EC933-G-AU – Lecture 4

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The Intertemporal Approach to the Current Account: Analytical Introduction of Time

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Plan of talk

- **introduction**

1. **simple** *SOE* 2-period model: *exo*genous RIR
2. **extended** *SOE* 2-period model: adding *f(k), i, g*
3. back to the *current account* as **intERtemporal trade**
4. **simple** *2-country* 2-period model: *endo*genous RIR
5. **elasticity of intERtemporal substitution** (in *consumption*)
6. **saving** and **RIR**: *substitution*, *income* and *wealth* effects

- **wrap-up**

Aim and learning outcomes

- **aim:** start introducing the microfoundations of open-economy macromodels by focusing on the *analytics* of their **time** dimension
- **learning outcomes**
  - derive and interpret in *partial* and/or *general* equilibrium
    - standard *intertemporal* Euler equations
    - the effects of *temporary* vs *permanent* output shocks on RIR and CA
    - the elasticity of intertemporal substitution (in consumption)
    - the *equilibrium* real interest rate
    - the effects – *substitution*, *income* and *wealth* – of RIR changes on saving/consumption and CA under constant EIS and isoelastic period utility
  - analyse the basic set-up of microfounded *dynamic* OEMs

Simple 2-period SOE real model: assumptions

1. 2 countries, SOE (*H*) and RoW (*F*)
2. that last for 2 periods, labelled *1* and *2*
3. a single, perishable (i.e. nonstorable) good available to consume
4. no production (function), i.e. an endowment model
5. no investment
6. no government spending
7. no money, i.e. real model
8. SOE takes the real interest rate (RIR), *r*, the only (relative) price in the model as given, i.e. RIR is *exogenous*

- **objective:** to understand *how a country can gain from intER-temporal trade* (not intRAtemporal trade), rearranging the timing of its consumption through international borrowing and lending

2-period SOE real model: utility

- **lifetime /intertemporal/ utility**, for *individual j*

$$U_l^j \equiv u\left(c_1^j\right) + \beta u\left(c_2^j\right), \qquad 0 < \beta < 1 \qquad \beta \equiv \tfrac{1}{1+\delta}$$

- comprised of *invariant*, *increasing* and *concave* **period utility**

$$u_1(\bullet) = u_2(\bullet) = u(\bullet) \qquad u'(c^j) \equiv \frac{du\left(c^j\right)}{dc^j} > 0$$

$$u''(c^j) \equiv \frac{du'\left(c^j\right)}{dc^j} = \frac{d\frac{du\left(c^j\right)}{dc^j}}{dc^j} = \frac{d\left[du\left(c^j\right)\right]}{dc^j dc^j} = \underbrace{\frac{d^2u\left(c^j\right)}{d\left(c^j\right)^2}}_{\text{usual notation}} < 0$$

$$\lim_{c^j \to 0} u'(c^j) = \infty$$

Simple 2-period SOE real model: consumer's problem and its FONCs

- objective $\max_{c_1^j, c_2^j} U_l^j \equiv \max_{c_1^j, c_2^j} u\left(c_1^j\right) + \beta u\left(c_2^j\right)$
- PV *lifetime /intertemporal/* budget constraint $c_1^j + \frac{c_2^j}{1+r} = y_1^j + \frac{y_2^j}{1+r}$
- Lagrangian $\mathcal{L}\left(c_1^j, c_2^j; \lambda\right) \equiv \underbrace{u\left(c_1^j\right) + \beta u\left(c_2^j\right)}_{\text{objective}} + \lambda \underbrace{\left(y_1^j + \frac{y_2^j}{1+r} - c_1^j - \frac{c_2^j}{1+r}\right)}_{\text{constraint}}$
- FONCs (for local extremum)

$$\frac{\partial \mathcal{L}\left(c_1^j, c_2^j; \lambda\right)}{\partial c_1^j} = 0 \Rightarrow u'\left(c_1^j\right) = \lambda \qquad \frac{\partial \mathcal{L}\left(c_1^j, c_2^j; \lambda\right)}{\partial c_2^j} = 0 \Rightarrow \beta u'\left(c_2^j\right) = \frac{\lambda}{1+r}$$

$$\frac{\partial \mathcal{L}\left(c_1^j, c_2^j; \lambda\right)}{\partial \lambda} = 0 \Rightarrow c_1^j + \frac{c_2^j}{1+r} = y_1^j + \frac{y_2^j}{1+r}$$

Simple 2-period SOE real model: Euler equation and consumption smoothing

- *intertemporal* **Euler equation**, also written as *relative* price

$$u'\left(c_1^j\right) = (1+r)\beta u'\left(c_2^j\right) \qquad \underbrace{\frac{\beta u'\left(c_2^j\right)}{u'\left(c_1^j\right)}}_{\text{MRS in consumption}} = \underbrace{\frac{1}{1+r}}_{\text{market discount factor}} \equiv p$$

- *special* case: **consumption smoothing** equilibrium

$$\text{if } \delta = r \text{, then } \beta \equiv \frac{1}{1+\delta} = \frac{1}{1+r} \equiv p \text{, i.e. } \beta = \frac{1}{1+r}$$

$$u'\left(c_1^j\right) = u'\left(c_2^j\right) \qquad c_1^j = c_2^j = \overline{c}^j = const$$

$$\overline{c}^j + \frac{\overline{c}^j}{1+r} = y_1^j + \frac{y_2^j}{1+r} \qquad \overline{c}^j = \frac{(1+r)y_1^j + y_2^j}{2+r}$$

Simple 2-period SOE real model: reinterpretation of the current account

$$CA_t \equiv \Delta NFA_t \equiv \underbrace{b_{t+1} - b_t}_{\text{national (dis)saving}}$$

$$CA_t \equiv \Delta NFA_t \equiv \underbrace{\overbrace{\underbrace{y_t}_{\equiv \text{GDP (or national } \textit{output}\text{)}} + r_t b_t}^{\equiv \text{GNP (or national } \textit{income}\text{)}} - \overbrace{c_t}^{\equiv \text{domestic absorption}}}_{\equiv \text{national (dis)saving}}$$

- combining definitions

$$CA_t \equiv \Delta NFA_t \equiv b_{t+1} - b_t = y_t + r_t b_t - c_t$$

- 2-period SOE: Fig. 1.1, p. 8, Obstfeld and Rogoff (1996)

$$CA_2 = -CA_1 \qquad \underbrace{CA_1 + CA_2}_{\textit{cumulative}\text{ CA}} \equiv \underbrace{b_3 - b_1}_{\textit{change}\text{ in NFA}} = \underbrace{0}_{\textit{if } b_1 = b_3 = 0}$$

Simple 2-period SOE real model: temporary vs permanent shocks, RIR and CA

- *autarky* RIR and *autarky* relative price of consumption across time

$$\frac{\beta u'(y_2)}{u'(y_1)} = \frac{1}{1+r_A} \equiv p_A$$

- **gains** from *intERtemporal* trade when *autarky* RIR $\neq$ *world* RIR
  - *source* of gains: differences across countries, reflected in the deviation of autarky RIRs from the world RIR
  - *magnitude* of gains: the bigger the difference, the greater the gains
- effects on CA of temporary vs permanent shocks on endowments (or output, or income): the *special* case of $\delta = r$, i.e. $\beta = 1/(1+r)$, as a *benchmark* =>

$$\frac{u'(y_2)}{u'(y_1)} = \frac{1+r}{1+r_A}$$

- now evident that $r_A \neq r \Leftrightarrow y_1 \neq y_2$
- *initial* expectation: $y_1 = y_2 = \overline{y} = const$
  - **temporary** *positive* shock: $y_1 > \overline{y}$ but $y_2 = \overline{y}$ => $r_A \downarrow$ => CA**S** in period *1*
  - **permanent** *positive* shock: $y_1 = y_2 > \overline{y}$ => r_A = const => CA does *not* change

Adding (1) production, (2) investment and (3) government spending to the simple SOE model => Fig. 1.3, p. 20, O-R

$$y \equiv F(k) \qquad y \equiv F(k, \overline{n}) \qquad k_{t+1} = k_t + i_t$$

budget line: $c_2 = y_2 - i_2 - g_2 - (1+r)(c_1 + i_1 + g_1 - y_1)$ slope of budget line: $\frac{\partial c_2}{\partial c_1} = -(1+r)$

PPF: $$c_2 = \underbrace{F\left[\overbrace{k_1 + \overbrace{F(k_1) - c_1 - g_1}^{=i_1}}^{=k_2}\right]}_{=y_2} + \overbrace{\overbrace{k_1}^{\text{inherited capital}} + \overbrace{\underbrace{F(k_1)}_{=y_1} - c_1 - g_1}^{=i_1}}^{=k_2}$$

slope of PPF: $$\frac{\partial c_2}{\partial c_1} = F'[k_1 + F(k_1) - c_1 - g_1][k_1 + F(k_1) - c_1 - g_1]' - 1 =$$

$$= F' \underbrace{[k_1 + F(k_1) - c_1 - g_1]}_{=k_2} (-1) - 1 = -[1 + F'(k_2)]$$

Extended 2-period SOE real model: the current account as saving less investment

- deriving CA from the transition equation for **wealth** (*financial* and *physical*)

$$\Delta \mathcal{W}_{t+1} \equiv b_{t+1} + k_{t+1} - (b_t + k_t) = y_t + rb_t - c_t - g_t$$

$$CA_t \equiv b_{t+1} - b_t = \underbrace{\underbrace{\underbrace{y_t}_{\equiv GDP_t} + rb_t}_{\equiv GNP_t} - c_t - g_t}_{\equiv s_t} - \underbrace{(k_{t+1} - k_t)}_{\equiv i_t}$$

- the *interpretation* of CA as the difference b/n saving and investment emphasises that it is fundamentally an **intertemporal phenomenon**

Extended 2-period SOE real model: deriving the intertemporal budget constraint

- to derive the intertemporal BC in the *extended* model, write CA in both periods

$$CA_1 \equiv b_2 - \underbrace{b_1}_{=0} = y_1 + r\underbrace{b_1}_{=0} - c_1 - g_1 - i_1 = y_1 - c_1 - g_1 - i_1$$

$$CA_2 \equiv \underbrace{b_3}_{=0} - b_2 = y_2 + rb_2 - c_2 - g_2 - i_2$$

- solve 2[nd] equation for b_2 and substitute back into 1[st] equation

$$\underbrace{\frac{-y_2+c_2+g_2+i_2}{1+r}}_{b_2} = y_1 - c_1 - g_1 - i_1 \qquad c_1 + i_1 + \frac{c_2+i_2}{1+r} = y_1 - g_1 + \frac{y_2-g_2}{1+r}$$

- to *simplify* further, assume $i_2 = \underbrace{k_3}_{=0} - k_2 = -k_2$ (which is *natural* in this model with *terminal* period 2)

Extended 2-period SOE real model: optimisation problem, FONCs and interpretation

- using the intertemporal BC to eliminate c_2 from U_1 transforms representative consumer's **optimisation problem** into

$$\max_{c_1, i_1} \quad u(c_1) + \beta u\left\{ (1+r)[F(k_1) - c_1 - g_1 - i_1] + F\underbrace{(i_1 + k_1)}_{=k_2} - g_2 + \underbrace{i_1 + k_1}_{=k_2 = -i_2} \right\}$$

- k_1 is *given*, by history, so it is not subject to choice on date 1
- the **two** corresponding **FONCs** are then (1) the Euler equation w.r.t. c_1 we saw in the beginning and (2) the FONC w.r.t. i_1 below:

$$\frac{\partial U_1}{\partial i_1} = \beta u'(c_2) \cdot [(1+r)\cdot(-1) + F'(k_2)\cdot 1 + 1] = 0, \text{ hence } F'(k_2) = r$$

- **interpretation** (given *separation* of investment from consumption decisions, satisfied under (1) *S*OE, (2) single *tradable* good, (3) *perfect* capital market)
  - usual (*closed*-economy): ***MPK* = *RIR***, in *equilibrium*
  - in the context of the present (*open*-economy) model: **investment** should continue to the point at which its *marginal return* **equalises** that of the **foreign loan**

Simple 2-period 2-country global economy real model: RIR endogeneity in general equilibrium

- set-up: now 2 **large** economies, *H* and *F* (instead of SOE-RoW)
- **objective**: *how the world RIR* (taken as *exogenous* up to here) *is endogenously determined in general equilibrium*
- **abstract from** (1) production, (2) investment and (3) government spending
- **impose** *parallel (symmetric)* structure on the two countries
- equilibrium in the *global* **output** (or rather endowment) *market* requires equal supply and demand on each date *t*:

$$\underbrace{y_t + y_t^*}_{\text{supply}} = \underbrace{c_t + c_t^*}_{\text{demand}} \qquad y_t + y_t^* - c_t - c_t^* = 0 \qquad s_t + s_t^* = 0 \qquad CA_t + CA_t^* = 0$$

- using **Walras law**, reduce the two *interdependent* markets – for output *today* and output *tomorrow* – to **one market**
  - **Fig. 1.5, p. 24, in O-R** shows how the **equilibrium RIR** is determined for *given* present and future endowments
  - **key lesson** is: $r_A < r < r_A^*$

Elasticity of intertemporal substitution (I)

- start from *across-date Euler equation* and **take natural logs**

$$\ln\left[\frac{\beta u'(c_2)}{u'(c_1)}\right] = \ln\left[\frac{1}{1+r}\right] \qquad \ln(1+r) = \ln u'(c_1) - \ln u'(c_2) - \underbrace{\ln\beta}_{=const}$$

- **totally differentiate** result

$$\frac{d\ln(1+r)}{d(1+r)} d(1+r) = \frac{d\ln u'(c_1)}{dc_1} dc_1 - \frac{d\ln u'(c_2)}{dc_2} dc_2$$

$$\frac{1}{1+r}\underbrace{\frac{1+r}{1+r}}_{=1} d(1+r) = \frac{u''(c_1)}{u'(c_1)}\underbrace{\frac{c_1}{c_1}}_{=1} dc_1 - \frac{u''(c_2)}{u'(c_2)}\underbrace{\frac{c_2}{c_2}}_{=1} dc_2$$

$$d\ln(1+r) = \frac{c_1 u''(c_1)}{u'(c_1)} d\ln c_1 - \frac{c_2 u''(c_2)}{u'(c_2)} d\ln c_2$$

Elasticity of intertemporal substitution (II)

- define the **elasticity of the marginal utility of consumption**

$$\varepsilon_{u'(c)} \equiv -\frac{\frac{du'(c)}{u'(c)}}{\frac{dc}{c}} = -\frac{du'(c)}{u'(c)}\frac{c}{dc} = -\underbrace{\frac{du'(c)}{dc}}_{\equiv u''(c)}\frac{c}{u'(c)} = -\frac{cu''(c)}{u'(c)}$$

- define **its inverse** as the **elasticity of intertemporal substitution**

$$\sigma(c) \equiv \frac{1}{\varepsilon_{u'(c)}} \equiv -\frac{u'(c)}{cu''(c)} \qquad \sigma(c) = \sigma = const$$

- when EIS is constant, the last equation on previous slide becomes

$$d\ln(1+r) = \underbrace{\frac{c_1 u''(c_1)}{u'(c_1)}}_{\equiv -\frac{1}{\sigma}} d\ln c_1 - \underbrace{\frac{c_2 u''(c_2)}{u'(c_2)}}_{\equiv \frac{1}{\sigma}} d\ln c_2$$

$$d\ln\left(\frac{c_2}{c_1}\right) = \sigma d\ln(1+r)$$

- *high* σ ⇔ *high* **sensitivity** of *relative* consumption to RIR *change*

RIR and saving/consumption decisions: (1) substitution, (2) income and (3) wealth effects

- EIS *constant* in the special case of the **isoelastic** class of *period* **utility** functions

$$u(c) = \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \quad \sigma > 0$$

- for isoelastic utility the Euler equation $u'(c_1) = (1+r)\beta u'(c_2)$ reduces to

$$c_1^{-\frac{1}{\sigma}} = (1+r)\beta c_2^{-\frac{1}{\sigma}} \qquad\qquad (1+r)^{\sigma}\beta^{\sigma} c_1 = c_2$$

- now susbtitute c_2 into the intertemporal BC

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \qquad\qquad c_1 + \frac{(1+r)^{\sigma}\beta^{\sigma} c_1}{1+r} = y_1 + \frac{y_2}{1+r}$$

- and solve for c_1: $c_1 = \frac{1}{1+(1+r)^{\sigma-1}\beta^{\sigma}}\left(y_1 + \frac{y_2}{1+r}\right)$ => effects from change in RIR
  - when $\sigma > 1$ **substitution effect** dominates: people willing to substitute consumption
  - when $\sigma < 1$ **income effect** dominates; when $\sigma = 1$ (**log**-consumption) **no** RIR effect
  - **wealth effect**: comes from the change in lifetime income, $y_1 + \frac{y_2}{1+r}$
  - theory: no clear prediction about how a change in RIR will affect consumption/saving

Concluding wrap-up

- **What have we learnt?**
  - distinguish b/n OEMs with *exogenous* vs *endogenous* RIR
  - derive and interpret
    - standard *intertemporal* Euler equations
    - the effects of *temporary* vs *permanent* output shocks on RIR and CA
    - the elasticity of intertemporal substitution (in consumption)
    - the effects – *substitution*, *income* and *wealth* – of RIR changes on saving/consumption and CA under *constant* EIS and *isoelastic* utility
  - summarise/analyse the *usual* set-up of microfounded OEMs
- **Where we go next**: to the *basics* of modelling **uncertainty**, in the framework we have started to develop