EC933-G-AU INTERNATIONAL FINANCE – LECTURE 7

MICROFOUNDED (OPTIMISING) MODELS OF EXCHANGE RATES UNDER FLEXIBLE PRICES

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ABSTRACT. We analyse in this lecture the Lucas (1982) dynamic stochastic general equilibrium model (DSGEM), an optimising model of exchange rates under *flexible* prices and *complete* asset markets. It is an excellent example of the methodological steps involved in constructing a coherent microfounded model with progressively increasing complexity of versions. The Lucas (1982) model captures in a condensed but clear way nearly all important aspects of our course thus far, as well as almost all essential features an open-economy analytical framework needs to incorporate. That is why it is of considerable significance in demonstrating how a consistent macroeconomic analysis could be performed. Before focusing on the Lucas (1982) DSGEM, we first briefly motivate the optimising approach from the perspective of the Lucas (1976) critique. After considering in detail the Lucas (1982) model, we also briefly note some further developments in the related literature, in part extending the Lucas (1982) approach to more realistic settings which could be calibrated and simulated so that comparisons with relevant real-world economy characteristics are possible. We view in this light the real business cycle (RBC) literature of the 1980s and the international (real) business cycle (I(R)BC) literature of the 1990s.

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This set of lecture notes is preliminary and incomplete. It is based on parts of the four textbooks suggested as essential and supplementary reading for my graduate course in international finance at Essex as well as on the related literature (see the course outline and reading list at http://courses/essex.ac.uk/ec/ec933/). The notes are intended to be of some help to the students attending the course and, in this sense, many aspects of them will be clarified during lectures. The present second draft may be developed and completed in future revisions. The responsibility for any errors and misinterpretations is, of course, only mine. Comments are welcome, preferably by e-mail at mihailov@essex.ac.uk and/or a_mihailov@hotmail.com.

Contents

1. Lucas (1976) Critique

[... Lucas (1976) paper to be summarised in class ...]

2. Lucas (1982) Dynamic Stochastic General Equilibrium Model

The Lucas (1982) paper is a theoretical study of the determination of prices, interest rates and exchange rates in an infinitely-lived two-country world which is subject both to stochastic endowment shocks and to monetary instability. In the words of Lucas (1982) himself, the paper is highly ambitious in some respects and very modest in others. It is ambitious in integrating monetary theory with the apparatus of financial economics, and particularly in "replicating all of the classical results of monetary theory as well as the main formulas for securities pricing that the theory of finance produces" by also "suggesting modifications to the latter theory suited to an unstable monetary environment". It is modest because "many, perhaps most, of the central substantive questions of monetary economics are left unanswered".

The paper consists of *three* models, building upon one another, with each introducing some additional feature into the considered environment. The main concern is with "alternative monetary arrangements". Yet it is convenient to begin with an analysis of a *barter* equilibrium, extended later to accommodate money and exchange rates. Thus, the model Lucas (1982) starts with is a two-country real model of a barter world economy, essentially a variation on Lucas (1978). The second model introduces a *single* world currency, motivated by a cash-in-advance (CiA) constraint. When the money supply is constant, the real aspects of the equilibrium in the monetary model replicate those of the barter economy; when the money supply is stochastic, the formulas for securities prices require modification. The third model begins by introducing *two* national currencies together with a free market or *flexible* exchange rate system under which currencies may be traded prior to shopping for goods. It next looks at a *fixed* exchange rate regime version. The *normative* conclusion from *comparing* the alternative exchange rate regimes reproduces the equivalence result of Helpman (1981): a key finding in the Lucas (1982) DSGEM is that with perfectly *flexible prices* the *nominal* exchange-rate regime does *not* matter, even under uncertainty, for optimal *real* allocations.

2.1. **Barter Two-Country Economy.**

2.1.1. *Assumptions.*

- complete information;
- rational expectations;
- no market imperfections;
- no nominal rigidities;
- two countries, which we – for coherence in this course – denote[1] $H(ome)$ and $F(oreign)$, populated by a large number of individuals with
  - identical utility functions, $u(\cdot)$;
  - identical (real) wealth, $\mathcal{W}_t \equiv \mathcal{W}_t^*$;
  - and constant population normalised to 1;

⇔ *representative* agent (household): the simplest way to "aggregate";

- "firms" are "fruit trees": pure endowment streams that generate a (i) homogeneous, (ii) nonstorable, (iii) country-specific good, using (iv) no labour or (v) capital inputs, and (vi) the number of firms in each country is also normalised to 1;
  - let i_t and i_t^* denote[2] the exogenous H and F "output" in t;
  - the evolution of output is given by the *net*[3] growth rate g_t and g_t^*: $i_t \equiv (1 + g_t)\, i_{t-1}$ and $i_t^* \equiv (1 + g_t^*)\, i_{t-1}^*$, where g_t and g_t^* are random variables which evolve according to a stochastic process[4] that is *known* to agents;

[1] The corresponding notation is country 1 and country 2 in Lucas (1982) original paper and "US" and "EU" in Mark's (2001) textbook interpretation.

[2] x_t and y_t in the original notation by Lucas, reproduced in Mark's textbook.

[3] Written in *gross* terms in Lucas (1982) and in Mark (2001).

[4] Characterised, in principle, by a *probability density* function (pdf) or the corresponding *cumulative distribution* function (CDF).

- each "firm" issues one *perfectly divisible* share of common stock which is traded at a competitive stock market; "firms" then pay out all their revenue from the sales of output as dividends to shareholders;
- in the *barter* economy, dividends form the *sole* source of income for individuals;
- let i_t be the numéraire good and q_t be the (relative) price of i_t^* in terms of i_t;
- e_t and e_t^* are the ex-dividend market values of the H and F firm, respectively;
- the H household consumes c_{it} units of the H-good and c_{i^*t} units of the F-good; it also holds ω_{it} shares of the H-firm and ω_{i^*t} shares of the F-firm; symmetrically, the F household consumes c_{it}^* units of the H-good and $c_{i^*t}^*$ units of the F-good and holds ω_{it}^* shares of the H-firm and $\omega_{i^*t}^*$ shares of the F-firm.

2.1.2. *Constraints.* The H agent *brings* into period t *wealth* ($\equiv$ dividends from the revenue from sales of H and F output + the *ex*-dividend value of shares held in the H and F firm, in this version of the model) valued (in terms of the numéraire good i) at

$$\mathcal{W}_t = \omega_{it-1} \underbrace{(i_t + e_t)}_{with\text{-dividend value of } H \text{ firm}} + \omega_{i^*t-1} \underbrace{(q_t i_t^* + e_t^*)}_{with\text{-dividend value of } F \text{ firm}} . \tag{2.1}$$

The H individual then *allocates* current *wealth*

$$\mathcal{W}_t \geq \underbrace{\omega_{it} e_t + \omega_{i^*t} e_t^*}_{\text{insurance: purchases of new shares}} + \underbrace{c_{it} + q_t c_{i^*t}}_{\text{consumption}}. \tag{2.2}$$

Equating (2.1) to (2.2) gives[5] the H *consolidated* budget constraint

$$\omega_{it-1}(i_t + e_t) + \omega_{i^*t-1}(q_t i_t^* + e_t^*) = \omega_{it} e_t + \omega_{i^*t} e_t^* + c_{it} + q_t c_{i^*t}. \tag{2.3}$$

2.1.3. *Objective and First-Order Conditions.* Let $u(c_{it}, c_{i^*t})$ and $u(c_{it}^*, c_{i^*t}^*)$ denote the (current-)*period* utility of the representative household in H and F, respectively, and $0 < \beta < 1$ be the subjective discount factor, *equal* across countries (and individuals). The H agent problem then is to choose sequences of consumption and stock purchases, $\{c_{it+k}, c_{i^*t+k}, \omega_{it+k}, \omega_{i^*t+k}\}_{k=0}^{\infty}$, to maximise the expected lifetime utility

$$\underbrace{E_t \underbrace{\left[\sum_{k=0}^{\infty} \beta^k \underbrace{u(c_{it+k}, c_{i^*t+k})}_{H period\ t+k \text{ utility}} \right]}_{H lifetime \text{ utility}}}_{H expected \text{ lifetime utility}} . \tag{2.4}$$

subject to (2.3). As we know (from lecture 5), one way to proceed (described in Mark's textbook) is to transform the constrained optimisation problem into an unconstrained one,[6] by expressing c_{it+k} from (2.3) in terms of the other variables in the consolidated budget constraint and then substituting it in the utility expression (2.4). Once this is done, the objective function becomes

$$u \left[\underbrace{\omega_{it-1}(i_t + e_t) + \omega_{i^*t-1}(q_t i_t^* + e_t^*) - \omega_{it} e_t - \omega_{i^*t} e_t^* - q_t c_{i^*t}}_{c_{it}}, c_{i^*t} \right] +$$

$$+ E_t \beta u \left[\underbrace{\omega_{it}(i_{t+1} + e_{t+1}) + \omega_{i^*t}\left(q_{t+1} i_{t+1}^* + e_{t+1}^*\right) - \omega_{it+1} e_{t+1} - \omega_{i^*t+1} e_{t+1}^* - q_{t+1} c_{i^*t+1}}_{c_{it+1}}, c_{i^*t+1} \right] +$$

[5]It is optimal for (2.2) to hold with equality.

[6]Another is, as we have also done (in lecture 4), to form the Lagrangian, see Lucas (1982) original paper.

(2.5)

$$+E_t\beta^2 u\left[\underbrace{\omega_{it+1}\left(i_{t+2}+e_{t+2}\right)+\omega_{i^*t+1}\left(q_{t+2}i^*_{t+2}+e^*_{t+2}\right)-\omega_{it+2}e_{t+2}-\omega_{i^*t+2}e^*_{t+2}-q_{t+2}c_{i^*t+2}}_{c_{it+2}},c_{i^*t+2}\right]+...$$

Now imposing the first-order (necessary) conditions (FO(N)Cs) for a (local) optimum – i.e., differentiating with respect to the choice /control, decision/ variables c_{i^*t}, ω_{it} and ω_{i^*t} and setting the results to zero – and rearranging yields the Euler equations:

$$c_{i^*t}: \qquad q_t u_1\left(c_{it},c_{i^*t}\right)=u_2\left(c_{it},c_{i^*t}\right)\Leftrightarrow\frac{u_2\left(c_{it},c_{i^*t}\right)}{u_1\left(c_{it},c_{i^*t}\right)}=q_t \tag{2.6}$$

$$\omega_{it}: \qquad e_t u_1\left(c_{it},c_{i^*t}\right)=\beta E_t\left[u_1\left(c_{it+1},c_{i^*t+1}\right)\left(i_{t+1}+e_{t+1}\right)\right] \tag{2.7}$$

$$\omega_{i^*t}: \qquad e^*_t u_1\left(c_{it},c_{i^*t}\right)=\beta E_t\left[u_1\left(c_{it+1},c_{i^*t+1}\right)\left(q_{t+1}i^*_{t+1}+e^*_{t+1}\right)\right] \tag{2.8}$$

As made clear earlier in this course (i.e., in lectures 4 and 5), these Euler equations are also called *optimality* or *efficiency* conditions: they must hold if the agent is behaving optimally /efficiently/. (2.6) is the standard intRAtemporal optimality condition (for the H agent) which equates the relative price between goods i and i^* to their marginal rate of substitution. More precisely, it states that reallocating consumption by adding a unit of c_{i^*} increases utility by $u_2(\cdot)$; but this is "financed" by giving up q_t units of c_i, each unit of which costs $u_1(\cdot)$ units of utility for a total utility cost of $q_t u_1(\cdot)$. If the individual is behaving optimally, no such reallocations of the consumption plan yield a net gain in utility. (2.7) is the intERtemporal Euler equation for purchases of domestic equity (for the H agent). Its LHS is the utility cost of the marginal purchase of domestic equity: to buy incremental shares of the domestic firm costs the individual e_t units of c_{it}, each unit of which lowers utility by $u_1(\cdot)$. The RHS of (2.7) is the utility expected to be derived from the payoff of the marginal investment. If the individual is behaving optimally, no such reallocations between consumption and saving can yield a net increase in utility. (2.8) has an analogous interpretation to (2.7), but w.r.t. purchases of foreign equity (by the H representative household).

Symmetric Euler equations hold, of course, for the F representative agent.

2.1.4. *Market Clearing.* A set of four *adding up /market clearing/* constraints /assumptions/ on outstanding equity shares and the exhaustion of output in H and F consumption complete the specification of Lucas (1982) barter model:

$$\omega_{it}+\omega^*_{it}=1; \tag{2.9}$$

$$\omega_{i^*t}+\omega^*_{i^*t}=1; \tag{2.10}$$

$$c_{it}+c^*_{it}=i_t; \tag{2.11}$$

$$c_{i^*t}+c^*_{i^*t}=i^*_t. \tag{2.12}$$

2.1.5. *Arrow-Debreu Planner's Problem: Centralised Social Optimum.* The assumptions of *complete* markets and the *competitive* setting make possible to solve the model as an optimisation problem that confronts *a fictitious social planner*. The *dynamic stochastic* barter economy above can thus be *reformulated* in terms of a *static competitive* general equilibrium /Arrow (1953, 1964) - Debreu (1959)/ model, the properties of which have been well studied. To pin down the definition of a "good" in such models, we characterise each good as being not only a H-firm or a F-firm "product" but also by the time t and state of nature s of delivery.

To simplify, we follow Mark's textbook by assuming *just two* possible realisations of the H *and* F "output" (in fact, stochastic endowment) at *each* date[7], that is, a high value (i_h and i_h^*) and a low value (i_l and i_l^*) for both countries, which makes a total of *four (symmetric) states of nature each period*, namely $s_1 \equiv (i_h, i_h^*)$, $s_2 \equiv (i_h, i_l^*)$, $s_3 \equiv (i_l, i_h^*)$ and $s_4 \equiv (i_l, i_l^*)$, with $(s_1, s_2, s_3, s_4) \equiv \mathcal{S}$, the *set* of all possible states of the world that is the *same* each period (i.e., a *stationary distribution* in a more general sense). In such a way, all possible future outcomes and the corresponding unique Arrow-Debreu goods are completely spelled out. This reformulation of what constitutes a good corresponds to a complete system of forward markets. Instead of waiting for nature to reveal itself over time, one could have people meet once and for all in the "beginning of the world" and contract for all future trades. After trades in future contingencies have been agreed, time starts to evolve but people do not make any further decisions, they simply fulfill their contractual obligations depending on which particular state of the *known* set has *materialised* at each date. The point of such a reformulation is, as stressed, that the dynamic stochastic economy has been transformed into a static general equilibrium model. And it is known from static general equilibrium analysis that the *solution to the social planner's problem* is a *Pareto optimal* allocation; moreover, from the fundamental theorems of welfare economics it follows that the Pareto optimum *supports* (i.e., can be *replicated* by) a *competitive* equilibrium. Hence, the social optimum solution would also describe (or coincide with) the equilibrium for the *market* economy.

To solve the *planner's problem*, we have to assume the *weights* ϕ and $1-\phi$ attached to (the importance of) the H and F country (i.e., representative agent, in the present setting), respectively. Then, the planner's problem is simply to allocate the i and i^* endowments optimally between the H and F individual(s) in each period, by maximising a (*global*-economy) *social welfare* function

$$\underbrace{E_t \left\{ \sum_{k=0}^{\infty} \beta^k \underbrace{\left[\phi \underbrace{u\left(c_{it+k}, c_{i^*t+k}\right)}_{H period\ t+k \text{ utility}} + (1-\phi) \underbrace{u\left(c^*_{it+k}, c^*_{i^*t+k}\right)}_{F period\ t+k \text{ utility}} \right]}_{global \text{ period } t+k \text{ welfare} \equiv \text{ weighted } national \text{ period } t+k \text{ utility}} \right\}}_{\text{global social welfare} \equiv \text{ global } lifetime \text{ utility}},$$

subject to the *resource* constraints (2.11) and (2.12). Since the goods are *non-storable*, the above problem reduces to a *timeless* one of maximising

$$\underbrace{\phi u\left(c_{it}, c_{i^*t}\right) + (1-\phi) u\left(c^*_{it}, c^*_{i^*t}\right)}_{global \text{ period } t \text{ welfare} \equiv \text{ weighted } national \text{ period } t \text{ utility}} .$$

subject to (2.11) and (2.12). The Euler equations for this problem are

$$\phi u_1\left(c_{it}, c_{i^*t}\right) = (1-\phi) u_1\left(c^*_{it}, c^*_{i^*t}\right) \Longleftrightarrow$$

$$\Longleftrightarrow \quad \underbrace{u_1\left(c_{it}, c_{i^*t}\right)}_{\text{MUC of } H \text{ agent w.r.t. } H \text{ good}} = \frac{1-\phi}{\phi} \underbrace{u_1\left(c^*_{it}, c^*_{i^*t}\right)}_{\text{MUC of } F \text{ agent w.r.t. } H \text{ good}} . \tag{2.13}$$

$$\phi u_2\left(c_{it}, c_{i^*t}\right) = (1-\phi) u_2\left(c^*_{it}, c^*_{i^*t}\right).$$

$$\Longleftrightarrow \quad \underbrace{u_2\left(c_{it}, c_{i^*t}\right)}_{\text{MUC of } H \text{ agent w.r.t. } F \text{ good}} = \frac{1-\phi}{\phi} \underbrace{u_2\left(c^*_{it}, c^*_{i^*t}\right)}_{\text{MUC of } F \text{ agent w.r.t. } F \text{ good}} . \tag{2.14}$$

Equations (2.13) and (2.14) are the *optimal* (or *efficient*) *risk sharing* conditions. Risk sharing is efficient *when consumption is allocated so that* the marginal utility of consumption with respect to both goods of the H individual is *proportional*, and therefore *perfectly correlated*, to the

[7]For a more general case, with the corresponding notation, see the original paper by Lucas (1982).

marginal utility of the F individual. The weight ϕ can be interpreted as a measure of *size* of the Home country in the market (i.e., not planner's) version of the world economy. Since it was assumed at the outset that the agents have *equal* wealth, it is logical to also assume now that they are *equally* important to the social planner, i.e., we set $\phi = \frac{1}{2}$. Then the Pareto optimal allocation is to *split* the available output of i and i^* *equally*:

$$c_{it} = c^*_{it} = \frac{i_t}{2} \text{ and } c_{i^*t} = c^*_{i^*t} = \frac{i^*_t}{2}. \tag{2.15}$$

Having thus determined the optimal quantities, to get the market solution one looks for the competitive equilibrium that supports this Pareto optimum. Models that can be solved like this are called *Arrow-Debreu models.*

2.1.6. *Arrow-Debreu Planner's Problem: Decentralised Market Equilibrium.* If households (consumers) owned only their own country's firms (producing units), they would be exposed to *country-specific (idiosyncratic)* risk. Being risk-*averse*, they would prefer to avoid (*diversify away*) such (*insurable*) risk. This can be done by holding a *perfectly diversified* portfolio of assets. In the present context, a diversification plan (strategy) that perfectly insures against country-specific risk and which replicates the social optimum is for each representative individual to hold stock in *half* of each country's representative firm (hence, output, sales and finally, dividend):

$$\omega_{it} = \omega^*_{it} = \omega_{i^*t} = \omega^*_{i^*t} = \frac{1}{2}. \tag{2.16}$$

This is called a *"pooling" equilibrium*, because the implicit insurance scheme at work is that agents agree in advance to pool their risk by *sharing* the realised output *equally.*

2.1.7. *Solution under CRRA Utility.* To obtain an explicit solution, as we know from earlier lectures (4 and 5), one needs to further adopt a particular functional form for utility. A usual special case in similar circumstances – recall lecture 5 – is a constant relative risk aversion (CRRA) utility function defined over (real) consumption, the latter now itself composed of the two goods *aggregated* by a Cobb-Douglas index:

$$u\left(c_{it}, c_{i^*t}\right) \equiv \frac{c_t^{1-\gamma}}{1-\gamma}, \tag{2.17}$$

with

$$c_t \equiv c_{it}^{\theta} c_{i^*t}^{1-\theta}. \tag{2.18}$$

Then

$$u_1\left(c_{it}, c_{i^*t}\right) = \theta \frac{c_t^{1-\gamma}}{c_{it}},$$

$$u_2\left(c_{it}, c_{i^*t}\right) = (1-\theta)\frac{c_t^{1-\gamma}}{c_{i^*t}},$$

and the Euler equations derived earlier now become

$$q_t = \frac{1-\theta}{\theta}\frac{i_t}{i^*_t}, \tag{2.19}$$

$$\frac{e_t}{i_t} = \beta E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{1-\gamma}\left(1 + \frac{e_{t+1}}{i_{t+1}}\right)\right], \tag{2.20}$$

$$\frac{e^*_t}{q_t i^*_t} = \beta E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{1-\gamma}\left(1 + \frac{e^*_{t+1}}{q_{t+1} i^*_{t+1}}\right)\right]. \tag{2.21}$$

From (2.19), the RER q_t is determined by relative output levels. (2.20) and (2.21) are stochastic difference equations in the "dividend/price" (inverted) ratios $\frac{e_t}{i_t}$ and $\frac{e_t^*}{q_t i_t^*}$. Iterating forward, each one of them can be expressed as the PDV of future consumption growth raised to the power of $1-\gamma$. Once an *assumption* is made *about the stochastic processes* governing output, an *explicit solution* can be arrived at.[8]

There is no *actual* asset trading in the Lucas (1982) model. Agents hold their investments forever and never rebalance their portfolios. It is said accordingly that the asset prices produced by this model are, in fact, *shadow* prices: these must be respected in order for agents to willingly hold the outstanding equity shares according to (2.16).

2.2. **Single-Currency Two-Country Monetary Economy.**

2.2.1. *Introducing Money through Cash-in-Advance Constraint.* In a first extension of the real, barter economy model, Lucas (1982) introduces a *single* world currency. This environment could be thought of as representing a *two-sector closed* economy. One of the difficulties in justifying the role of money into a model like the described above is that people in that barter economy get along fine without any fiat currency. To get around this problem, Lucas (1982) prohibits barter in the monetary economy and imposes a *cash-in-advance* (CiA) constraint.[9] In such a set up, the *timing of events* becomes an important issue, so we sketch it next, as adopted in Lucas (1982) and summarised in Mark's textbook.

2.2.2. *Timing of Events.*

(1) *endowment (output)* shock realisations i_t and i_t^* are revealed.
(2) μ_t, the exogenous stochastic *net* rate of change in the *money* stock, M_t, is also revealed.[10] Thus M_t evolves according to $M_t \equiv (1+\mu_t)\, M_{t-1}$. The economy-wide *increment* (*first difference*) $\Delta M_t \equiv M_t - M_{t-1} \equiv (1+\mu_t)\, M_{t-1} - M_{t-1} = \mu_t M_{t-1}$, is distributed *evenly* to all H and F individuals, so that each representative agent (in the present setting) receives $\frac{\Delta M_t}{2} = \frac{\mu_t M_{t-1}}{2}$.
(3) A *centralised* securities market opens, and agents allocate their wealth between *stock* purchases (i.e., *insurance*) and the cash they will need to purchase *goods* (i.e., *consumption*). To distinguish between the aggregate money stock M_t and its two components selected respectively by the H and F representative agent, we introduce the notation[11] M_{Ht} and M_{Ft}. The *securities* market closes.
(4) *Decentralised* goods trading then takes place in the "shopping mall": *each household splits* into "worker-seller" (e.g., the husband) and "shopper" (e.g., the wife); the *shopper* takes the cash from the securities market trading and buys i and i^* goods from *other* "stores" in the mall (shoppers are not allowed to buy from their own stores); the H-country *worker-seller* collects the i-good endowment and offers it for sale at an i-good store in the "mall", and similarly does the F-country worker-seller. The *goods* market closes.
(5) The *cash* value of the goods *sales* is distributed to stockholders as *dividends.* Stockholders carry these nominal dividend payments into the next period.

2.2.3. *Why No Extra Cash Is Carried Across Periods and Why CiA Binds.* Now the state of the world is summarised by a *triplet*, not by a pair of shock realisations as in the barter economy. The additional third shock is the growth rate of the stock of *money.* So $s \equiv (g_t, g_t^*, \mu_t)$ and is revealed *prior to* trading. Therefore, the representative household can *precisely* determine the amount of money that it needs to finance its current-period consumption plan. As a result, it is not necessary to carry *extra* cash from one period to the next. If the (shadow) *nominal* interest rate is always *positive*, it is *optimal* for households to use up *all* their cash intended for consumption.

[8] Mark's (2001) textbook, sections 4.4 and 4.5, demonstrates this by calibrating the Lucas (1982) model.

[9] Also called "finance constraint" or "Clower constraint" (due to a 1967 paper by Clower on the microfoundations of money).

[10] The notation in Lucas (1982) and Mark's (2001) textbook referes to the corresponding *gross* rate of money growth λ_t.

[11] m_t and m_t^*, respectively, in Lucas (1982) and Mark's (2001) textbook interpretation.

To formally derive the H-agent problem, let P_t be the *nominal* (or *money*) price of good i_t, i.e., its price in terms of the *single* currency in circulation in the present version of the model. Now (2.1) becomes (cf. (2.1))

$$\mathcal{W}_t = \underbrace{\frac{P_{t-1}}{P_t}\left(\omega_{it-1}i_{t-1} + \omega_{i^*t-1}q_{t-1}i^*_{t-1}\right)}_{\text{dividends}} + \underbrace{\omega_{it-1}e_t + \omega_{i^*t-1}e^*_t}_{\text{ex-dividend share values}} + \underbrace{\frac{\Delta M_t}{2P_t}}_{\text{money transfer}}. \tag{2.22}$$

In the securities market, the H-household allocates wealth between cash to finance shopping plans and equity (cf. (2.2)):

$$\mathcal{W}_t \geq \underbrace{\frac{M_{Ht}}{P_t}}_{\text{cash to buy consumption}} + \underbrace{\omega_{it}e_t + \omega_{i^*t}e^*_t}_{\text{insurance: purchases of new shares}}. \tag{2.23}$$

This is so because, under the CiA constraint with a positive nominal RIR,

$$M_{Ht} = \underbrace{P_t\left(c_{it} + q_t c_{i^*t}\right)}_{\text{nominal consumption}} \Leftrightarrow \frac{M_{Ht}}{P_t} = \underbrace{c_{it} + q_t c_{i^*t}}_{\text{real consumption}}. \tag{2.24}$$

That is, the CiA constraint is said to *bind* (i.e., to hold with *equality*). Substituting (2.24) into (2.23) eliminates M_{Ht} and gives a simpler expression for the consolidated budget constraint (cf. (2.3))

$$\frac{P_{t-1}}{P_t}\left(\omega_{it-1}i_{t-1} + \omega_{i^*t-1}q_{t-1}i^*_{t-1}\right) + \omega_{it-1}e_t + \omega_{i^*t-1}e^*_t + \frac{\Delta M_t}{2P_t} =$$

$$= \omega_{it}e_t + \omega_{i^*t}e^*_t + c_{it} + q_t c_{i^*t}. \tag{2.25}$$

2.2.4. *Objective and First-Order Conditions.* Maximising the same expected lifetime utility (2.4) as in the barter economy under the now modified constraint (2.25) above, the Euler equation for real consumption remains unchanged (see (2.6)) but the equity pricing formulas, which are the other two Euler equations, are now modified by a term *inverse* to the *gross* inflation rate, which could be interpreted as an *inflation premium* (cf. (2.7) and (2.8)):

$$\omega_{it}: \qquad e_t u_1\left(c_{it}, c_{i^*t}\right) = \beta E_t\left[u_1\left(c_{it+1}, c_{i^*t+1}\right)\left(\frac{P_t}{P_{t+1}}i_t + e_{t+1}\right)\right], \tag{2.26}$$

$$\omega_{i^*t}: \qquad e^*_t u_1\left(c_{it}, c_{i^*t}\right) = \beta E_t\left[u_1\left(c_{it+1}, c_{i^*t+1}\right)\left(\frac{P_t}{P_{t+1}}q_t i^*_t + e_{t+1}\right)\right]. \tag{2.27}$$

The inflation premium, $\frac{P_t}{P_{t+1}}$, arises because the nominal dividends of the current period must be carried over into the next period, at which time their value can potentially be eroded by an inflation shock.

The F-household solves an analogous problem.

2.2.5. *Market Clearing.* The introduction of a *money* into the previously barter economy has resulted in a fifth market clearing condition (or adding up constraint), which complements the four other (recall equations (2.9)–(2.10)):

$$M_t \equiv M_{Ht} + M_{Ft}. \tag{2.28}$$

To solve the model, one aggregates the CiA constraints in H, $M_{Ht} = P_t\left(c_{it} + q_t c_{i^*t}\right)$, and F, $M_{Ft} = P_t\left(c^*_{it} + q_t c^*_{i^*t}\right)$, and uses the market clearing conditions /adding up constraints/ to get

$$M_t = P_t\left(i_t + q_t i^*_t\right), \tag{2.29}$$

which is the *quantity* (theory of money) equation for the *world* economy, with *unitary* velocity. Since the single currency used in both national economies does not generate a new idiosyncratic (country-specific) shock, the equilibrium established for the barter economy (i.e., *equal and constant* portfolio shares) is still the *perfect risk-pooling* equilibrium, described by equations (2.15) and (2.16).

Solution under CRRA Utility. Under the CRRA utility function (2.17), the RER q_t is the same expression (2.19). When one substitutes the latter into (2.29), the inverse gross inflation rate is given by

$$\frac{P_t}{P_{t+1}} = \frac{M_t}{M_{t+1}} \frac{i_{t+1}}{i_t}. \tag{2.30}$$

(2.19) and (2.30) can be used together to rewrite the equity pricing formulas as (cf. (2.20) and (2.21))

$$\frac{e_t}{i_t} = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} \left(\frac{M_t}{M_{t+1}} + \frac{e_{t+1}}{i_{t+1}} \right) \right], \tag{2.31}$$

$$\frac{e_t^*}{q_t i_t^*} = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} \left(\frac{M_t}{M_{t+1}} + \frac{e_{t+1}^*}{q_{t+1} i_{t+1}^*} \right) \right]. \tag{2.32}$$

To price *nominal* bonds, one looks for the *shadow* price of a *hypothetical* nominal bond such that the public willingly keeps it in *zero net supply.* Let B_t be the *nominal price* of a *zero-coupon discount bond* that pays (with certainty) 1 unit of currency at the end of the period. The utility cost of buying the nominal bond is then $u_1 \left(c_{it}, c_{i^*t} \right) \frac{Bt}{P_t}$. In equilibrium, this is offset by the discounted expected marginal utility of the pay-off of 1 monetary unit, $\beta E_t \left[u_1 \left(c_{it+1}, c_{i^*t+1} \right) \frac{1}{P_{t+1}} \right]$. Under the CRRA utility function (2.17) one obtains

$$B_t = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} \frac{M_t}{M_{t+1}} \right]. \tag{2.33}$$

If ι_t is the nominal interest rate, then $B_t = \frac{1}{1+\iota_t}$. Nominal interest rates will be positive in all states of nature if $B_t < 1$, and this is likely to be true when the endowment growth rates g_t and g_t^* and the monetary growth rate μ_t are positive.

2.3. **Two-Currency Two-Country Monetary Economy.** In a next extension to the analysed set-up, Lucas (1982) proceeds to the core of his paper, namely addressing the comparison of *exchange rate regimes.* But, first of all, to be able to define the nominal exchange rate, he introduces a *second* national currency. From now on, we therefore assign the previous M_t notation to the *Home* country, whereas the currency of the *Foreign* country will be – as it is the custom[12] – distinguished by an asterisk and thus denoted by M_t^*.

Having introduced a second monetary unit requires an additional assumption: we need to specify *in which national currency* consumers have *to pay* when buying the (foreign) goods they consume and the (foreign) assets in which they save. We shall explore the interesting implications of this issue further in the course, when discussing pricing-to-market (PTM). For the time being, we simply adopt the assumption in Lucas (1982) – in fact, the same as in the *traditional* open-economy literature before the mid-1980s – according to which H goods (i) can only be purchased by payment in the H-currency (M) while F goods (i^*) can only be purchased by payment in the F-currency (M^*). Similarly, H equity (e) can only be bought with H-money (M) and F equity (e^*) with F-money (M^*). Finally, H-firms pay dividends (i) in H-currency (M) and F-firms pay dividends (i^*) in F-currency (M^*). Agents can *acquire* the *foreign currency* needed for consumption or saving from foreign dividends and during securities market trading.

2.3.1. *Flexible Exchange Rate Regime.*

[12] This does not prevent Lucas (1982), as well as Mark (2001) in his textbook treatment of the paper, to use N_t where we would prefer M_t^*.

Additional Assumptions. Lucas (1982) first looks into a perfectly *flexible* exchange rate regime. Let P_t be the H-currency price of the H-good i, P_t^* the F-currency price of the F-good i^*, and S_t the spot exchange rate expressed in the usual way as the H-currency price of F-money. At t, M_t is the outstanding stock (in circulation) of the H-currency, and M_t^* is the outstanding stock (in circulation) of the F-currency. These *money supplies* are *random* (exogenous) and evolve over time according to the *same* stochastic process as in the preceding section, but instead of having a single currency we now have *two* national monies: $\Delta M_t \equiv M_t - M_{t-1} \equiv (1+\mu_t)\, M_{t-1} - M_{t-1} = \mu_t M_{t-1}$ and $\Delta M_t^* \equiv M_t^* - M_{t-1}^* \equiv (1+\mu_t^*)\, M_{t-1}^* - M_{t-1}^* = \mu_t^* M_{t-1}^*$, with μ_t and μ_t^* denoting the *net* rates of change in M and M^*, respectively.

Since households also buy goods abroad (their *love for diversity* is imposed via the specification of preferences – see above), they face *foreign purchasing power risk*. That is why introducing the second currency creates a new country-specific risk, which the risk-averse households would wish to hedge. The complete markets paradigm allows markets to develop whenever there is a demand for a product, and the product that individuals desire in the present context are claims to future H-currency and F-currency transfers. Let r_t be the price of a claim to all future H-currency transfers in terms of the numéraire good i. Similarly, let r_t^* be the price of a claim to all future F-currency transfers in terms of i. Let also there be one *perfectly divisible* claim outstanding for each of these two *monetary transfer streams*. Lucas (1982) now assumes that the H-agent holds ψ_{Mt} claims on the H-currency stream and ψ_{M^*t} claims on the F-currency stream. Analogously, the F-agent holds ψ_{Mt}^* claims on the H-currency stream and $\psi_{M^*t}^*$ claims on the F-currency stream. A final assumption is that *initially* the H-agent is endowed with claims *only* on his *national* currency, i.e., his initial endowment is $\psi_M = 1, \psi_{M^*} = 0$; symmetrically, the F-agent begins with the following endowment of monetary transfer claims: $\psi_M^* = 0, \psi_{M^*}^* = 1$. From the initial period onwards, agents are *free to trade* their claims on monetary transfer streams.

With the features discussed above, the wealth with which each agent *enters* the current period becomes more complicated (cf. (2.22)):

$$\mathcal{W}_t = \underbrace{\frac{P_{t-1}}{P_t}\omega_{it-1}i_t + \frac{S_t P_{t-1}^*}{P_t}\omega_{i^*t-1}i_t^*}_{\text{dividends}} + \underbrace{\frac{\psi_{Mt-1}\Delta M_t}{P_t} + \frac{\psi_{M^*t-1}\Delta M_t^*}{P_t}}_{\text{monetary transfers}} +$$

$$(2.34) \qquad + \underbrace{\underbrace{\omega_{it-1}e_t + \omega_{i^*t-1}e_t^*}_{\text{market value of ex-dividend shares}} + \underbrace{\psi_{Mt-1}r_t + \psi_{M^*t-1}r_t^*}_{\text{market value of monetary transfer claims}}}_{\text{market value of securities}} .$$

During securities market trading, this wealth is *allocated* as follows:

$$(2.35) \qquad \mathcal{W}_t \geq \underbrace{\frac{M_{Ht}}{P_t} + \frac{S_t M_{Ht}^*}{P_t}}_{\text{cash to buy consumption}} + \underbrace{\omega_{it}e_t + \omega_{i^*t}e_t^*}_{\textit{output}\text{ insurance: purchases of new shares}} + \underbrace{\psi_{Mt}r_t + \psi_{M^*t}r_t^*}_{\textit{money}\text{ insurance: purchases of new claims}} .$$

Uncertainty is *resolved* – i.e., the current-period values of the shocks g_t, g_t^*, μ_t, μ_t^* governing the evolution of domestic and foreign output i_t, i_t^* and the money stocks M_t, M_t^* are revealed – *before* trading occurs. That is why households acquire the *exact* amount of H and F currency they need to finance their current-period consumption plans. In equilibrium, therefore, the two CiA constraints for the H-household will *bind*:

$$(2.36) \qquad M_{Ht} = P_t c_{it},$$

$$(2.37) \qquad M_{Ht}^* = P_t^* c_{i^*t},$$

which can be used to eliminate M_{Ht} and M_{Ht}^* from (2.35) and rewrite the latter equation as:

$$\mathcal{W}_t = \underbrace{c_{it} + \frac{S_t P_t^*}{P_t} c_{i^*t}}_{\text{consumption: goods}} + \underbrace{\omega_{it} e_t + \omega_{i^*t} e_t^*}_{\text{saving: new equity}} + \underbrace{\psi_{Mt} r_t + \psi_{M^*t} r_t^*}_{\text{insurance: new monetary transfer claims}} . \tag{2.38}$$

The *consolidated* budget constraint of the H-individual now becomes:

$$\frac{P_{t-1}}{P_t}\omega_{it-1} i_{t-1} + \frac{S_t P_{t-1}^*}{P_t}\omega_{i^*t-1} i_{t-1}^* +$$

$$+\omega_{it-1} e_t + \omega_{i^*t-1} e_t^* + \psi_{Mt-1} r_t + \psi_{M^*t-1} r_t^* +$$

$$+\frac{\psi_{Mt-1}\Delta M_t}{P_t} + \frac{\psi_{M^*t-1} S_t \Delta M_t^*}{P_t} =$$

$$= \omega_{it} e_t + \omega_{i^*t} e_t^* + \psi_{Mt} r_t + \psi_{M^*t} r_t^* + c_{it} + \frac{S_t P_{t-1}^*}{P_t} c_{i^*t}. \tag{2.39}$$

Maximising the same expected lifetime utility (2.4) as in the barter economy under the now modified constraint (2.39) above, the associated Euler equations are:

$$c_{i^*t}: \qquad \frac{S_t P_t^*}{P_t} u_1(c_{it}, c_{i^*t}) = u_2(c_{it}, c_{i^*t}) \Longleftrightarrow \frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})} = \frac{S_t P_t^*}{P_t} \equiv RER_t. \tag{2.40}$$

Compare it with (2.6), the corresponding formula for both the barter and the single currency monetary economies.

$$\omega_{it}: \qquad e_t u_1(c_{i^*t}, c_{i^*t}) = \beta E_t \left[u_1(c_{it+1}, c_{i^*t+1}) \left(\frac{P_t}{P_{t+1}} i_t + e_{t+1} \right) \right].$$

This is exactly the same expression as the corresponding equation in the single currency (two-sector) monetary economy, namely (2.26).

$$\omega_{i^*t}: \qquad e_t^* u_1(c_{i^*t}, c_{i^*t}) = \beta E_t \left[u_1(c_{it+1}, c_{i^*t+1}) \left(\frac{S_t P_t^*}{P_{t+1}} i_t^* + e_{t+1} \right) \right]. \tag{2.41}$$

Compare it with (2.8) and (2.27) above.

We now have *two new* Euler equations, arising from *hedging foreign currency risk*:

$$\psi_{Mt}: \qquad r_t u_1(c_{it}, c_{i^*t}) = \beta E_t \left[u_1(c_{it+1}, c_{i^*t+1}) \left(\frac{\Delta M_{t+1}}{P_{t+1}} i_t + r_{t+1} \right) \right], \tag{2.42}$$

$$\psi_{M^*t}: \qquad r_t^* u_1(c_{it}, c_{i^*t}) = \beta E_t \left[u_1(c_{it+1}, c_{i^*t+1}) \left(\frac{S_{t+1}\Delta M_{t+1}^*}{P_{t+1}} i_t + r_{t+1}^* \right) \right]. \tag{2.43}$$

An analogous optimisation problem for the F representative household results, of course, in symmetrical Euler equations.

Market Clearing. The introduction of a *second* national currency into the previously one-money economy has resulted in a sixth market clearing condition (or adding up constraint), which complements the five such constraints introduced thus far (we shall not re-write them here, see equations (2.9)–(2.10) and (2.28)):

$$M_t^* \equiv M_{Ht}^* + M_{Ft}^*. \tag{2.44}$$

The CiA and adding-up constraints, taken together, give a unit-velocity quantity of money equation for each country

$$M_t = P_t i_t, \tag{2.45}$$

$$M_t^* = P_t^* i_t^*, \tag{2.46}$$

which can be used to eliminate the endogenous nominal price levels from the Euler equations. The equilibrium with *perfect risk-pooling* of country-specific risks is given by

$$\omega_{it} = \omega_{it}^* = \omega_{i^*t} = \omega_{i^*t}^* = \psi_{Mt} = \psi_{Mt}^* = \psi_{M^*t} = \psi_{M^*t}^* = \frac{1}{2}. \tag{2.47}$$

In this equilibrium, both the H and F representative household own:

- half of the domestic endowment (output) stream;
- half of the foreign endowment (output) stream;
- half of all future domestic monetary transfers;
- half of all future foreign monetary transfers.

In short, the world resources are *split equally* between the H and F representative agents, subjected to *country-specific* endowment (output) and monetary risks (uncertainty): the pooling equilibrium thus supports the symmetric allocation

$$c_{it} = c_{it}^* = \frac{i_t}{2} \text{ and } c_{i^*t} = c_{i^*t}^* = \frac{i_t^*}{2}. \tag{2.48}$$

Now to solve for the *nominal* exchange rate, S_t, we use the *real* exchange rate equation (2.40) and the two quantity theory equations (2.45) and (2.46) derived earlier:

$$\frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})} = \frac{S_t P_{t-1}^*}{P_t},$$

$$\frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})} = \frac{S_t M_t^* i_t}{M_t i_t^*},$$

$$S_t = \frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})} \frac{M_t}{M_t^*} \frac{i_t^*}{i_t}. \tag{2.49}$$

This is the *microfounded* nominal exchange rate determination equation that comes out from the Lucas (1982) *optimising* DSGE model. Note that, as in the *ad-hoc* monetary model with flexible prices we studied in lecture 3, the fundamental determinants of the NER are relative money supplies, $\frac{M_t}{M_t^*}$, and relative output, $\frac{i_t^*}{i_t}$: a higher money stock domestically than abroad and/or a lower output (endowment) domestically than abroad depreciates the national currency (that is, increases the its exchange rate, S_t). These two similarities between the NER determination equations in the two models are, however, supplemented by two differences. First, in the Lucas model the exchange rate depends also on preferences, $\frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})}$: a higher marginal utility for the foreign good, $u_2(c_{it}, c_{i^*t})$, relative to the marginal utility for the domestic good, $u_1(c_{it}, c_{i^*t})$, depreciates the domestic currency as well, which is not the case in the monetary model. Second, the NER does not explicitly depend on expectations about the future in the Lucas model, but depends in the monetary model. We have thus seen that providing microfoundations does confirm some earlier results based on ad-hoc models, yet it also adds new results (or – in other cases – modifies older ones). Microfounded models yield much insight into the mechanisms driving such (macro)results too, and finally justify the latter from the perspectives of the "first principles" of rational behaviour. In that consists one of the key valuable lessons from building up macroeconomic research on microeconomic foundations.

Solution under CRRA Utility. Under the CRRA utility function (2.17), the RER, q_t, is the same expression (2.19). When one substitutes the latter from (2.40), using the CRRA utility, the NER becomes

$$S_t = \frac{1-\theta}{\theta} \frac{M_t}{M_t^*}.$$

In addition to the equity Euler equations (2.31) and (2.32), which remain the same, there are now *two new* ones, relating to each of the two currencies in circulation:

$$\frac{r_t}{i_t} = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} \left(\frac{\Delta M_{t+1}}{M_{t+1}} + \frac{r_{t+1}}{i_{t+1}} \right) \right], \tag{2.50}$$

$$\frac{r_t^*}{i_t} = \beta E_t \left[\frac{1-\theta}{\theta} \left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} \left(\frac{\Delta M_{t+1}^*}{M_{t+1}^*} + \frac{r_{t+1}^*}{i_{t+1}} \right) \right]. \tag{2.51}$$

Finally, the equilibrium price of the *Home* bond remains as it was in the single-currency model version, (2.33), and the equilibrium price of the *Foreign* bond is symmetric:

$$B_t^* = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} \frac{M_t^*}{M_{t+1}^*} \right]. \tag{2.52}$$

Similarly to what was derived in the single-currency model version for the H economy, we now have corresponding definitions for the F economy: if ι_t^* is the F nominal interest rate, then $B_t^* = \frac{1}{1+\iota_t^*}$. Nominal interest rates will thus be positive in all states of nature if $B_t < 1$ and $B_t^* < 1$, which is likely to be true when the endowment growth rates g_t and g_t^* and the monetary growth rates μ_t and μ_t^* are positive.

2.3.2. *Fixed Exchange Rate Regime.* In a final section of his paper, Lucas (1982) sets the objective to find a symmetric, perfectly pooled equilibrium in which the exchange rate is maintained at a constant level through central bank intervention in the foreign exchange market. This is the new feature we add at this stage to the model developed until now. The timing and monetary conventions as well as the market structure are otherwise kept unchanged.

If the NER is to be fixed, some agency has to ensure and implement this fixity. Lucas (1982) assigns this role to a single, central authority, holding reserves of both currencies. This institution also trades in the spot currency markets so as to maintain the exchange rate S_t at some constant level $\overline{S}$. Lucas (1982) points out that to analyse such a regime under *rational expectations*, it is necessary

- either to assume that the behaviour of this central authority, in combination with the behaviour of monetary and real shocks in the two countries, is consistent with the permanent maintenance of the pegged level $\overline{S}$;
- or to incorporate into the analysis the possibility of deviations and of consecutive speculative activity.

His choice is then the first, much simpler, alternative. Let the authority begin and also end a given period with total reserves of H-currency value R_0, possibly after receiving new currency transfers from one or both countries. Let its holdings after all securities trading is completed be R in H-currency and R^* in F-currency, so that

$$R_0 = R + \overline{S} R^*. \tag{2.53}$$

Under the hypothesis that nominal interest rates are uniformly positive, provisionally maintained here by Lucas (1982), the two quantity theory equations (2.45) and (2.46) will continue to hold, but with and replaced by the quantities and of these currencies remaining in private circulation. Then the equilibrium NER (2.49) becomes

$$\overline{S} = \frac{M-R}{M^*-R^*} \frac{u_2\left(c_{it}, c_{i^*t}\right)}{u_1\left(c_{it}, c_{i^*t}\right)} \frac{i_t^*}{i_t}. \tag{2.54}$$

Now given $\overline{S}$, the realisations of output (endowment) shocks leading to the quantities of goods i_t and i_t^* available for consumption and the realisations of monetary shocks leading to the quantities of money and available to agents, (2.53) and (2.54) are two equations in two unknowns, the end-of-period reserve levels in each currency, R and R^*. *Viability* of the *peg* regime then requires that $R > 0$ and $R^* > 0$ for all possible states of nature. It can be shown that these two inequalities are equivalent to

$$R_0 > \overline{S}M^* - M\frac{u_2\left(c_{it}, c_{i^*t}\right)}{u_1\left(c_{it}, c_{i^*t}\right)}\frac{i_t^*}{i_t} \tag{2.55}$$

and

$$R_0 > M - \frac{\overline{S}M^*}{\frac{u_2(c_{it}, c_{i^*t})}{u_1(c_{it}, c_{i^*t})}\frac{i_t^*}{i_t}}. \tag{2.56}$$

To interpret the above conditions, Lucas (1982) assumes that the positive random variable $\frac{u_2(c_{it},c_{i^*t})}{u_1(c_{it},c_{i^*t})}\frac{i_t^*}{i_t}$ ranges in value from *zero* to *infinity.* Then for (2.55) and (2.56) to hold for all states of nature, the stabilising authority must hold reserves of H-currency value exceeding *both* the H-currency value of F currency outstanding, $\overline{S}M^*$, (inequality (2.55)) *and* all outstanding H currency, M, (inequality (2.56)). Tighter bounds on the range of $\frac{u_2(c_{it},c_{i^*t})}{u_1(c_{it},c_{i^*t})}\frac{i_t^*}{i_t}$ would permit smaller reserves. With *constant* money supply, a sufficiently large (constant) reserve level R_0 can always be selected. With M_t and M_t^* *drifting over time*, even if the drifts are perfectly correlated, *no constant* reserve level R_0 can maintain (2.55) and (2.56) forever. Lucas (1982), p. 354, concludes: "... then, the maintenance of fixed exchange rate requires coordination in the monetary policies of the two countries and of the stabilising authority. At the same time, there may remain a good deal of latitude for independent monetary policies on a period-by-period basis."

With (2.55) and (2.56) maintained, the rest of the analysis is precisely the same as in the single-currency world economy. In other words, once there is a sufficient level of reserves and coordination of monetary policies to ensure credibility, i.e., that (2.55) and (2.56) are never violated, the peg versus float debate does not matter for the equilibrium allocation of real consumption with complete markets and flexible prices. As Lucas (1982), pp. 354-355, writes, "In summary, then, it is possible to devise a pegged exchange rate regime under which the Pareto-optimal resource allocation obtained under flexible rate system is replicated exactly, provided only that the authority responsible for maintaining the fixed rate is armed with sufficient reserves."

3. Real Business Cycle Research in Closed Economy

[... to be summarised in class: sections 4.4, 4.5 and 5.1 in Mark (2001) provide a compact introduction to the RBC approach and sections 7.4.3.1–7.4.3.4 in Obstfeld and Rogoff (1996) offer another textbook treatment; Kydland and Prescott (1982) is the seminal paper that started this literature; King, Plosser and Rebelo (1988 a, b) discuss in greater detail the technical aspects of RBC research, itself viewed as extending the basic neoclassical growth model ...]

4. International (Real) Business Cycle Models

[... to be summarised in class: section 5.2 in Mark (2001), section 7.4.3.5 in Obstfeld and Rogoff (1996) and Baxter (1995) discuss the essential features and techniques of I(R)BC research; Backus, Kehoe and Kydland (1992) is the paper that started this literature; Baxter and Crucini (1995) focuses on the solution algorithm of these models ...]

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