EC933-G-AU INTERNATIONAL FINANCE – LECTURE 3

MACROECONOMIC THEORIES OF BALANCE OF PAYMENTS ADJUSTMENT: STOCK AND STOCK-FLOW APPROACHES

ALEXANDER MIHAILOV

ABSTRACT. This chapter continues to review the major theories of balance of payments (BoP) adjustment and exchange-rate determination, moving further to stock and stock-flow models, such as the monetary approach and the portfolio (or asset-markets) approach developed in the 1960s and the 1970. Section 1 begins, in subsection 1.1, with the monetary model under *fixed* exchange rates, often called the monetary approach to the *balance of payments*, and then considers, in subsection 1.2, the monetary model under *flexible* exchange rates, sometimes termed the monetary approach to the *exchange rate* or, simply, the *monetary model*. Subsections 1.3 and 1.4 present the portfolio approach to the balance of payments (under *peg*) and to the exchange rate (under *float*), respectively, the latter also called the *portfolio balance model*. Section 2 briefly summarises the versions of the stock-flow approach to BoP adjustment and NER determination in partial equilibrium and in general equilibrum, indicating references for further study.

Date: 1 November 2005 (First draft: 22 October 2004).

This set of lecture notes is preliminary and incomplete. It is based on parts of the four textbooks suggested as essential and supplementary reading for my graduate course in international finance at Essex as well as on the related literature (see the course outline and reading list at http://courses/essex.ac.uk/ec/ec933/). The notes are intended to be of some help to the students attending the course and, in this sense, many aspects of them will be clarified during lectures. The present second draft may be developed and completed in future revisions. The responsibility for any errors and misinterpretations is, of course, only mine. Comments are welcome, preferably by e-mail at mihailov@essex.ac.uk and/or a_mihailov@hotmail.com.

Contents

1. Stock Approaches to BOP Adjustment and NER Determination

1.1. **The Monetary Approach to the Balance of Payments (Peg).** The monetary approach to the balance of payments was developed in the 1960s, in part within the research department of the IMF under the intellectual leadership of Jacques Polak, in the context of the Bretton Woods system of *fixed* exchange rates. With the collapse of the latter, the approach was also extended to cover the case of *flexible* exchange rates, thus becoming one of the major theories of NER determination. The monetary approach to the balance of payments is described in a comprehensive manner in the volume edited by Frenkel and Johnson (1976). In this and the next subsections, we present its two versions, for a regime of peg or float, respectively. Our summary is based mostly on Mark (2001), chapter 3, and Gandolfo (2001), parts of chapters 12, 13 and 15.

No matter that the monetary approach to BoP found its accomplished form in the 1970s, its proponents have claimed that the intellectual origins of this theory go back to Hume (1752), to writings of Wheatly and Ricardo in the 18-th century and to Cassel's (1918, 1921) revival of ideas of the Salamanca School in the 16-th century related to the proposition of purchasing power parity (PPP). The Cassellian approach to PPP has later been articulated by Samuelson (1964) in the context of a generalisation to the law of one price (LOP) ensured by commodity arbitrage for all internationally traded goods. PPP is definitely violated in the short run, but tends to be a long-run equilibrium concept to which nominal exchange rates gravitate. That is why it is often an ingredient in many economic models, including the monetary approach, especially as a long-term condition.

1.1.1. *Origins: The Classical (Humean) Price-Specie-Flow Mechanism.* In summary, the *classical* theory of balance of payments adjustment under the *gold* standard, building on ideas of Hume (1752), is as follows. A *surplus* in the BoP (or, more precisely, the current account) causes an *inflow* of gold. Because of the strict connection between gold reserves and the amount of money in circulation under the gold standard, originating in the *quantity theory of money* considered as valid, the increase of gold tends to increase the *price level.* Hence, as the goods of the surplus country become relatively more expensive in the international market, exports reduce, and for the same reason, but with inverse effect, imports increase. The increased demand for imports by residents and the reduced demand for exports by nonresidents lead to a gradual reduction of the initial trade surplus. An analogous, but inverse, logic applies to the case of BoP (that is, CA) *deficit.* Thus, the so-called *price-specie(=gold)-flow mechanism* ensures *automatically* BoP equilibrium under the gold standard.

A different perspective on the same process is based on the notion of the *optimum /desired/ distribution of the stock of specie* (that is, gold) among all countries in the world. Any BoP disequilibrium, which is a *flow* disequilibrium, is therefore determined by the underlying *stock* disequilibrium, i.e., a distribution of the stock of gold available in the world which does *not* coincide with the optimal (or desired) one, ensuring BoP equilibrium.

The important point of Hume's price-specie-flow mechanism is that the ultimate cause of balance of payments disequilibria is to be found in money stock disequilibria, namely in a divergence of the quantity of money (gold) in existence and the optimum or desired quantity (to be held) by each country's residents. It is from such considerations that the monetary approach to the BoP took its source.

1.1.2. *Main Assumptions.*

(1) *PPP* is a key assumption in the monetary approach, justified as an aggregation of the law of one price (LOP) in the markets for goods and services.
(2) *UIP* is assumed to hold as well.
(3) In contrast to the *flow* approaches to BoP adjustment we studied thus far in the course, all prices are assumed *flexible* under the monetary approach.
(4) Again, opposite to the *flow* models explored earlier, the focus of the monetary approach is on conditions for *stock* equilibrium in the *money* market: the BoP is essentially a monetary phenomenon, and should therefore be analysed in terms of adjustment of money stocks.
(5) Production is assumed at the level of *full* employment, so real income is *fixed.*

(6) A *stable money demand* function is taken to exist.
(7) We would consider below the *small* open-economy case.

Despite being an *ad-hoc*, i.e., not a microfounded, model, the monetary approach to *BoP* adjustment (under *fixed* exchange rates) or to *NER* determination (under *flexible* exchange rates) is still widely used for policy analysis. The reason is that, as we shall see later in this course, many of its predictions are confirmed by more complicated *optimising* models, both in flexible-price and sticky-price environments.

As is customary in the literature, the model will be presented in a notation distinguishing between *levels* of the variables, denoted by *upper*-case letters, and corresponding (natural) *logarithms* of the variables, denoted by *lower*-case letters (except for interest rates, which are themselves (net) rates of growth and are in levels).

1.1.3. *The Monetary Model under Peg.*
Set-Up and Derivation of Key Result. Using the notation we have introduced so far in previous lectures, we start by writing the definition of the *money supply*, in *levels*:

$$MS_t \equiv \mu MB_t \equiv \mu \left(DC_t + IR_t \right), \tag{1.1}$$

where $\mu \equiv \frac{E[MS_t]}{E[MB_t]}$ is the *money* multiplier, assumed *constant*.

We next proceed to a *logarithmic expansion* of the money supply and its components about their *mean* values. A first-order expansion of (1.1) about mean values could be done as follows. Taking expectations from both sides of (1.1),

$$E\left[MS_t\right] \equiv \mu E\left[DC_t\right] + \mu E\left[IR_t\right],$$

subtracting the above equation from (1.1) and rearranging, yields:

$$MS_t - E\left[MS_t\right] \equiv \mu \left(DC_t - E\left[DC_t\right]\right) + \mu \left(IR_t - E\left[IR_t\right]\right).$$

Dividing through by $E\left[MS_t\right]$,

$$\frac{MS_t - E\left[MS_t\right]}{E\left[MS_t\right]} \equiv \frac{\mu \left(DC_t - E\left[DC_t\right]\right)}{E\left[MS_t\right]} + \frac{\mu \left(IR_t - E\left[IR_t\right]\right)}{E\left[MS_t\right]},$$

substituting for μ and rewriting, using $\theta \equiv \frac{E[IR_t]}{E[MB_t]}$, progressively gives:

$$\frac{MS_t - E\left[MS_t\right]}{E\left[MS_t\right]} \equiv \frac{\frac{E[MS_t]}{E[MB_t]} \left(DC_t - E\left[DC_t\right]\right)}{E\left[MS_t\right]} + \frac{\frac{E[MS_t]}{E[MB_t]} \left(IR_t - E\left[IR_t\right]\right)}{E\left[MS_t\right]},$$

$$\frac{MS_t - E\left[MS_t\right]}{E\left[MS_t\right]} \equiv \frac{\left(DC_t - E\left[DC_t\right]\right)}{E\left[MB_t\right]} + \frac{\left(IR_t - E\left[IR_t\right]\right)}{E\left[MB_t\right]},$$

$$\frac{MS_t - E\left[MS_t\right]}{E\left[MS_t\right]} \equiv \frac{\left(DC_t - E\left[DC_t\right]\right)}{\frac{E[DC_t]}{E[DC_t]} E\left[MB_t\right]} + \frac{\left(IR_t - E\left[IR_t\right]\right)}{\frac{E[IR_t]}{E[IR_t]} E\left[MB_t\right]},$$

$$\frac{MS_t - E\left[MS_t\right]}{E\left[MS_t\right]} \equiv \frac{\left(DC_t - E\left[DC_t\right]\right)}{\frac{E[DC_t]}{\frac{E[DC_t]}{E[MB_t]}}} + \frac{\left(IR_t - E\left[IR_t\right]\right)}{\frac{E[IR_t]}{\frac{E[IR_t]}{E[MB_t]}}},$$

$$\frac{MS_t - E\left[MS_t\right]}{E\left[MS_t\right]} \equiv \frac{\left(DC_t - E\left[DC_t\right]\right)}{\frac{E[DC_t]}{\frac{E[MB_t]-E[IR_t]}{E[MB_t]}}} + \frac{\left(IR_t - E\left[IR_t\right]\right)}{\frac{E[IR_t]}{\theta}},$$

$$\frac{MS_t - E\left[MS_t\right]}{E\left[MS_t\right]} \equiv \frac{\left(DC_t - E\left[DC_t\right]\right)}{\frac{E[DC_t]}{1-\frac{E[IR_t]}{E[MB_t]}}} + \theta \frac{\left(IR_t - E\left[IR_t\right]\right)}{E\left[IR_t\right]},$$

$$\frac{MS_t - E\left[MS_t\right]}{E\left[MS_t\right]} \equiv \frac{\left(DC_t - E\left[DC_t\right]\right)}{\frac{E[DC_t]}{1-\theta}} + \theta \frac{\left(IR_t - E\left[IR_t\right]\right)}{E\left[IR_t\right]},$$

$$\frac{MS_t - E\left[MS_t\right]}{E\left[MS_t\right]} \equiv \left(1-\theta\right) \frac{\left(DC_t - E\left[DC_t\right]\right)}{E\left[DC_t\right]} + \theta \frac{\left(IR_t - E\left[IR_t\right]\right)}{E\left[IR_t\right]}.$$

Now, since for a random variable Z_t, $\frac{Z_t - E[Z_t]}{E[Z_t]} \approx \ln(Z_t) - \ln(E[Z_t])$, apart from an arbitrary constant, we can write the *money supply* in the corresponding logarithms:

$$m_t^s \equiv (1-\theta)\, d_t + \theta r_t. \tag{1.2}$$

A standard transactions motive gives rise to the following *demand for money* function, written in logarithms too:

$$m_t^d - p_t = \phi y_t - \lambda \iota_t + \epsilon_t, \tag{1.3}$$

with $0 < \phi < 1$ being the income elasticity of money demand, $\lambda > 0$ the interest *semi*-elasticity (since the interest rate is *not* in natural logs, as noted earlier) of money demand and the error term assumed independently and identically distributed (iid) with zero mean and constant variance, $\epsilon_t \overset{iid}{\sim} \left(0, \sigma_\epsilon^2\right)$.

We next use PPP and UIP, as assumed within the monetary approach framework.

Since the exchange rate is *fixed*, say at $\overline{s}$, *PPP* implies that the domestic price level in the SOE is determined by the exogenous foreign (RoW) price level:

$$p_t = \overline{s} + p_t^*. \tag{1.4}$$

Moreover, assuming a *credible* peg, market participants do not expect a change in the NER, i.e., $E_t[S_{t+1}] = S_t = \overline{S} = const$ for any t (hence $\ln \frac{E_t[S_{t+1}]}{S_t} = \ln \frac{\overline{S}}{\overline{S}} = \ln 1 = 0$), so that *UIP* reduces to:

$$\iota_t = \iota_t^*, \tag{1.5}$$

which implies that the interest rate in the SOE is equal to the exogenous foreign (RoW) interest rate.

We finally assume that the money market is continuously in equilibrium, so that we can equate the *supply* of money (1.2) to the *demand* for money (1.3):

$$(1-\theta)\, d_t + \theta r_t = \underbrace{p_t}_{=\overline{s}+p_t^*} + \phi y_t - \lambda \underbrace{\iota_t}_{=\iota_t^*} + \epsilon_t.$$

Rearranging, one obtains:

$$\theta r_t = \underbrace{\overline{s} + p_t^* + \phi y_t - \lambda \iota_t^* + \epsilon_t}_{=m_t^d} - (1-\theta)\, d_t. \tag{1.6}$$

Analysis and Interpretation. Equation (1.6) summarises the main insights of the monetary approach to the balance of payments, or, which is the same, of the monetary model (of the balance of payments) under *peg*. It is evident from the money demand function (1.3) that if the SOE in question experiences any of the following:

- positive income growth, y_t;
- declining interest rates, ι_t;
- rising prices, p_t;

the demand for *nominal* money balances, m_t^d, will grow.

Equation (1.6) further shows that if this increased demand for money is not satisfied by an accommodating increase in domestic credit, d_t, so that $m_t^d > (1-\theta)\, d_t$ in (1.6), the public will obtain the additional money it desires to hold by running a(n overall) BoP *surplus*, i.e., an *increase* in international reserves, r_t; if, on the other hand, the central bank engages in a domestic credit expansion that exceeds the growth of money demand, so that $m_t^d < (1-\theta)\, d_t$ in (1.6), the public will eliminate the excess supply of money (it does not wish to hold) by spending or investing it abroad and thus running a(n overall) BoP *deficit*, i.e., a *decrease* in international reserves. Thus, the *money supply*, $m_t^s \equiv (1-\theta)\, d_t + \theta r_t$, in the monetary model under *peg* is *endogenous*, that is, determined by expression (1.6).

1.2. **The Monetary Approach to the Exchange Rate (Float).** As we said, the monetary approach to the balance of payments was developed in the context of a *fixed* exchange rate regime. As such, it is also referred to as the monetary model of the *balance of payments*.

Changing the assumption of a peg with the alternative of a *flexible* exchange rate regime transforms the above approach, or model, into what has become known as the monetary approach to the *(nominal) exchange rate*, or, which is the same, the *monetary model* (of exchange rate determination).[1]

1.2.1. *The Monetary Model under Float.* Bearing in mind the common underlying framework, it is not surprising that the similarities between the monetary model under *peg* and under *float* are many indeed. However, there are also differences, originating in the alternative assumption about the NER regime: these differences are evident in the expressions for PPP and UIP below, which still hold but in a modified analytical form, as well as in the *pair* of money demand equations, with *identical* parameters for the two countries modelled.
Set-Up and Derivation of Key Results. Under *float*, the *money supply* is *exogenous*. Equilibrium in the *money market* in *Home* and *Foreign* is then given by the pair of equations:

$$\underbrace{m_t - p_t}_{\text{supply of real balances in } H} = \underbrace{\phi y_t - \lambda \iota_t}_{\text{demand for real balances in } H}, \tag{1.7}$$

$$\underbrace{m_t^* - p_t^*}_{\text{supply of real balances in } F} = \underbrace{\phi y_t^* - \lambda \iota_t^*}_{\text{demand for real balances in } F}. \tag{1.8}$$

International *capital market* equilibrium is determined by *UIP*, now allowing for expected depreciation under the *float* regime (i.e., with $\ln \frac{E_t[S_{t+1}]}{S_t} = \ln E_t[S_{t+1}] - \ln S_t \equiv E_t[s_{t+1}] - s_t$):

$$\iota_t - \iota_t^* = E_t[s_{t+1}] - s_t. \tag{1.9}$$

PPP relates the price levels in the two economies through the exchange rate, given that LOP holds for the individual products ensuring equilibrium in the market for *goods* via arbitrage:

$$s_t = p_t - p_t^*. \tag{1.10}$$

A usual simplification of the notation at this point, which allows certain interpretation as to what constitutes the *(economic) fundamentals* of the exchange rate, is:

$$f_t \equiv (m_t - m_t^*) - \phi(y_t - y_t^*). \tag{1.11}$$

Substituting (1.7), (1.8) and then (1.9) and (1.11) into (1.10) and rearranging, we get:

$$s_t = \underbrace{m_t + \phi y_t - \lambda \iota_t}_{=p_t,\text{ from (1.7)}} - \underbrace{m_t^* + \phi y_t^* - \lambda \iota_t^*}_{=p_t^*,\text{ from (1.8)}},$$

$$s_t = \underbrace{(m_t - m_t^*) - \phi(y_t - y_t^*)}_{\equiv f_t,\text{ from (1.11)}} + \lambda \underbrace{(\iota_t - \iota_t^*)}_{=E_t[s_{t+1}]-s_t,\text{ from (1.9)}},$$

$$s_t = f_t + \lambda(E_t[s_{t+1}] - s_t).$$

Now solving for s_t:

$$s_t = f_t + \lambda E_t[s_{t+1}] - \lambda s_t,$$

$$(1+\lambda)s_t = f_t + \lambda E_t[s_{t+1}],$$

[1]Obstfeld and Rogoff (1996), pp. 526-530, also call the monetary model the Cagan model, in honour of an early contribution by Cagan (1956) with application to hyperinflation.

$$s_t = \underbrace{\frac{1}{1+\lambda}}_{\equiv\gamma} f_t + \underbrace{\frac{\lambda}{1+\lambda}}_{\equiv\psi=\lambda\gamma} E_t\left[s_{t+1}\right],$$

$$s_t = \gamma f_t + \psi E_t\left[s_{t+1}\right]. \tag{1.12}$$

(1.12) is the basic first-order stochastic difference equation of the monetary model. Mark (2001) notes that it serves the same function as an "Euler equation" in optimising models. It states that expectation of future values of the exchange rate are embodied in the current exchange rate. From the composition of the fundamental f_t in it, it is clear that high *relative* money growth in *Home*, $m_t - m_t^* > 0$, leads to a depreciation of the *Home* currency, $s_t \uparrow$, while high *relative* income growth, $y_t - y_t^* > 0$, tends to appreciate it, $s_t \downarrow$.
General Forward-Looking Solution. Equation (1.12) can be solved forward for the exchange rate, under an (implicit or explicit) assumption of rational expectations. This is done by first advancing time in (1.12) by one period,

$$s_{t+1} = \gamma f_{t+1} + \psi E_{t+1}\left[s_{t+2}\right],$$

then taking expectations conditional on time t information,

$$E_t\left[s_{t+1}\right] = \gamma E_t\left[f_{t+1}\right] + \psi E_t\left[E_{t+1}\left[s_{t+2}\right]\right],$$

and using the law of iterated expectations, to obtain

$$E_t\left[s_{t+1}\right] = \gamma E_t\left[f_{t+1}\right] + \psi E_t\left[s_{t+2}\right],$$

by which we substitute $E_t\left[s_{t+1}\right]$ back into (1.12) to get

$$s_t = \gamma f_t + \psi\underbrace{\left(\gamma E_t\left[f_{t+1}\right] + \psi E_t\left[s_{t+2}\right]\right)}_{=E_t\left[s_{t+1}\right]}.$$

Further rearranging, one could write:

$$s_t = \gamma E_t\left[f_t\right] + \psi\gamma E_t\left[f_{t+1}\right] + \psi^2 E_t\left[s_{t+2}\right],$$

$$s_t = \gamma\left(E_t\left[f_t\right] + \psi E_t\left[f_{t+1}\right]\right) + \psi^2 E_t\left[s_{t+2}\right],$$

$$s_t = \gamma\sum_{j=0}^{1}\psi^j E_t\left[f_{t+j}\right] + \psi^2 E_t\left[s_{t+2}\right].$$

We can now repeat the same procedure by advancing time in (1.12) by two, three, four periods and so forth, say up to k periods ahead. The result will be similar to the above equation, yet more general:

$$s_t = \gamma\sum_{j=0}^{k}\psi^j E_t\left[f_{t+j}\right] + \psi^{k+1} E_t\left[s_{t+k+1}\right]. \tag{1.13}$$

No-Bubbles Solution. If now we let $k \to \infty$, the second term in (1.13) will vanish, becoming negligible asymptotically: recall that $0 < \psi \equiv \frac{\lambda}{1+\lambda} < 1$ because we have, by definition, restricted the interest semi-elasticity of money demand, λ, to be positive ($\lambda > 0$).

$$\lim_{k\to\infty}\psi^k E_t\left[s_{t+k}\right] = 0. \tag{1.14}$$

Equation (1.14) is called in similar models a *transversality condition* (TVC). By imposing it, we restrict the rate at which the exchange rate can grow asymptotically to obtain the unique *fundamentals* (or *no-bubbles*) solution:

$$s_t = \gamma \sum_{j=0}^{k} \psi^j E_t \left[f_{t+j} \right]. \tag{1.15}$$

(1.15) expresses the current period (t) exchange rate as the *present discounted value* (PDV) of expected future values of the fundamentals. It is thus similar to the present value model in finance, a popular theory of asset pricing, where in place of the NER one has the stock price in the formula and in place of the fundamentals one has the firm's dividends. Because of this similarity with asset pricing theory, the monetary model (under float) is sometimes referred to as the *asset(-market) approach* to the exchange rate. From the perspective of this latter approach, it is natural to expect the exchange rate to be more volatile than fundamentals, just like the prices of assets such as stocks are more volatile than the corresponding dividends.
Rational Bubbles. If the TVC does not hold, the exchange rate behaviour will be governed in part by an (asymptotically) explosive bubble, b_t, which will eventually dominate the fundamentals. To see why, assume that the bubble evolves according to a first-order autoregressive process (AR1), where the autoregressive coefficient (measuring persistence) is defined to be $\frac{1}{\psi}$ and thus exceeds 1 ($\frac{1}{\psi} > 1$ since $0 < \psi < 1$), which means that the bubble process is explosive:

$$b_t = \frac{1}{\psi} b_{t-1} + \xi_t, \tag{1.16}$$

where $\xi_t \overset{iid}{\sim} N\left(0, \sigma_\xi^2\right)$. We can now add the bubble process (1.16) to the fundamentals solution (1.15) to obtain back the general solution of the first-order stochastic equation for the exchange rate we started with, (1.12), but now assuming a bubble:

$$\widehat{s}_t = s_t + b_t. \tag{1.17}$$

One can check that $\widehat{s}_t$ violates the TVC, by simply substituting (1.17) into (1.14):

$$\lim_{k\to\infty} \psi^k E_t \left[\widehat{s}_{t+k}\right] = \underbrace{\lim_{k\to\infty} \psi^k E_t \left[s_{t+k}\right]}_{=0} + \underbrace{\lim_{k\to\infty} \psi^k E_t \left[b_{t+k}\right]}_{=b_t} = b_t \neq 0 \tag{1.18}$$

To understand why $b_t = \lim_{k\to\infty} \psi^k E_t \left[b_{t+k}\right]$ in the expression just above, solve forward (1.16):

$$\psi b_t = b_{t-1} + \xi_t,$$

$$b_{t-1} = \psi b_t - \xi_t.$$

Taking expectation at $t-1$:

$$E_{t-1}\left[b_{t-1}\right] = \psi E_{t-1}\left[b_t\right] - \underbrace{E_{t-1}\left[\xi_t\right]}_{=0},$$

$$b_{t-1} = \psi E_{t-1}\left[b_t\right].$$

Or, which is the same,

$$b_t = \psi E_t\left[b_{t+1}\right].$$

Hence,

$$b_t = \psi E_t \left[\overbrace{\psi E_{t+1}\left[b_{t+2}\right]}^{=b_{t+1}} \right],$$

$$b_t = \psi^2 E_t\left[b_{t+2}\right],$$

and so on, up to lead k:

$$b_t = \psi^{t+k} E_t \left[b_{t+k} \right].$$

Now it is clear that $\lim_{k\to\infty} \psi^{t+k} E_t \left[b_{t+k} \right] = \lim_{k\to\infty} b_t = b_t$.

No matter that it violates the TVC, $\widehat{s}_t = s_t + b_t$ is a solution to the model because it solves (1.12): to check, substitute (1.17) into (1.12):

$$\underbrace{s_t + b_t}_{=\widehat{s}_t} = \gamma f_t + \psi E_t \underbrace{[s_{t+1} + b_{t+1}]}_{=\widehat{s}_{t+1}},$$

$$s_t + b_t = \gamma \underbrace{\frac{s_t - \psi E_t \left[s_{t+1} \right]}{\gamma}}_{=f_t,\text{ from (1.12)}} + \psi E_t \left[s_{t+1} \right] + \psi E_t \underbrace{\left[\frac{1}{\psi} b_t + \xi_{t+1} \right]}_{=b_{t+1},\text{ from (1.16)}},$$

$$s_t + b_t = s_t - \psi E_t \left[s_{t+1} \right] + \psi E_t \left[s_{t+1} \right] + \psi \frac{1}{\psi} b_t,$$

$$s_t + b_t = s_t + b_t.$$

However, even being a solution to (1.12), $\widehat{s}_t = s_t + b_t$ is an *unstable* one, since the bubble will eventually dominate the fundamentals, as we saw in (1.18), and will thus drive the exchange rate arbitrarily far away from them. Because the bubble arises in a model where people are assumed (by working out the forward-looking solutions to the stochastic differential equation for the behaviour of the exchange rate) to hold rational expectations, it is known as a *rational bubble*. It may be the case that forex markets are occasionally driven by bubbles, but real-world experience suggests that such bubbles eventually pop. This is a reason not to focus too much on the solution with a (rational) bubble.

1.2.2. *Analysis and Interpretation.* The monetary model is a useful first approximation in providing intuition about *nominal* exchange rate *dynamics*. We shall consider other, richer dynamic models of the exchange rate in further lectures. In the monetary model, as in the traditional approach to international finance, the exchange rate is the relative price of two national currencies. However, these monies are themselves two *assets*, *stocks* of which are held by economic agents. The forward-looking (rational expectations) PDV solution of the monetary model emphasises exactly this *asset-market* approach to the NER.

A key problem with the monetary model is that it cannot explain the dynamics of the *real* exchange rate. But the reason to start looking into exchange rate determination with the monetary approach is that NER behaviour is considered to be highly correlated to RER movements, so NER variability may have real consequences. The correlation between the NER and the RER is not necessarily high, however, and this depends crucially on price-setting (PCP vs CCP) assumptions under sticky prices in the open economy, topics to which we shall return in more detail in our next lectures.

1.3. **The Portfolio Approach to the Balance of Payments (Peg).** Similarly to the monetary approach, the portfolio approach to the balance of payments has got a peg and a float version. But differently from the monetary approach, it focuses on stock adjustment of assets *other* than, or *in addition to*, money. One such model featuring asset stock adjustment in partial equilibrium is described, for instance, in section 13.2, pp. 186-190, of Gandolfo's textbook. We only present here its main equations and implications.

Three assets are assumed available to agents to hold their wealth in, domestic money, M, domestic bonds, B, and foreign bonds, B^* (under a *fixed* exchange rate normalised at $S = 1$):

$$\mathcal{W} \equiv M^d + B^d + B^{*d}, \tag{1.19}$$

so that the fractions of each stock of asset in the total stock of wealth sum up to 1:

$$1 \equiv \frac{M^d}{\mathcal{W}} + \frac{B^d}{\mathcal{W}} + \frac{B^{*d}}{\mathcal{W}}. \tag{1.20}$$

The choice of a fraction (i.e., a share) of each asset in the *Home* agent wealth portfolio, hence the *demand* for each asset relative to wealth, is assumed to depend on the nominal interest rate in *Home*, ι, and *Foreign*, ι^*, and on domestic real income, y for *Home*:

$$\frac{M^d}{\mathcal{W}} = h(\iota, \iota^*, y), \qquad \frac{B^d}{\mathcal{W}} = g(\iota, \iota^*, y), \qquad \frac{B^{*d}}{\mathcal{W}} = f(\iota, \iota^*, y). \tag{1.21}$$

The three functions are not independent of each other insofar once two of them are known, the third is also determined given the balance constraint in (1.19) or (1.20). Furthermore, the functions are assumed to have certain plausible properties.[2]

Equilibrium in each of the three asset markets is then imposed by equating supply to demand:

$$M^s = \underbrace{h(\iota, \iota^*, y)\,\mathcal{W}}_{M^d}, \qquad B^s = \underbrace{g(\iota, \iota^*, y)\,\mathcal{W}}_{B^d}, \qquad B^{*s} = \underbrace{f(\iota, \iota^*, y)\,\mathcal{W}}_{B^{*d}}. \tag{1.22}$$

Again, when any two of the above equations are satisfied, the third one is necessarily satisfied too. This follows from Walras law, according to which when n markets are connected by a balance constraint, such as (1.19) or (1.20) in the present context, if any $n-1$ of them are in equilibrium, then the n-th is necessarily in equilibrium.

The stock of wealth is the same, no matter whether one views it from the demand side, as in (1.19), or from the supply side, so we write:

$$\mathcal{W} \equiv M^s + B^s + B^{*s}. \tag{1.23}$$

From (1.21), the demand side could also be written:

$$\mathcal{W} = \underbrace{h(\iota, \iota^*, y)\,\mathcal{W}}_{M^d} + \underbrace{g(\iota, \iota^*, y)\,\mathcal{W}}_{B^d} + \underbrace{f(\iota, \iota^*, y)\,\mathcal{W}}_{B^{*d}}. \tag{1.24}$$

Now subtracting (1.24) from (1.23), we obtain the formal statement of Walras law, in terms of the sum of the excess supply/demand of all asset markets:

$$[M^s - h(\iota, \iota^*, y)\,\mathcal{W}] + [B^s - g(\iota, \iota^*, y)\,\mathcal{W}] + [B^{*s} - f(\iota, \iota^*, y)\,\mathcal{W}] = 0. \tag{1.25}$$

Any two equations from the equilibrium conditions, (1.22), plus the wealth constraint viewed from the supply side, (1.23), thus form a system of three equations from which one can determine the three *endogenous* variables, namely the domestic interest rate, ι, the stock of foreign bonds, B^*, and the stock of national money, M, held by residents in *Home*. The four *exogenous* variables are the stock of domestic bonds, B, and y, ι^* and $\mathcal{W}$. The model has a usual *graphical* interpretation (see figures 13.1, p. 188, and 13.2, p. 189, in Gandolfo's textbook: to be discussed in class if time allows).

1.4. **The Portfolio Approach to the Exchange Rate (Float).** In its simplest version, the portfolio approach to the exchange rate, also called the *portfolio balance model* (PBM),[3] can be illustrated within the framework of Frankel (1983), where *bonds* – home and foreign – are the only assets considered.[4]

A small-open economy is assumed, and the assumption is also taken to imply that domestic bonds are held *solely* by residents, as the SOE-issued bonds are of no interest to the rest of the world. The model can be extended to allowing holdings of (domestic) bonds by *nonresidents*, which will not change the essence of results provided that the residents of any country hold a *bigger* proportion of *domestic* bonds in their portfolio, i.e., under the so-called hypothesis of *preferred local habitat* ensuring the empirically relevant *home bias* in bond holdings.

If *perfect substitutability* existed between the available assets, home and foreign bonds here, then *UIP* should hold, i.e., $\iota = \iota^* + \frac{E[\Delta S]}{S}$, with $\frac{E[\Delta S]}{S}$ denoting the expected change of the

[2]See Gandolfo (2001), pp. 186-187.

[3]For a compact treatment, see section 4.1.5 in the book by Sarno and Taylor (2002), pp. 115-123.

[4]Introducing *money* would not substantially alter the results, as noted by Gandolfo (2001), p. 233.

nominal exchange rate over a given time interval. But with *imperfect substitutability*, there should be a *divergence* between ι and $\iota^* + \frac{E[\Delta S]}{S}$, which will, in fact, determine the allocation of wealth, ceteris paribus.

Given the above simplifying assumptions, the wealth allocation constraint for the domestic representative agent of the SOE can be written as:

$$\mathcal{W} \equiv B^d + SB^{*d}, \tag{1.26}$$

where the demands for home bonds, B^d, and foreign bonds, B^{*d}, are expressed (according to portfolio choice theory in the present context) as:

$$B^d = g\left(\iota - \iota^* - \frac{E[\Delta S]}{S}\right)\mathcal{W},$$
$$SB^{*d} = f\left(\iota - \iota^* - \frac{E[\Delta S]}{S}\right)\mathcal{W}.$$

Note that $g(...) + f(...) = 1$ because of (1.26). Now imposing equilibrium in both asset markets, we get:

$$B^d = B^s, \qquad B^{*d} = B^{*s}.$$

One can then substitute the supply of bonds in the respective demand functions above and then divide the resulting foreign bond equilibrium expression by the domestic bond equilibrium expression, which yields:

$$\frac{SB^{*s}}{B^s} = \underbrace{\frac{f\left(\iota - \iota^* - \frac{E[\Delta S]}{S}\right)}{g\left(\iota - \iota^* - \frac{E[\Delta S]}{S}\right)}}_{\equiv \varphi\left(\iota - \iota^* - \frac{E[\Delta S]}{S}\right)}.$$

Hence, one obtains the equilibrium exchange rate in the context of the (simple) portfolio approach to NER determination, summarised in the present subsection:

$$S = \frac{B^s}{B^{*s}}\varphi\left(\iota - \iota^* - \frac{E[\Delta S]}{S}\right). \tag{1.27}$$

Equation (1.27) presents the nominal exchange rate as the relative price of two stocks of *assets*. It is determined by the relative quantities of domestic and foreign bonds, for any given interest rate differential corrected for expectations about exchange-rate variability. The basic idea behind the above key equation of the portfolio approach to exchange rate determination is that the NER adjusts instantaneously so as to keep international asset markets in equilibrium. The main limitations of the simplified PE model we sketched are that it does not go deeper into what determines the interest rate differential, nor it considers possible interactions between the current account and the capital account.

2. Stock-Flow Approaches to BoP Adjustment and NER Determination

The stock(-flow) approaches to the BoP of the late 1960s and early 1970s essentially extended the Tobin-Markowitz theory of *portfolio* equilibrium to the *open* economy. A key point in them is that international capital movements are no longer considered as pure *flows* but as *deriving* from underlying adjustment of *stocks* of assets. In this line of literature, variations of the standard *portfolio problem* we introduced in lecture 1 are modelled either in *partial* equilibrium (PE), where macrovariables such as national income and the current account are *exogenous* (that is, given by certain assumptions); or in *general* equilibrium (GE), where the main macroeconomic aggregates are *endogenous* (that is, determined within the particular model).

2.1. **Asset Stock Adjustment in Partial Equilibrium.**

Due to time limitations and for a better focus of our course, we would not discuss here the relevant models. Only initial references are provided below for those interested.

2.1.1. *Asset Stock Adjustment in Partial Equilibrium under Peg.* Following Gandolfo (2001), we already illustrated key ideas in the simpler version of the framework of stock(-flow) adjustment assuming *fixed* exchange rates in subsection 1.3.

2.1.2. *Asset Stock Adjustment in Partial Equilibrium under Float.* For the case of a *flexible* exchange rate regime, a good reference is Branson and Henderson (1985).

2.2. **Portfolio and Macroeconomic (General) Equilibrium.** Again, due to time limitations and for a better focus of the course, we would not discuss the models in question. The interested reader may explore them alone, starting from the references provided below.

2.2.1. *Portfolio and Macroeconomic Equilibrium under Peg.* A simpler version of the general equilibrium framework of stock-flow adjustment assuming a *peg* regime is sketched, following O'Connell (1984), in section 13.3, pp. 191-197, in Gandolfo (2001).

2.2.2. *Portfolio and Macroeconomic Equilibrium under Float.* A version of the same type of framework under *float*, based on Branson and Buiter (1983), is summarised in Gandolfo's textbook, section 13.4, pp. 197-209.

References

[1] Branson, William and Wilhem Buiter (1983), "Monetary and Fiscal Policy and Flexible Exchange Rates", chapter 9 in Bhandari, J. S. and Putnam, B. H. (eds.), *Inflation and Employment in Open Economies*, Amsterdam: North Holland.

[2] Branson, William and Dale Henderson (1985), "The Specification and Influence of Asset Markets", chapter 15 in Jones, R. W. and Kenen, P. B. (eds.), *Handbook of International Economics*, Vol. II, Amsterdam: North Holland.

[3] Cagan, Phillip (1956), "The Monetary Dynamics of Hyperinflation", in Friedman, M. (ed.), *Studies in the Quantity Theory of Money*, Chicago: University of Chicago Press.

[4] Cassel, Gustav (1918), "Abnormal Deviations in International Exchanges", *Economic Journal* 28, 413-415.

[5] Cassel, Gustav (1921), *The World's Monetary Problems*, London: Constable.

[6] Frankel, Jeffrey (1983), "Monetary and Portfolio Models of Exchange Rate Determination", in Bhandari, J. S. and Putnam, B. H. (eds.), *Economic Interdependence and Flexible Exchange Rates*, Cambridge (Mass.): MIT Press, 84-115.

[7] Frenkel, Jacob A. and H. G. Johnson (1976), *The Monetary Approach to the Balance of Payments*, London: Allen & Unwin.

[8] Gandolfo, Giancarlo (2001), *International Finance and Open-Economy Macroeconomics*", Springer.

[9] Hume, David (1752), "Of the Balance of Trade", in Hume, D., *Political Discourses*, Edinburgh; reprinted in Hume, D. (1955), *Writings on Economics*, edited by Rotwein, E., London: Nelson, 60ff.

[10] Mark, Nelson (2001), *International Macroeconomics and Finance: Theory and Econometric Methods*, Blackwell.

[11] O'Connell, J. (1984), "Stock Adjustment and the Balance of Payments", *Economic Notes* 1, 136-144.

[12] Samuelson, Paul A. (1964), "Theoretical Notes on Trade Problems", *Review of Economics and Statistics* 46, 145-154.

[13] Sarno, Lucio and Mark Taylor (2002), *The Economics of Exchange Rates*, Cambridge University Press.

Department of Economics, University of Essex, Wivenhoe Park, Colchester CO4 3SQ, UK

E-mail address: `mihailov@essex.ac.uk`

URL: `http://www.essex.ac.uk/economics/people/staff/mihailov.shtm`