EC933-G-AU INTERNATIONAL FINANCE – LECTURE 8

NEW OPEN-ECONOMY MACROECONOMICS

ALEXANDER MIHAILOV

ABSTRACT. This lecture introduces to an influential and broadly expanding recent literature in international finance, the so-called New Open-Economy Macroeconomics (NOEM). It is impossible to study NOEM in just a lecture, it would normally take at least one course to cover the major branches of this actively ongoing research. Our purpose here will thus be to sketch the main issues in the NOEM field and methodology, without going into the details of the many papers we only suggest as references for further study. We would, naturally, present the NOEM approach by focusing in this lecture on the Obstfeld and Rogoff (1995) "redux" model of *microfounded* exchange rate *dynamics* under *sticky* prices in the *currency of producers*, the article that initiated that whole literature. Analysing in detail the "redux" model will be (partly) representative of the topics usually addressed and methods commonly employed by other (later) NOEM papers.

Date: 6 December 2005 (First draft: 5 December 2004).

This set of lecture notes is preliminary and incomplete. It is based on parts of the four textbooks suggested as essential and supplementary reading for my graduate course in international finance at Essex as well as on the related literature (see the course outline and reading list at http://courses/essex.ac.uk/ec/ec933/). The notes are intended to be of some help to the students attending the course and, in this sense, many aspects of them will be clarified during lectures. The present second draft may be developed and completed in future revisions. The responsibility for any errors and misinterpretations is, of course, only mine. Comments are welcome, preferably by e-mail at mihailov@essex.ac.uk and/or a mihailov@hotmail.com.

Contents

1. New Neo-Classical Synthesis in Closed Economy	3
2. Obstfeld-Rogoff (1995) Redux Model	3
2.1. Model Specification	3
2.2. Log-Linear Approximation to the Model	9
2.3. Flex-Price Equilibrium in the Linearised Model	11
2.4. Sticky-Price Equilibrium in the Linearised Model	15
2.5. Concluding Comments	17
3. New Open-Economy Macroeconomics	17
3.1. NOEM Models of Exchange Rate Dynamics	17
3.2. Explicitly Stochastic NOEM Models	18
References	19

1. New Neo-Classical Synthesis in Closed Economy

[... to be summarised in class, based on Goodfriend and King (1997) ...]

2. Obstfeld-Rogoff (1995) Redux Model

In a way, the Obstfeld and Rogoff (1995) "redux" model of exchange rate dynamics¹ could be viewed as extending to an open-economy setting the main ideas of the NNS literature for a closed economy we summarised above.

In another perspective, the redux model can be regarded as *providing microfoundations to the Mundell-Fleming-Dornbusch tradition of open-economy analysis under sticky prices.* As pointed out by Obstfeld and Rogoff (1996), p. 605, the Dornbusch (1976) perfect foresight extension of the essentially static Keynesian approach to modelling nominal exchange rates due to Mundell (1960, 1961a,b, 1963, 1964) and Fleming (1962) suffers, on a theoretical plane, from at least three methodological drawbacks:

- (1) an important one is the *lack of explicit choice-theoretic foundations* of the overshooting model, in particular, of aggregate supply (or output), which as we saw in lecture 6 was simply postulated /assumed/ in it: thus the Dornbusch (1976) model cannot predict how incipient gaps between aggregate demand and output are resolved when prices are set in advance and fail to clear markets;
- (2) another drawback is that the overshooting model is ill-equipped to capture current account dynamics or the effects of government spending, as it does not account for private or government *intertemporal budget constraints*;
- (3) finally, the Dornbusch (1976) model lack of microfoundations deprives it of any *natural* welfare metric by which to evaluate alternative macroeconomic policies.

The Obstfeld and Rogoff (1995) "redux" model addresses essentially these three kinds of failure by underpinning the realistic implications of price stickiness with the dynamic foundations of choice-theoretic economics.

2.1. Model Specification.

2.1.1. General Set-Up.

- the "world economy" in the model is represented by the unit interval [0, 1]; it consists of a continuum of agents, each of whom is the monopolistic producer of a single differentiated good;
- (2) there are two "countries", H(ome) and F(oreign):²
 - (a) producer-agents on the subinterval [0, n] reside in Home, and we index them, as well as the corresponding differentiated goods they produce, by i, so that $i \in [0, n]$;
 - (b) analogously, producer-agents $i^* \in (n, 1]$ live in Foreign;³
 - (c) n thus provides an index of the relative size of the two economies;
- (3) endogeneity of output: there are no capital or investment in the redux model, yet this is not an endowment economy because *labour supply is elastic*, i.e., chosen in individual intertemporal labour-leisure optimisation (as will become clearer in a moment, when we introduce the specification of utility).
- 2.1.2. Individual Preferences. Individual preferences are assumed:
 - (1) *identical* across agents, hence a representative agent model;
 - (2) symmetric in the two countries.

In this set-up, the representative national consumers, designated by the indexes $j \in [0, n]$ in *Home* and $j^* \in (n, 1]$ in *Foreign*, solve analogous maximisation problems. In fact, they are consumer-producers (or "yeoman-farmers") in the "redux" framework.

 $^{^1\}mathrm{Obstfeld}$ and Rogoff (1994) was the working paper version.

²Recall that Dornbusch (1976) considers the small open economy case.

³In their original work, Obstfeld and Rogoff (1994, 1995, 1996) use a notation which is similar, in principle, to ours here but slightly different: they use a common index for both countries, z, to account for producers and their respective single differentiated goods in *Home*, if $z \in [0, n]$, as well as in *Foreign*, when $z \in (n, 1]$.

The period, and hence, intertemporal /lifetime/ utility of the typical (that is, representative) Home – or Foreign, distinguished by an asterisk (*) in the notation – agent j is defined over (i) a consumption index, (ii) real money balances and (iii) effort expended in production:

(2.1)
$$U^{j} \equiv \sum_{t=0}^{\infty} \beta^{t} \left\{ \ln c_{t}^{j} + \chi \ln \frac{M_{t}^{j}}{P_{t}} - \frac{\kappa}{2} \left(y_{t}^{j} \right)^{2} \right\},$$

where

(2.2)
$$c^{j} \equiv \left[\int_{0}^{n} \left(c_{i}^{j}\right)^{\frac{\theta-1}{\theta}} di + \int_{n}^{1} \left(c_{i^{*}}^{j}\right)^{\frac{\theta-1}{\theta}} di^{*}\right]^{\frac{\theta}{\theta-1}},$$

the first (additive) term in the (period) utility function, itself time-separable within lifetime utility (2.1), is a *real* consumption Dixit-Stiglitz (1977) index (or aggregator), representing the *j*-th *Home* individual consumption of *Home* goods *i* and *Foreign* goods i^* . $\theta > 1$ will turn out to be, as will become evident a little bit later, the price elasticity of demand faced by each monopolist.⁴

The price deflator on nominal money balances is the consumption-based (money) price index corresponding to (2.2). If P_i is the *Home*-currency price of *Home* good i and P_{i^*} is the *Home*-currency price of *Foreign* good i^* , then the respective *Home* aggregate (money) price level will be defined by:

(2.3)
$$P \equiv \left[\int_{0}^{n} (P_{i})^{1-\theta} di + \int_{n}^{1} (P_{i^{*}})^{1-\theta} di^{*}\right]^{\frac{1}{1-\theta}}$$

(2.3) can also be *derived* formally using cost minimisation à la Dixit-Stiglitz (1977) to buy a *unit* of real consumption. The problem is a standard one, as mentioned in footnote 4, p. 662, in Obstfeld and Rogoff (1996) and as demonstrated in chapter 4 of that same textbook. It can be represented as

$$\min_{c_z} Z = \int_0^1 P_z c_z dz$$

subject to

$$\left[\int_{0}^{1} \left(c_{z}\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} = 1$$

In our case, with the somewhat modified notation w.r.t. the original model, $z \equiv i$ for $i \in [0, n]$ and $z \equiv i^*$ for $i^* \in (n, 1]$.

The analogous *Foreign* price level Dixit-Stiglitz index will be:

(2.4)
$$P^* \equiv \left[\int_{0}^{n} (P_i^*)^{1-\theta} \, di + \int_{n}^{1} (P_{i^*}^*)^{1-\theta} \, di^*\right]^{\frac{1}{1-\theta}}.$$

Another new feature we encounter in this course here is the *money-in-the-utility* (MiU) function approach to modelling money in microfounded general equilibrium. Recall that in Lucas (1982) money entered the model through the cash-in-advance (CiA) constraint on the transactions "technology". Money is now introduced as a direct component (i.e., the second term) of

⁴Obstfeld and Rogoff (1996) explain in footnote 2, p. 661, that when this elasticity is less than 1, the marginal revenue is negative: they therefore require $\theta > 1$ to ensure interior equilibrium with a positive level of output.

the utility function (2.1). Obstfeld and Rogoff restrict the representative agents in the redux model to hold real money balances in *their domestic* currency only.

The third term in the utility function (2.1) captures the disutility from effort /work, labour/ an individual experiences in having to produce more output (or, implicitly and inversely, his/her utility of leisure). To see it, suppose, for example, that the disutility from effort l is given by $-\phi l$ and that the production function is $y = Al^{\alpha}$ (with $0 < \alpha < 1$).

Inverting the production function yields

$$l = \left(\frac{y}{A}\right)^{\frac{1}{\alpha}}.$$

Then, the *disutility from effort* will be

$$-\phi l = -\phi \left(\frac{y}{A}\right)^{\frac{1}{\alpha}} = -\frac{\phi}{A^{\frac{1}{\alpha}}}y^{\frac{1}{\alpha}}.$$

Let $\kappa = \frac{2\phi}{A^{\frac{1}{\alpha}}}$: (NB!) observe from this assumption that a *rise* in productivity A in the redux model can thus be captured by a *fall* in the parameter κ ! The expression above can be written as:

$$-\phi l = -\phi \left(\frac{y}{A}\right)^{\frac{1}{\alpha}} = -\frac{\phi}{A^{\frac{1}{\alpha}}} y^{\frac{1}{\alpha}} = -\frac{1}{2} \frac{2\phi}{A^{\frac{1}{\alpha}}} y^{\frac{1}{\alpha}} = -\frac{1}{2} \kappa y^{\frac{1}{\alpha}} = -\frac{\kappa}{2} y^{\frac{1}{\alpha}}.$$

Let also⁵ $\alpha = \frac{1}{2}$. Now the interpretation of the output term in (2.1) as the disutility of effort becomes evident:

$$-\frac{\kappa}{2}y^{\frac{1}{\alpha}} = -\frac{\kappa}{2}y^2$$

2.1.3. Consumption-Based PPP. To derive a relationship between the aggregate price levels in H and in F, P and P^* , no impediments to trade are assumed, so that the law of one price (LOP) holds for each individual good:

(2.5)
$$\frac{P_i}{S} = P_i^* \text{ and } P_{i^*} = SP_{i^*}^*,$$

where P_i is the Home-currency price of Home good i, P_i^* is the Foreign-currency price of the same Home good i, P_{i^*} is the Home-currency price of Foreign good i^* , P_i^* is the Foreign-currency price of the same Foreign good i^* , and the nominal (spot) exchange rate S is defined (in the usual way) as the Home currency price of Foreign currency.

Under LOP, we can re-write the definitions of the H and F price levels (2.3) and (2.4) as:

(2.6)
$$P = \left[\int_{0}^{n} (P_{i})^{1-\theta} di + \int_{n}^{1} (SP_{i^{*}}^{*})^{1-\theta} di^{*}\right]^{\frac{1}{1-\theta}}$$

(2.7)
$$P^* = \left[\int_{0}^{n} \left(\frac{P_i}{S}\right)^{1-\theta} di + \int_{n}^{1} \left(P_{i^*}^*\right)^{1-\theta} di^*\right]^{\frac{1}{1-\theta}}$$

Dividing both sides of the first equation by the NER, S, or – equivalently – multiplying both sides of the second equation by S convinces that purchasing power parity (PPP) also holds, as a consequence of LOP, in the redux model, so that:

$$(2.8) P = SP^*.$$

(NB!) PPP holds in the Obstfeld-Rogoff redux model because:

 $^{^{5}}$ As Obstfeld and Rogoff (1996), p. 662, assume. They note, at the same time, that it is easy to relax this restriction in the following analysis, but doing so would complicate some expressions without modifying any of the main qualitative conclusions from the redux model.

- preferences are identical across countries; and
- there are no departures from the law of one price.

(NB!) But relative prices of various individual goods need not remain constant: changes in the terms of trade (\equiv the relative price of H and F tradables) do play an essential role in the model!

2.1.4. Individual Budget Constraints. To complete the specification of the individual's problem, we need to write down the agent's budget constraint. In doing so, Obstfeld and Rogoff (1996) assume that the only internationally traded asset is a riskless *real* bond, b_t , denominated in the *composite* consumption good. Thus,

- asset markets are *incomplete* in the sense that the only financial instrument available is the riskless real bond;⁶
- there exists, at the same time, an *integrated* world capital market where both countries can borrow or lend, by trading in bonds.

Under these assumptions about the asset structure of the redux economy, the (current-) period budget constraint for the representative Home individual j can be written in nominal terms as

(2.9)
$$M_{t-1}^{j} + P_t \left(1 + r_t\right) b_t^{j} + P_{jt} y_t^{j} - P_t c_t^{j} - P_t \tau_t \ge M_t^{j} + P_t b_{t+1}^{j},$$

where r_t denotes the (net) real interest rate on bonds between t-1 and t, y_t^j is the output of good j, for which agent j is the sole producer, and P_{jt} is its H-currency price. Obstfeld and Rogoff (1996), p. 663, emphasise that P_{jt} need not be the same for all j because each product is differentiated. Symmetric H producers will, however, find it optimal (as will be seen later) to choose the same prices for their distinct products in equilibrium. M_{t-1}^j is agent j's holdings of nominal money balances entering period t, and τ_t denotes net lump-sum transfers (taxes less subsidies) payable in the composite good c_t . The representative H agent – and, of course, symmetrically, the representative F agent – thus brings into the current period nominal wealth in the form of money balances and maturing bonds; obtains a monetary transfer $(-\tau_t)$ and chooses output (or alternatively, the price) of good j so as to maximise utility (2.1); receives income from the sales of output and allocates this income across consumption, next-period money and bond holdings (i.e., savings) observing the (nominal) budget constraint (2.9).

Dividing through by the price level and rearranging (2.9), one can rewrite the nominal budget constraint above in *real* terms, which are indeed relevant in the optimisation problem of the representative individual (consumer-producer) in *Home* (and, symmetrically, in *Foreign*):

$$c_t^j + \frac{M_t^j}{P_t} + \tau_t + b_{t+1}^j \le \frac{P_{jt}}{P_t} y_t^j + (1+r_t) b_t^j + \frac{M_{t-1}^j}{P_t}$$

From the definition of *inflation* we have:

$$\pi_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}},$$
$$\pi_t \equiv \frac{P_t}{P_{t-1}} - 1,$$
$$1 + \pi_t \equiv \frac{P_t}{P_{t-1}},$$

$P_t \equiv \left(1 + \pi_t\right) P_{t-1}.$

The last equation will be further used to substitute for P_t in the last term of the budget constraint in real terms above, which is thus modified to:

⁶Obstfeld and Rogoff (1996), p. 663, make the point that it would not be difficult to reformulate the redux model to incorporate greater diversity of assets, "but the assumption of complete asset markets would seem incongruous alongside the nominal rigidities" to be introduced later on. This "bonds-only formulation", the authors claim, is also more natural and consistent with their objective to provide microfoundations for the Mundell-Fleming-Dornbusch set-up.

(2.10)
$$c_t^j + \frac{M_t^j}{P_t} + \tau_t + b_{t+1}^j \le \frac{P_{jt}}{P_t} y_t^j + (1+r_t) b_t^j + \frac{1}{1+\pi_t} \frac{M_{t-1}^j}{P_{t-1}}.$$

2.1.5. *Government Budget Constraint*. Since *Ricardian equivalence* holds in redux set-up, Obstfeld and Rogoff also assume that:

- (1) there is no government spending;
- (2) the government runs a *balanced* budget *each* period;
- (3) all seigniorage revenues are rebated to the public in the form of transfers:

In nominal terms,

$$0 = P_t \tau_t + \underbrace{(M_t - M_{t-1})}_{\equiv \Delta M_t},$$

in *real* terms,

(2.11)

$$0 = \tau_t + \frac{M_t - M_{t-1}}{P_t},$$

or, equivalently,

$$-\tau_t = \frac{M_t - M_{t-1}}{P_t}.$$

2.1.6. Demand Curve Facing Each Monopolist. Given the constant elasticity of substitution (CES) consumption index (2.2), Home individual j's demand for any (Home- or Foreign-made) good z is derived in the following – now standard Dixit-Stiglitz (1977) – way:

Maximizing

$$c = \left[\int_{0}^{1} \left(c_{z}\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$

subject to the nominal budget constraint

$$\int_{0}^{1} P_z c_z dz = Z,$$

where Z is any *fixed* total nominal expenditure on goods, one can show that, for any *two* goods z and z':

$$c_z = \left(\frac{P_z}{P_{z'}}\right)^{-\theta} c_{z'}$$

Plugging this expression into the preceding budget constraint and using the Dixit-Stiglitz (1977) price index (2.3) gives the representative *Home* agent's demand for good z:

$$c_z = \left(\frac{P_z}{P}\right)^{-\theta} \frac{Z}{P} = \left(\frac{P_z}{P}\right)^{-\theta} c,$$

where the second equality uses the fact that P is, by definition in the Dixit-Stiglitz (1977) context, the (minimum) money cost of *one unit* of composite (real) consumption.

Thus, by analogy, we can obtain the Home individual j's optimal demand for any good z:

$$c_z^j = \left(\frac{P_z}{P}\right)^{-\theta} c^j,$$

and the *Foreign* individual j^* 's optimal demand for the same good z:

$$c_z^{j^*} = \left(\frac{P_z^*}{P^*}\right)^{-\theta} c^{j^*}.$$

It is now evident, from the Dixit-Stiglitz (1977) aggregators above, why θ is the (relative) price *elasticity of substitution* across the differentiated goods.

Integrating demand for good z across all agents (that is, in our symmetric case, taking a population-weighted average of *Home* and *Foreign* demands and skipping the j and j^{*} superscripts), and making use of equations (2.5) and (2.8) – which imply that for any good z, $\frac{P_z}{P} = \frac{P_z^*}{P^*}$ – the total world demand for good z is:

(2.12)
$$y_z^d = \left(\frac{P_z}{P}\right)^{-\theta} c^W,$$

where world consumption is given by:

$$c^{W} \equiv \int_{0}^{n} c_{z}^{j} dz + \int_{n}^{1} c_{z}^{j^{*}} dz = nc + (1-n) c^{*}.$$

(2.12) is a downward-sloping world demand curve the monopolistic producer (of any good z) faces in the redux set-up.

2.1.7. FONCs for the Representative Home Agent's Problem. To solve the model, Obstfeld and Rogoff first express

$$P_{jt}y_t^j = P_t \left(y_t^j\right)^{\frac{\theta-1}{\theta}} \left(c_t^W\right)^{\frac{1}{\theta}}$$

from the downward-sloping world demand curve (2.12) and then substitute the above expression in the period budget constraint in real terms (2.10). The resulting expression is further used to solve (2.10) for c_t and to substitute back in the intertemporal utility function (2.1), yielding the unconstrained maximisation problem⁷ for the Home representative agent j:⁸

$$\max_{y_t^j, M_t^j, b_t^j} U_t^j = \sum_{t=0}^{\infty} \beta^t \left\{ \ln \left[(1+r_t) b_t^j + \frac{M_{t-1}^j}{P_t} + \left(y_t^j \right)^{\frac{\theta-1}{\theta}} \left(c_t^W \right)^{\frac{1}{\theta}} - \tau_t - b_{t+1}^j - \frac{M_t^j}{P_t} \right] + \chi \ln \frac{M_t^j}{P_t} - \frac{\kappa}{2} \left(y_t^j \right)^2 \right\}$$
(2.13)

In performing the maximisation, the representative *Home* individual j takes c_t^W as given. The FO(N)Cs w.r.t. the choice variables are:

(2.14)
$$b_t^j: \quad c_{t+1}^j = \beta \left(1 + r_{t+1}\right) c_t^j \Leftrightarrow \frac{c_{t+1}^j}{c_t^j} = \beta \left(1 + r_{t+1}\right),$$

(2.15)
$$y_t^j: \qquad \kappa y_t^j = \frac{\theta - 1}{\theta} \frac{1}{c_t^j} \left(\frac{y_t^j}{c_t^W}\right)^{-\frac{1}{\theta}} \Leftrightarrow \left(y_t^j\right)^{\frac{\theta + 1}{\theta}} = \frac{\theta - 1}{\kappa \theta} \frac{1}{c_t^j} \left(c_t^W\right)^{\frac{1}{\theta}},$$

(2.16)
$$M_t^j: \qquad \frac{M_t^j}{P_t} = \chi c_t^j \frac{1 + \iota_{t+1}}{\iota_{t+1}} \Leftrightarrow \frac{P_t}{M_t^j} = \frac{1}{\chi} \frac{1}{c_t^j} \frac{\iota_{t+1}}{1 + \iota_{t+1}},$$

where ι_{t+1} is the *nominal* interest rate for *Home*-currency loans between t and t+1, *defined* as usual by the so-called Fisher equation:

 $^{^{7}}$ As we know from previous lectures, an alternative route is to use the *Lagrangian multiplier* method, as done, for instance, in Walsh (1998), chapter 6.

⁸The unconstrained optimisation problem for the *Foreign* representative agent j^* is symmetric.

(2.17)
$$1 + \iota_{t+1} = \frac{P_{t+1}}{P_t} \left(1 + r_{t+1}\right).$$

(2.14) is standard Euler equation for the optimal consumption path (for the case where IES is 1, i.e., for *log*-utility);

(2.15) is the optimal labour-leisure trade-off: the ratio of the marginal disutility from effort to the MUC must equal the marginal product of work;

(2.16) is a microfounded money demand function (for *log*-utility): the ratio of the marginal utility of money to MUC must equal $\frac{\iota_{t+1}}{1+\iota_{t+1}}$;

As noted earlier in this course, the FONCs w.r.t. the control variables do not fully characterise equilibrium. We have to also consider the period budget constraint (2.9) and a standard *transversality condition* (TVC):

(2.18)
$$\lim_{T \to \infty} \qquad \qquad \underbrace{\prod_{v=t+1}^{T} \frac{1}{(1+r_v)}}_{t=t+1} \qquad \left(b_{t+T+1} + \frac{M_{t+T}}{P_{t+T}}\right) = 0$$

 $\equiv R_{t,t+T}^{-1}$: market discount factor for date T consumption on date t < T

Furthermore, general equilibrium implies certain *market clearing* assumptions, which we present next.

2.1.8. *Global Equilibrium*. Global (general) equilibrium in the redux model is defined by 15 equations:

- the 3 FONCs, the TVC and the period budget constraint for the representative agent's decision problem in *each* country (as presented above for *Home*) \Leftrightarrow 10 symmetric equations;
- plus LOP-PPP and the following 4 additional, *market-clearing* conditions: 2 for each *domestic money* market as well as 2 for the *global* (i) *bond* and (ii) *goods* markets ⇔ 5 other equations.

In addition to domestic *money* market clearing in H and in F, the global *bond* market-clearing condition (these are, in fact the 3 *asset* market equilibrium conditions in the redux) imposes *zero* net foreign assets:

$$(2.19) nb_{t+1} + (1-n)b_{t+1}^* = 0$$

Given the 3 asset market clearing conditions as well as LOP and PPP, Obstfeld and Rogoff (1996) derive an aggregate global goods market clearing condition. They use the period budget constraints in real terms, (2.10) for *Home* and the corresponding equation for *Foreign*, take a population-weighted average of these real budget constraints and finally impose the bond market clearing condition (2.19) together with the *Home* government budget constraint (2.11) and its *Foreign* analogue to obtain:

(2.20)
$$c_t^W \equiv nc_t + (1-n)c_t^* = n\frac{P_{Ht}}{P_t}y_{Ht} + (1-n)\frac{P_{Ft}^*}{P_t^*}y_{Ft}^* \equiv y_t^W,$$

where P_H or P_F^* and y_H or y_F^* are price and output of the representative *Home* or *Foreign* good. Equation (2.20) thus states that world real consumption equals world real income.

2.2. Log-Linear Approximation to the Model. Because of monopoly pricing and endogenous output, the redux model does not yield simple closed-form solutions for general paths of the exogenous variables. To analyse the effects of exogenous shocks, one has to

- (1) either *simulate* it *numerically*, for *large* disturbances;
- (2) or examine a *linearised* version of (or, rather, approximation to) the model around a well-defined *flexible*-price steady state (SS), for *small* shocks and with the aim of seeing through the *intuition* behind the redux.

The most convenient SS corresponds to the case where all exogenous variables – that is, the policy variables in the redux: money supplies in H and in F – are constant. Because consumption and output are constant in this SS, the RIR is tied down by the consumption Euler equation (2.14) and is given by:

$$\beta \left(1+r \right) = 1,$$

Hence

$$r = \frac{1-\beta}{\beta} \equiv \delta = const$$

Recall from earlier lectures that the subjective discount factor $\beta \equiv \frac{1}{1+\delta}$ and the subjective discount (time preference) rate $\delta \equiv \frac{1-\beta}{\beta}$ are linked through their definition. In a symmetric SS – i.e., with all (Home) individuals being the same –, the representative Home agent's budget constraint (2.10) reduces to a simpler expression, (2.21) below. We show why: in the steady state, $M_t = M_{t-1} = M = const$ and $P_t = P_{t-1} = P = const$, hence $-\tau = \frac{M_t - M_{t-1}}{P_t} = 0$ and $\pi = \frac{P_t - P_{t-1}}{P_{t-1}} = 0$. Now re-write (2.10), with equality required for optimality and duly modified to reflect the above SS values,

$$c + \frac{M}{P} + \underbrace{\tau}_{=0} + b = \frac{P_H}{P} y_H + (1+r) b + \frac{1}{1 + \underbrace{\pi}_{=0}} \frac{M}{P},$$

cancel respective terms and rearrange

$$c + \frac{M}{P} + b = \frac{P_H}{P}y_H + b + rb + \frac{M}{P},$$

to obtain

(2.21)
$$c = \frac{P_H}{P} y_H + \delta b,$$

where b is the SS real stock of bonds held by all *Home* individuals. (2.21) says that SS real *consumption* in H equals real *income* (the real value of *output sold* plus income from *bond* holding). From the bond-market clearing condition (2.19) one could determine b^* , the SS real stock of bonds held by all *Foreign* individuals, as

$$b^* = -\frac{n}{1-n}b$$

and then express the SS real consumption in F, c^* :

(2.22)
$$c^* = \frac{P_F^*}{P^*} y_F^* - \frac{n}{1-n} \delta b$$

Obstfeld and Rogoff (1996), p. 668, note that, in general, there is no simple closed-form solution for the SS we describe, but one does exist when *initial* foreign assets are *zero*:

$$b = 0$$
 and, thus, $b^* = -\frac{n}{1-n} \underbrace{b}_{=0} = 0$

Moreover, Obstfeld and Rogoff (1996), p. 669, restrict attention to a *perfect foresight* setting (excepting the *initial* shocks), which reminds of Dornbusch (1976) overshooting model we studied in lecture 6.

The log-linear approximation (or log-linearisation) to the redux model, i.e., its version expressed in terms of *percentage deviations* around the SS (with any "hatted" variable below defined as $\hat{x}_t \equiv d \ln x_t \equiv \ln \frac{x_t}{x_0}$, where x_0 is its *initial* SS value) is thus given by:⁹

 $^{^{9}}$ We would not have space and time here to linearise the redux model equation by equation. We would only note that the procedure usually involves total differentiation and is similar to the way we derived the intertemporal elasticity of substitution in lecture 4 and the coefficient of relative risk aversion in lecture 5.

(2.23)
$$\widehat{P}_t = n\widehat{P}_{Ht} + (1-n)\left(\widehat{S}_t + \widehat{P}_{Ft}^*\right)$$

(2.24)
$$\widehat{P}_t^* = n\left(\widehat{P}_{Ht} - \widehat{S}_t\right) + (1-n)\,\widehat{P}_{Ft}^*$$

(2.25)
$$\widehat{y}_t = \theta \left(\widehat{P}_t - \widehat{P}_{Ht} \right) + \widehat{c}_t^W,$$

(2.26)
$$\widehat{y}_t^* = \theta \left(\widehat{P}_t^* - \widehat{P}_{Ft}^* \right) + \widehat{c}_t^W,$$

(2.27)
$$n\hat{c}_t + (1-n)\,\hat{c}_t^* = \hat{c}_t^W \equiv n\hat{y}_t + (1-n)\,\hat{y}_t^* = \hat{y}_t^W,$$

(2.28)
$$\widehat{c}_{t+1} = \widehat{c}_t + \frac{\delta}{1+\delta}\widehat{r}_{t+1}.$$

(2.29)
$$\widehat{c}_{t+1}^* = \widehat{c}_t^* + \frac{\delta}{1+\delta}\widehat{r}_{t+1},$$

(2.30)
$$(\theta+1)\,\widehat{y}_t = -\theta\widehat{c}_t + \widehat{c}_t^W,$$

(2.31)
$$(\theta+1)\,\widehat{y}_t^* = -\theta\widehat{c}_t^* + \widehat{c}_t^W,$$

(2.32)
$$\widehat{M}_t - \widehat{P}_t = \widehat{c}_t - \frac{\widehat{r}_{t+1}}{1+\delta} - \frac{\widehat{P}_{t+1} - \widehat{P}_t}{\delta},$$

(2.33)
$$\widehat{M}_t^* - \widehat{P}_t^* = \widehat{c}_t^* - \frac{\widehat{r}_{t+1}}{1+\delta} - \frac{P_{t+1}^* - P_t^*}{\delta},$$

(2.23) and (2.24) express the domestic and foreign price levels as weighted averages of prices of *H*- and *F*-produced goods in common currencies;

(2.25) and (2.26) express the demand for each country's output as a function of world consumption and relative price;

(2.27) defines world consumption (and world output);

(2.28) and (2.29) are the Euler conditions (for optimal intertemporal allocation of consumption): the change in consumption equals the rate of real return;

(2.30) and (2.31) are implied by optimal production decisions (labour-leisure trade-offs);

(2.32) and (2.33) are microfounded real money demand functions; (NB!): while both countries face the same real interest rate, nominal interest rates may differ if expected inflation is different! Thus, we have 11 equations to solve for the equilibrium path of 11 endogenous variables:

- output $(\widehat{y}_t, \widehat{y}_t^*)$; consumption $(\widehat{c}_t, \widehat{c}_t^*, \widehat{c}_t^W)$;
- prices and the NER $(\widehat{P}_{Ht}, \widehat{P}_t, \widehat{P}_{Ft}, \widehat{P}_t^*, \widehat{S}_t);$ and the *real* interest rate $(\widehat{r}_{t+1}).$

Subtracting (2.23) from (2.24) implies PPP:

(2.34)
$$\widehat{S}_t = \widehat{P}_t - \widehat{P}_t^*.$$

Note as well that addition of n times (2.25) and (1 - n) times (2.26) yields the goods market clearing relationship – the RHS of (2.27) – equating world consumption to world production!

2.3. Flex-Price Equilibrium in the Linearised Model.

2.3.1. Monetary Policy and Welfare. To analyse the welfare implications of monetary policy, Obstfeld and Rogoff (1996) define it as one-time (permanent) unanticipated change in (the level of) nominal money supply.

The classical real-monetary dichotomy (as in the closed-economy case) is then evident from the structure of the flexible-price model we described up to here: with prices and the NER free to adjust immediately to changes in either Home or Foreign money supply, the equilibrium values of all real variables can be determined independently of money supply and money demand factors: (2.23) and (2.24) imply $n\left(\hat{P}_{Ht} - \hat{P}_t\right) + (1-n)\left(\hat{P}_{Ft}^* - \hat{P}_t^*\right) = 0$ and then this equation plus world demand, world consumption, optimal individual consumption and labour-leisure schedules, i.e., (2.25) through (2.31), suffice to determine real equilibrium, while money demand functions, (2.32) and (2.33), determine the price paths and the PPP equation, (2.34), the path of the nominal exchange rate!

2.3.2. Exchange-Rate Dynamics. Subtracting money demand functions, (2.33) from (2.32):

$$(2.35) \qquad \qquad \widehat{M}_t - \widehat{M}_t^* - \underbrace{\left(\widehat{P}_t - \widehat{P}_t^*\right)}_{=\widehat{S}_t} = (\widehat{c}_t - \widehat{c}_t^*) - \frac{1}{\delta} \underbrace{\left[\left(\widehat{P}_{t+1} - \widehat{P}_t\right) - \left(\widehat{P}_{t+1}^* - \widehat{P}_t^*\right)\right]}_{=\widehat{S}_{t+1} - \widehat{S}_t}, \\ \underbrace{= \left(\widehat{P}_{t+1} - \widehat{P}_{t+1}^*\right) - \left(\widehat{P}_t - \widehat{P}_t^*\right)}_{=\widehat{S}_{t+1} - \widehat{S}_t}, \\ \widehat{M}_t - \widehat{M}_t^* - \widehat{S}_t = (\widehat{c}_t - \widehat{c}_t^*) - \frac{1}{\delta} \left(\widehat{S}_{t+1} - \widehat{S}_t\right)$$

and solving forward for \widehat{S}_t (in the no-bubbles case):

(2.36)
$$\widehat{S}_t = \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^s \left[\left(\widehat{M}_{t+s} - \widehat{M}_{t+s}^*\right) - \left(\widehat{c}_{t+s} - \widehat{c}_{t+s}^*\right) \right]$$

(2.36) states that the current NER depends on the current and future path of nominal money supply and real consumption (not output, as in ad-hoc macromodels) differentials! Note as well that from the SS condition $\beta (1 + r) = 1$ the discount factor in (2.36) is also related to the SS real interest rate, $r = \delta$.

A key feature of the redux model relates to its consumption Euler equations, (2.28) and (2.29). Looking at them convinces that:

$$\widehat{c}_{t+1} - \widehat{c}_{t+1}^* = \widehat{c}_t - \widehat{c}_t^*.$$

By analogy,

$$\hat{c}_{t+s} - \hat{c}_{t+s}^* = \dots = \hat{c}_{t+2} - \hat{c}_{t+2}^* = \hat{c}_{t+1} - \hat{c}_{t+1}^* = \hat{c}_t - \hat{c}_t^*, \qquad s > t.$$

Hence, we can also write (2.36) as

$$\begin{split} \widehat{S}_{t} &= \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{s} \left[\left(\widehat{M}_{t+s} - \widehat{M}_{t+s}^{*}\right) - \underbrace{\left(\widehat{c}_{t} - \widehat{c}_{t}^{*}\right)}_{=\widehat{c}_{t+s} - \widehat{c}_{t+s}^{*}} \right], \\ \widehat{S}_{t} &= \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{s} \left(\widehat{M}_{t+s} - \widehat{M}_{t+s}^{*}\right) - \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{s} \left(\widehat{c}_{t} - \widehat{c}_{t}^{*}\right), \\ \widehat{S}_{t} &= \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{s} \left(\widehat{M}_{t+s} - \widehat{M}_{t+s}^{*}\right) - \frac{\delta}{1+\delta} \left[\left(\frac{1}{1+\delta}\right)^{0} + \left(\frac{1}{1+\delta}\right)^{1} \dots \right] \left(\widehat{c}_{t} - \widehat{c}_{t}^{*}\right), \\ \widehat{S}_{t} &= \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{s} \left(\widehat{M}_{t+s} - \widehat{M}_{t+s}^{*}\right) - \frac{\delta}{1+\delta} \left[\frac{1}{1-\frac{1}{1+\delta}} \right] \left(\widehat{c}_{t} - \widehat{c}_{t}^{*}\right), \end{split}$$

$$\widehat{S}_{t} = \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{s} \left(\widehat{M}_{t+s} - \widehat{M}_{t+s}^{*}\right) - \frac{\delta}{1+\delta} \left[\frac{1}{\frac{\delta}{1+\delta}}\right] \left(\widehat{c}_{t} - \widehat{c}_{t}^{*}\right),$$

$$\widehat{S}_{t} = \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{s} \left(\widehat{M}_{t+s} - \widehat{M}_{t+s}^{*}\right) - \left(\widehat{c}_{t} - \widehat{c}_{t}^{*}\right),$$

$$\widehat{S}_{t} = -\left(\widehat{c}_{t} - \widehat{c}_{t}^{*}\right) + \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^{s} \left(\widehat{M}_{t+s} - \widehat{M}_{t+s}^{*}\right).$$
(2.37)

The current NER thus depends in the flex-price redux model version on the current and the future path of the nominal money supplies in the two countries and on current consumption differentials. This equilibrium equation lend itself to an easy and natural interpretation, insofar the exchange rate measures the price of one currency in terms of another: we can see in the above equation, or in (2.36), that an increase in one country's money supply relative to the other's depreciates its currency. As Walsh (1998), p. 667, stresses, since agents are forward-looking in their decision making, only the present discount value (PDV) of relative money supplies matters for the equilibrium NER in (2.36); in other words, the SS NER only depends on "the permanent money supply differential", an analogy with the *permanent income hypothesis* (PIH) in closed-economy macroeconomics suggested by Friedman (1957) and tested/modified by Hall (1978), among others!

Another parallel can be made more visible by letting $\hat{f}_{t+s} \equiv \left(\widehat{M}_{t+s} - \widehat{M}_{t+s}^*\right) - \left(\widehat{c}_{t+s} - \widehat{c}_{t+s}^*\right)$ in (2.36):

(2.38)
$$\widehat{S}_t = \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^s \widehat{f}_{t+s}$$

which is a dynamic equation for the nominal exchange rate (under perfect foresight) reminiscent of the monetary model, with \hat{f}_{t+s} again interpreted as the "fundamentals", but now consisting of real *consumption* (not *output*) differentials and the same old money supply differentials. This difference arises from the explicit microfoundations of the equilibrium exchange rate (determination) equation (2.36).

To see another point related to predicting exchange-rate dynamics, put clearly in Walsh (1998), p. 250, let us re-write (2.38) as:

$$\begin{split} \widehat{S}_t &= \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^s \widehat{f}_{t+s} = \\ &= \frac{\delta}{1+\delta} \left[\left(\frac{1}{1+\delta}\right)^0 \widehat{f}_t + \left(\frac{1}{1+\delta}\right)^1 \widehat{f}_{t+1} + \left(\frac{1}{1+\delta}\right)^2 \widehat{f}_{t+2} \dots \right] = \\ &= \frac{\delta}{1+\delta} \left[\widehat{f}_t + \left(\frac{1}{1+\delta}\right)^1 \widehat{f}_{t+1} + \left(\frac{1}{1+\delta}\right)^2 \widehat{f}_{t+2} \dots \right] = \\ &= \frac{\delta}{1+\delta} \left\{ \widehat{f}_t + \left(\frac{1}{1+\delta}\right) \left[\widehat{f}_{t+1} + \left(\frac{1}{1+\delta}\right)^1 \widehat{f}_{t+2} \dots \right] \right\} = \\ &= \frac{\delta}{1+\delta} \left\{ \widehat{f}_t + \left(\frac{1}{1+\delta}\right) \underbrace{\left[\left(\frac{1}{1+\delta}\right)^0 \widehat{f}_{t+1} + \left(\frac{1}{1+\delta}\right)^1 \widehat{f}_{t+2} \dots \right] \right\} = \\ &= \frac{\delta}{1+\delta} \left\{ \widehat{f}_t + \left(\frac{1}{1+\delta}\right) \underbrace{\left[\left(\frac{1}{1+\delta}\right)^0 \widehat{f}_{t+1} + \left(\frac{1}{1+\delta}\right)^1 \widehat{f}_{t+2} \dots \right] \right\} = \\ &= \frac{\delta}{1+\delta} \left[\widehat{f}_t + \left(\frac{1}{1+\delta}\right) \underbrace{\sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^s \widehat{f}_{t+1+s}} \right] = \end{split}$$

$$= \frac{\delta}{1+\delta}\widehat{f}_t + \frac{\delta}{1+\delta}\left(\frac{1}{1+\delta}\right)\sum_{s=0}^{\infty}\left(\frac{1}{1+\delta}\right)^s\widehat{f}_{t+1+s} = \\ = \frac{\delta}{1+\delta}\widehat{f}_t + \left(\frac{1}{1+\delta}\right)\underbrace{\frac{\delta}{1+\delta}\sum_{s=0}^{\infty}\left(\frac{1}{1+\delta}\right)^s f_{t+1+s}}_{=\widehat{S}_{t+1}},$$

so that

$$\widehat{S}_t = \frac{\delta}{1+\delta}\widehat{f}_t + \frac{1}{1+\delta}\widehat{S}_{t+1}.$$

Rearranging

$$(1+\delta)\,\widehat{S}_t = \delta\,\widehat{f}_t + \widehat{S}_{t+1}$$
$$\widehat{S}_t + \delta\,\widehat{S}_t = \delta\,\widehat{f}_t + \widehat{S}_{t+1}$$
$$\delta\,\widehat{S}_t - \delta\,\widehat{f}_t = \widehat{S}_{t+1} - \widehat{S}_t$$
$$\widehat{S}_{t+1} - \widehat{S}_t = -\delta\,\left(\widehat{f}_t - \widehat{S}_t\right)$$

and using, to substitute above, the definition of the fundamental, \hat{f}_t , and the equilibrium flex-price NER, equation (2.36), yields

$$\begin{split} \widehat{S}_{t+1} - \widehat{S}_t &= -\delta \left\{ \left(\widehat{M}_t - \widehat{M}_t^* \right) - \left(\widehat{c}_t - \widehat{c}_t^* \right) - \right. \\ &\left. - \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta} \right)^s \left[\left(\widehat{M}_{t+s} - \widehat{M}_{t+s}^* \right) - \underbrace{\left(\widehat{c}_{t+s} - \widehat{c}_{t+s}^* \right)}_{=\widehat{c}_t - \widehat{c}_t^*} \right] \right\} = \\ &= -\delta \left\{ \left(\widehat{M}_t - \widehat{M}_t^* \right) - \frac{\delta}{1+\delta} \left[\begin{array}{c} \left(\frac{1}{1+\delta} \right)^1 \left(\widehat{M}_{t+1} - \widehat{M}_{t+1}^* \right) + \left(\frac{1}{1+\delta} \right)^2 \left(\widehat{M}_{t+2} - \widehat{M}_{t+2}^* \right) + \dots \right] \right\} = \\ &= -\delta \left\{ \left(\widehat{M}_t - \widehat{M}_t^* \right) - \frac{\delta}{1+\delta} \left[\begin{array}{c} \left(\widehat{M}_{t+1} - \widehat{M}_{t+1}^* \right) + \left(\frac{1}{1+\delta} \right)^2 \left(\widehat{M}_{t+2} - \widehat{M}_{t+2}^* \right) + \dots \right] \right\} = \\ &= -\delta \left\{ \left(\widehat{M}_t - \widehat{M}_t^* \right) - \frac{\delta}{1+\delta} \left[\begin{array}{c} \left(\widehat{M}_t - \widehat{M}_t^* \right) + \left(\frac{1}{1+\delta} \right)^2 \left(\widehat{M}_{t+2} - \widehat{M}_{t+2}^* \right) + \dots \right] \right\} = \\ &= -\delta \left\{ \left(\underbrace{\widehat{M}_t - \widehat{M}_t^*} \right) - \frac{\delta}{1+\delta} \left(\widehat{M}_t - \widehat{M}_t^* \right) + \left(\frac{1}{1+\delta} \right)^1 \left(\widehat{M}_{t+2} - \widehat{M}_{t+2}^* \right) + \dots \right] \right\} = \\ &= -\delta \left\{ \left(\frac{1}{1+\delta} \right)^0 \left(\widehat{M}_{t+1} - \widehat{M}_{t+1}^* \right) + \left(\frac{1}{1+\delta} \right)^1 \left(\widehat{M}_{t+2} - \widehat{M}_{t+2}^* \right) + \dots \right] \right\} = \\ &= -\delta \left\{ \frac{1+\delta-\delta}{1+\delta} \left(\widehat{M}_t - \widehat{M}_t^* \right) - \frac{\delta}{1+\delta} \frac{1}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta} \right)^s \left(\widehat{M}_{t+1+s} - \widehat{M}_{t+1+s}^* \right) \right\} = \\ &= -\delta \left[\frac{1}{1+\delta} \left(\widehat{M}_t - \widehat{M}_t^* \right) - \frac{1}{1+\delta} \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta} \right)^s \left(\widehat{M}_{t+1+s} - \widehat{M}_{t+1+s}^* \right) \right], \end{aligned}$$
 so finally we get

so finally we get

$$(2.39) \qquad \widehat{S}_{t+1} - \widehat{S}_t = -\frac{\delta}{1+\delta} \left[\left(\widehat{M}_t - \widehat{M}_t^* \right) - \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta} \right)^s \left(\widehat{M}_{t+1+s} - \widehat{M}_{t+1+s}^* \right) \right].$$

(NB!): (2.39) shows clearly that if the *current* value of the money-supply differential, $\widehat{M}_t - \widehat{M}_t^*$, is *high relative to* the *permanent* money-supply differential, the second term in the square brackets $\frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^s \left(\widehat{M}_{t+1+s} - \widehat{M}_{t+1+s}^*\right)$, $\widehat{S}_{t+1} - \widehat{S}_t$ will be < 0, or – equivalently, $\widehat{S}_{t+1} < \widehat{S}_t$ so the NER \widehat{S}_t will \downarrow (i.e., the domestic currency will appreciate)!

An explicit solution for the nominal exchange rate in the flex-price case can be obtained if specific processes for the nominal money supplies are assumed. Among the simplest cases is to look at constant, deterministic growth paths in both countries, often given by equations like:

$$\widehat{M}_t = \widehat{M}_0 + \mu t$$
 and $\widehat{M}_t^* = \widehat{M}_0^* + \mu^* t$.

As Walsh (1998), p. 250, emphasises, strictly speaking (2.36) applies only to deviations around the steady state and not to money supply processes that include deterministic trends, like the two defined just above. However, it is very common to specify the money demand functions we used to derive (2.36) in terms of the log levels of the variables, perhaps adding a constant to represent SS levels. The advantage of interpreting (2.36) as holding for the log levels of the variables is that one can use it then to analyse shifts in the trend growth paths of the nominal money supplies, rather than just deviations around the trend. In doing so, it is important to keep in mind the limitations of such analysis. A conclusion from an example along these lines, in Walsh (1998) textbook, p. 251, is the well-known one from other models, microfounded as well as ad-hoc: if domestic money growth exceeds the foreign one ($\mu > \mu^*$), the NER will rise over time to reflect the falling value of the home currency relative to the foreign currency!

2.4. Sticky-Price Equilibrium in the Linearised Model.

2.4.1. *Rationale for and Specification of Price Rigidities.* As in the case of closed economies, flexprice models of the open economy appear unable to replicate the size and persistence of monetary shocks on real variables, and just as with closed-economy models, this can be remedied by the introduction of nominal rigidities!

The redux model assumes a simple (and symmetric for the two countries) pricing rule: domestic-currency prices of domestically-produced goods are set one period in advance and stay fixed for just one period; thereafter they adjust completely and both economies return to their SS; but during the one period in which prices are set, real output and consumption levels will be affected. Thus the presence of nominal rigidities leads to real effects of monetary disturbances (through the channels known from closed-economy models, but now also through a new channel)!

2.4.2. The New Channel of Monetary Transmission. Although domestic output price indices P_{Ht} and P_{Ft}^* are preset, the aggregate price level indices in each country P_t and P_t^* , as clear from (2.6) and (2.7), now fluctuate with the NER: e.g., a nominal depreciation raises the domestic general price index: (NB!) no distinction was made in closed-economy models between these two types of price indices (GDP deflator vs CPI), hence, this new channel of shock transmission in an open economy: NER movements alter the domestic currency price of foreign goods, allowing CPIs to move in response to monetary disturbances, even in the presence of nominal rigidities (whereas with nominal stickiness, the price level could not adjust immediately in a closed economy)!

2.4.3. Specification of a Monetary Shock. In period t the Home money supply rises unexpectedly relative to Foreign money supply. To smooth consumption, H agents may now lend, and Home can thus run a CA surplus: such a result alters the NFA position of the two economies and can affect the new SS equilibrium!

2.4.4. Monetary Policy, Exchange-Rate Dynamics and Welfare. Using the linearised model version, it now follows from the Euler equations that

$$\widehat{c}_{t+1} - \widehat{c}_{t+1}^* = \widehat{c}_t - \widehat{c}_t^*,$$

and since the economies are in the new SS after one period, i.e., at t + 1,

$$\hat{c}_{t+1} - \hat{c}_{t+1}^* \equiv \Theta = const$$

is the SS consumption differential between the two countries; but since we also have that

$$\widehat{c}_t - \widehat{c}_t^* = \widehat{c}_{t+1} - \widehat{c}_{t+1}^* = \Theta,$$

this relationship implies that relative consumption in the two economies *immediately jumps* in period t to the *new* SS value!

Relative money demand (2.35) can now be written as

$$\widehat{M}_t - \widehat{M}_t^* - \widehat{S}_t = -\Theta - \frac{1}{\delta} \left(\widehat{S}_{t+1} - \widehat{S}_t \right)$$

and solving forward for the NER (again, in the no-bubbles case) yields:

$$\widehat{S}_t = -\Theta + \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^s \left(\widehat{M}_{t+s} - \widehat{M}_{t+s}^*\right).$$

If one assumes, as before, that the change in $\widehat{M}_t - \widehat{M}_t^*$ is a *permanent one-time change*, then

$$\widehat{M}_{t+s} - \widehat{M}_{t+s}^* \equiv \Omega = const$$

so the NER becomes:

$$\begin{split} \widehat{S}_t &= -\Theta + \frac{\delta}{1+\delta} \sum_{s=0}^{\infty} \left(\frac{1}{1+\delta}\right)^s \Omega = \\ &= -\Theta + \frac{\delta}{1+\delta} \left[\left(\frac{1}{1+\delta}\right)^0 \Omega + \left(\frac{1}{1+\delta}\right)^1 \Omega + \left(\frac{1}{1+\delta}\right)^2 \Omega + \ldots \right] = \\ &= -\Theta + \frac{\delta}{1+\delta} \left[1 + \left(\frac{1}{1+\delta}\right)^1 + \left(\frac{1}{1+\delta}\right)^2 + \ldots \right] \Omega = \\ &= -\Theta + \frac{\delta}{1+\delta} \left[\frac{1}{1-\frac{1}{1+\delta}} \right] \Omega = \\ &= -\Theta + \frac{\delta}{1+\delta} \left[\frac{1}{\frac{1+\delta-1}{1+\delta}} \right] \Omega = \\ &= -\Theta + \frac{\delta}{1+\delta} \left[\frac{1}{\frac{1}{1+\delta}} \right] \Omega = \\ &= -\Theta + \frac{\delta}{1+\delta} \left[\frac{1}{\frac{\delta}{1+\delta}} \right] \Omega = \\ &= -\Theta + \frac{\delta}{1+\delta} \left[\frac{1+\delta}{\delta} \right] \Omega = \\ &= -\Theta + \frac{\delta}{1+\delta} \left[\frac{1+\delta}{\delta} \right] \Omega = \\ &= -\Theta + \Omega. \end{split}$$

Thus

(2.40)
$$\widehat{S}_t = \Omega - \Theta = const.$$

Since $\Omega - \Theta$ is a constant, (2.40) implies that NER *jumps immediately* to its *new* SS (following a *permanent* change in the *relative* money supplies! For this reason, we do *not* observe exchange rate overshooting, as in the Dornbusch (1976) model we studied in lecture 6.

If relative consumption levels did not adjust (i.e., if $\Theta = 0$), then the permanent change in \widehat{S}_t is just equal to the relative change in nominal money supplies, Ω , and $\widehat{M}_t \uparrow$ relative to \widehat{M}_t^* (i.e., $\Omega > 0$) produces a home currency depreciation!

If $\Theta \neq 0$, then changes in relative consumption affect relative money demand, from (2.32) and (2.33): e.g., if $\Theta > 0$, consumption as well as money demand in *Home* is higher than initially, and then equilibrium between *Home* money supply and money demand can be restored with a smaller increase in the *Home* price level: since P_H and P_F^* are (pre)fixed, the increase in Pnecessary to maintain real money demand and real money supply equilibrium is generated by a depreciation $(S \uparrow)$; the larger the rise in home consumption, the larger is the rise in money demand, and the smaller is the necessary rise in the NER!

With *endogenous* real consumption differential Θ , several steps are needed to determine its value. We shall not have space and time to explain them in this lecture. We would just summarise the main result in this version of the Obstfeld-Rogoff redux model. It is that (NB!) domestic monetary *expansion* leads to a domestic currency *depreciation* that is *less* than proportional to the increase in the money supply. This induces expansion in both domestic production and consumption. As consumption rises by less than income, the home country runs a CA surplus (lends) and accumulates NFA (claims against future income of the foreign country). This allows the expansionary country to maintain higher consumption forever!!!

2.5. Concluding Comments.

2.5.1. Theoretical Import of the Obstfeld-Rogoff Redux Model.

- introducing microfoundations and nominal rigidities within a traditional open-economy framework;
- hence, the responses of consumption, output (therefore, the current account), interest rates and the exchange rate to a monetary shock are consistent with optimising behaviour;
- thus, allowing for explicit welfare analysis of alternative monetary and exchange-rate policies.

2.5.2. Main Result and Transmission Mechanism.

- an *unanticipated* monetary disturbance can have a *permanent* impact on real consumption levels and, hence, welfare (explicitly derived from the utility metric) when prices are preset: domestic monetary expansion increases welfare!
- \widehat{M} \uparrow (money surprise) $\Rightarrow \widehat{S}$ \uparrow (depreciation) and \widehat{P} \uparrow (inflation) $\Rightarrow \widehat{y}$ \uparrow and \widehat{c} \uparrow (but less than \widehat{y} , because \widehat{c} is determined on the basis of permanent income) \Rightarrow CA surplus (the excess of output over consumption is exported) $\iff b \equiv \Delta NFA > 0$ (lending abroad \equiv accumulating foreign bonds, i.e., claims on future foreign output, as payment for Home exports) \Rightarrow welfare (consumption in the utility) rises permanently, even though the increase in output lasts only one period, as home permanent income has risen by the annuity value of its claim on future foreign output! \Rightarrow an incentive for monetary expansion! \Rightarrow "beggar-thy-neighbour" policy!
- 2.5.3. Implications.
 - policy coordination: a joint proportionate expansion leaves $\widehat{M} \widehat{M}^*$, the NER and, thus, $\widehat{c} \widehat{c}^*$ unchanged, but since output is inefficiently low with monopolistic competition, both countries have incentive to expand (\Rightarrow steady inflation \Rightarrow no welfare gains)!
 - small open-economy case (n is very low) \Rightarrow foreign variables are now exogenous \Rightarrow flexible vs fixed NER matters: the choice of exchange rate regime influences the way in which disturbances affect the small economy!

3. New Open-Economy Macroeconomics

[... to be summarised in class, based on Lane (2001), Mark (2001), Sarno and Taylor (2002), Canzoneri, Cumby and Diba (2002) and Vanhoose (2004) ...]

3.1. NOEM Models of Exchange Rate Dynamics. [... to be summarised in class ...]

3.1.1. Betts-Devereux Redux Extensions: Pricing to Market. [... Betts and Devereux (1996, 2000) to be summarised in class (if time allows) ...]

3.1.2. Corsetti-Pesenti Redux Extensions: Low (Unit) Cross-Country Substitutability. [... Corsetti and Pesenti (1997, 2001 a, b and 2002) to be summarised in class (if time allows) ...]

3.1.3. Devereux-Engel Redux Extensions: Exchange-Rate Regimes. [... Devereux and Engel (1998, 1999, 2000), Devereux (2000) and Engel (2000) to be summarised in class (if time allows) ...]

3.2. Explicitly Stochastic NOEM Models. [... to be summarised in class ...]

3.2.1. Obstfeld-Rogoff Directions for NOEM Research: Risk and Space. [... Obstfeld and Rogoff (1998, 2000 – risk and 2001 – space (transport and/or trade costs) to be summarised in class (if time allows) ...]

3.2.2. Early Stochastic NOEM Contributions: Bacchetta and van Wincoop. [... Bacchetta and van Wincoop (1998, 2000) to be summarised in class (if time allows) ...]

3.2.3. A Single-Period NOEM Model with Trade Costs and Inelastic Imports: Mihailov. [... Mihailov (2003 a, b) to be summarised in class (if time allows) ...]

3.2.4. A Multi-Period NOEM Model with Asset Structure and Intermediate Goods: Singh. [... Singh (2004) to be summarised in class (if time allows) ...]

References

- Bacchetta, Philippe and Eric van Wincoop (1998), "Does Exchange Rate Stability Increase Trade and Capital Flows?", National Bureau of Economic Research Working Paper No. 6704 (August).
- [2] Bacchetta, Philippe and Eric van Wincoop (2000), "Does Exchange Rate Stability Increase Trade and Welfare?", American Economic Review 90 (5, December), 1093-1109.
- Betts, Caroline and Michael B. Devereux (1996), "The Exchange Rate in a Model of Pricing to Market", European Economic Review 40 (3-5, April), 1007-1021.
- Betts, Caroline and Michael B. Devereux (2000), "Exchange Rate Dynamics in a Model of Pricing to Market", *Journal of International Economics* 50 (1, February), 215-244.
- [5] Canzoneri, Matthew, Robert Cumby and Behzad Diba (2002), "The Need for International Policy Coordination: What's Old, What's New, What's Yet to Come?", National Bureau of Economic Research Working Paper No. 8765 (February).
- [6] Corsetti, Giancarlo and Paolo Pesenti (1997), "Welfare and Macroeconomic Independence", National Bureau of Economic Research Working Paper No. 6307 (October).
- [7] Corsetti, Giancarlo and Paolo Pesenti (2001 a), "Welfare and Macroeconomic Independence", Quarterly Journal of Economics 116 (2, May), 421-445.
- [8] Corsetti, Giancarlo and Paolo Pesenti (2001 b), "International Dimensions of Optimal Monetary Policy", *National Bureau of Economic Research* Working Paper No. 8230 (April).
- [9] Corsetti, Giancarlo and Paolo Pesenti (2002), "International Dimensions of Optimal Monetary Policy", Centre for Economic Policy Research Discussion Paper No. 3349 (April).
- [10] Devereux, Michael B. (2000), "Monetary Policy Rules and Exchange Rate Flexibility in a Simple Dynamic General Equilibrium Model", University of British Columbia, mimeo (revised version of October).
- [11] Devereux, Michael B. and Charles Engel (1998), "Fixed versus Floating Exchange Rates: How Price Setting Affects the Optimal Choice of Exchange-Rate Regime", National Bureau of Economic Research Working Paper No. 6867 (December).
- [12] Devereux, Michael B. and Charles Engel (1999), "The Optimal Choice of Exchange-Rate Regime: Price Setting Rules and Internationalized Production", *National Bureau of Economic Research* Working Paper No. 6992 (March).
- [13] Devereux, Michael B. and Charles Engel (2000), "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility", National Bureau of Economic Research Working Paper No. 7665 (April).
- [14] Dixit, Avinash and Joseph Stiglitz (1977), "Monopolistic Competition and Optimum Product Diversity", American Economic Review 67 (3, June), 297-308.
- [15] Dornbusch, Rudiger (1976), "Expectations and Exchange Rate Dynamics", Journal of Political Economy 84 (6, December), 1161-1176.
- [16] Engel, Charles (2000), "Optimal Exchange Rate Policy: The Influence of Price Setting and Asset Markets", National Bureau of Economic Research Working Paper No. 7889 (September).
- [17] Friedman, Milton (1957), A Theory of the Consumption Function, Princeton University Press: Princeton, New Jersey.
- [18] Goodfriend, Marvin and Robert King (1997), "The New Neoclassical Synthesis and the Role of Monetary Policy", National Bureau of Economic Research Macroeconomics Annual 1997, 231-295.
- [19] Hall, Robert (1979), "Stochastic Implications of the Life Cycle Permanent Income Hypothesis: Theory and Evidence", Journal of Political Economy 86 (6, October), 971-987.
- [20] Lane, Philip (2001), "The New Open Economy Macroeconomics: A Survey", Journal of International Economics 54 (2, August), 235-266.
- [21] Lucas, Jr., Robert (1982), "Interest Rates and Currency Prices in a Two-Country World", Journal of Monetary Economics 10 (3, November), 335-359.
- [22] Mark, Nelson (2001), International Macroeconomics and Finance: Theory and Econometric Methods, Blackwell.
- [23] Mihailov, Alexander (2003 a), "Effects of the Exchange-Rate Regime on Trade under Monetary Uncertainty: The Role of Price Setting", *Essex Economics Discussion Paper* No. 566 (October), Department of Economics, University of Essex.
- [24] Mihailov, Alexander (2003 b), "When and How Much Does a Peg Increase Trade? The Role of Trade Costs and Import Demand Elasticity under Monetary Uncertainty", Essex Economics Discussion Paper No. 567 (October), Department of Economics, University of Essex.
- [25] Mundell, Robert A. (1960), "The Monetary Dynamics of International Adjustment under Fixed and Flexible Exchange Rates", Quarterly Journal of Economics 74, 227-257.
- [26] Mundell, Robert A. (1961 a), "The International Disequilibrium System", Kyklos 14, 152-170.
- [27] Mundell, Robert A. (1961 b), "Flexible Exchange Rates and Employment Policy", Canadian Journal of Economics and Political Science 27, 509-517.
- [28] Mundell, Robert A. (1963), "Capital Mobility and Stabilisation Policy under Fixed and Flexible Exchange Rates", Canadian Journal of Economics and Political Science 29 (November), 475-485.
- [29] Mundell, Robert A. (1964), "A Reply: Capital Mobility and Size", Canadian Journal of Economics and Political Science 30 (August), 421-431.
- [30] Obstfeld, Maurice and Kenneth Rogoff (1994), "Exchange Rate Dynamics Redux", National Bureau of Economic Research Working Paper No. 4693 (March).

- [31] Obstfeld, Maurice and Kenneth Rogoff, "Exchange Rate Dynamics Redux", Journal of Political Economy 103, 1995 (June), 624-660.
- [32] Obstfeld, Maurice and Kenneth Rogoff (1996), Foundations of International Macroeconomics, MIT Press.
- [33] Obstfeld, Maurice and Kenneth Rogoff (1998), "Risk and Exchange Rates", National Bureau of Economic Research Working Paper No. 6694 (August); a slightly revised version was circulated as mimeo in February 2001.
- [34] Obstfeld, Maurice and Kenneth Rogoff (2000), "New Directions for Stochastic Open Economy Models", Journal of International Economics 50 (1, February), 117-153.
- [35] Obstfeld, Maurice, and Kenneth Rogoff (2001), "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?", National Bureau of Economic Research Macroeconomics Annual 2000, Volume 15, 341-390.
- [36] Sarno, Lucio and Mark Taylor (2002), The Economics of Exchange Rates, Cambridge University Press, 2002.
- [37] Singh, Rajesh (2004), "Trade and Welfare Under Alternative Exchange Rate Regimes", *Iowa State University* Working Paper No. 04008 (February).
- [38] Vanhoose, David (2004), "The New-Open Economy Macroeconomics: A Critical Appraisal", Open Economies Review 15, 193-215.
- [39] Walsh, Carl (1998), Monetary Theory and Policy, MIT Press.

DEPARTMENT OF ECONOMICS, UNIVERSITY OF ESSEX, WIVENHOE PARK, COLCHESTER CO4 3SQ, UK *E-mail address*: mihailov@essex.ac.uk

 $\mathit{URL:}\ \texttt{http://www.essex.ac.uk/economics/people/staff/mihailov.shtm}$