n}{N_i}\nu_t.$$

Using this equation to substitute the nominal interest rate out of (4.4) gives

$$s_t - \overline{s} = \frac{1}{1+\alpha}\left(m_t - \overline{m} + \epsilon_t - \alpha\frac{n}{N_i}\nu_t\right). \tag{4.14}$$

Recall that α is the interest elasticity of real money balances.

Taking the variance of (4.14) and using (4.9) to substitute out the variance of the noise closes the characterisation of equilibrium with an expression for exchange rate variability:

$$var\left[s\right] = \frac{var\left[m+\epsilon\right]}{\left(1+\alpha\right)^2 - \lambda\alpha^2\left(\frac{n}{N_i}\right)^2}. \tag{4.15}$$

The variance of the exchange rate depends on both fundamentals (numerator above) and – the novelty in this model – noise (denominator): an exogenous increase in the number of noise traders (a) unambiguously *increases the variance of the exchange rate* (see equation (4.15)), which *tends to increase the risk premium* (see equation (4.13)); on the other hand, it (b) also increases the total number of traders demanding peso bonds, which lowers the risk premium (see again equation (4.13)). In the interpretation offered by Jeanne and Rose (2002), noise traders thus have two counteracting roles in the model: they both (a) *create risk* and (b) *share risk*! As a result, the impact of the extra noise traders on the equilibrium risk premium is non-monotonic (ambiguous): see Figure 1, p. 549, in Jeanne and Rose (2002). The authors claim that this ambiguity, and the fact that the risk premium can be increasing with the number of noise traders, lie at the heart of their model.

4.5. **Endogenous Entry of Noise Traders.** In section II.5 of the original article the composition of the pool of active traders is endogenised. While informed traders always enter the peso market in equilibrium, since they do not bear any entry cost, a noise trader j enters only if the gross benefit, GB, of diversifying her portfolio into peso bonds exceeds her cost of entry, c^j.

$$GB\left(\overline{\rho}, var\left(s\right)\right) = \frac{1}{2a\left(1+\lambda\right)} \frac{\overline{\rho}^2}{var\left(s\right)} + \frac{1}{2a} \log\left(1+\lambda\right) \geq c^j \tag{4.16}$$

Note that the partial derivatives in (4.16) have an intuitive interpretation: the benefit of entry, as assessed by noise traders, is increasing with the average risk premium, $\overline{\rho}$, and decreasing with exchange rate variability, $var\left(s\right)$.

But – as Jeanne and Rose (2002), p. 550, duly stress – in equilibrium both the average risk premium and the variance of the NER are functions of the number of noise traders that enter the peso-bond market. This *circularity* is then shown to generate *multiple* equilibria.

We would not have time to consider in our present course the model version with endogenous entry of noise traders in more detail, so it is left for individual study by those interested.[3]

4.6. **Policy Implications.** [... to be discussed in class ...]

The loss function which the government is assumed to minimise is:

$$\min L \equiv \omega var\left(p\right) + \left(1-\omega\right) var\left(\iota\right).$$

Under (pure) float, the minimisation problem is subject to the following constraint:

$$\left(p_t - \overline{p}\right) + \left(\iota_t - \overline{\iota}\right) = z_t,$$

where z_t, is the composite external shock given by:

$$z_t \equiv -\left(\epsilon_t + \frac{n}{N_i}\nu_t\right).$$

The composite shock, z_t, is the sum of two components: an exogenous fundamental component, the shock to PPP ϵ_t, and an endogenous nonfundamental component, noise $\frac{n}{N_i}\nu_t$.

So:

$$(P)\left\{\begin{array}{c} \min L \equiv \omega var\left(p\right) + \left(1-\omega\right) var\left(\iota\right) \\ \left(p_t - \overline{p}\right) + \left(\iota_t - \overline{\iota}\right) = z_t \end{array}\right. .$$

Under "stable NER" regime, a new constraint is added, as a second one, to the minimisation problem:

$$var\left(s\right) \leq \upsilon.$$

So now:

$$(P')\left\{\begin{array}{l} \min L \equiv \omega var\left(p\right) + \left(1-\omega\right) var\left(\iota\right) \\ \left(p_t - \overline{p}\right) + \left(\iota_t - \overline{\iota}\right) = z_t \\ var\left(s\right) \leq \upsilon \end{array}\right. .$$

[... to be discussed in class ...]

4.7. **Empirics.** [... to be discussed in class ...]

[3] The algebraic treatment is in the Appendix to Jeanne and Rose (2002).

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