Univesrity of Essex
Department of Economics

EC248-2-SP: Monetary Innovations and Central Banks, 2005-06

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CLASS 4 (8 FEBRUARY 2006): PROBLEM SET – SKETCH OF SOLUTIONS

Multiple Choice / Simple Computation Questions

3) With an interest rate of 5 percent, the present value of \$100 next year is approximately (Mishkin, Chapter 4)
(a) \$100.
(b) \$105.
(c) \$95.
(d) \$90.
Answer: (c).

$$PV = \frac{\overbrace{CF}^{\equiv FV}}{1+i} = \frac{100}{1+0.05} = \frac{100}{1.05} = \frac{10000}{105} = 95.238 \approx 95.$$

4) With an interest rate of 10 percent, the present value of a security that pays \$1,100 next year and \$1,460 four years from now is (Mishkin, Chapter 4)
(a) \$1,000.
(b) \$2,560.
(c) \$3,000.
(d) \$2,000.
Answer: (d).
In *general*, the formula to apply is

$$PV = \overbrace{\sum_{t=1}^{T} \frac{CF_t}{(1+i)^t}}^{\equiv FV} = \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + ... + \frac{CF_{T-1}}{(1+i)^{T-1}} + \frac{CF_T}{(1+i)^T}. \tag{1}$$

In our *special* case here, it reduces to

$$\begin{aligned} PV &= \sum_{t=1}^{4} \frac{CF_t}{(1+i)^t} = \frac{CF_1}{(1+i)^1} + \frac{0}{(1+i)^2} + \frac{0}{(1+i)^3} + \frac{CF_4}{(1+i)^4} = \\ &= \frac{1100}{(1+0.1)^1} + \frac{0}{(1+0.1)^2} + \frac{0}{(1+0.1)^3} + \frac{1460}{(1+0.1)^4} = \\ &= \frac{1100}{(1+0.1)^1} + \frac{1460}{(1+0.1)^4} = \frac{1100}{1.1} + \frac{1460}{1.1^4} \approx \\ &\approx \frac{11000}{11} + \frac{1460}{1.4641} \approx 1000 + \frac{146000}{146} \approx 1000 + 1000 \approx 2000. \end{aligned} \tag{2}$$

5) If a security pays \$110 next year and \$121 the year after that, what is its yield to maturity if it sells for \$200? (Mishkin, Chapter 4)
(a) 9 percent.
(b) 10 percent.
(c) 11 percent.
(d) 12 percent.
Answer: (b).

$$200 = \sum_{t=1}^{T} \frac{CF_t}{(1+i)^t} = \sum_{t=1}^{2} \frac{CF_t}{(1+i)^t} = \frac{110}{(1+i)^1} + \frac{121}{(1+i)^2}$$

At this stage, using discount tables or a calculator or computer *programme* , one will be able to solve for i. Otherwise the algebra is long and boring, and can be enourmously time-consuming and difficult, the more so for many periods of cash flows. We illustrate that "pencil and paper" solution for the simple special case here in what follows.

$$200 = \frac{(1+i)\,110}{(1+i)^2} + \frac{121}{(1+i)^2}$$

$$200 = \frac{(110+110i)+121}{(1+i)^2}$$

$$200\,(1+i)^2 = 110 + 110i + 121$$

$$200\left(1+2i+i^2\right) = 231 + 110i$$

$$200 + 400i + 200i^2 = 231 + 110i$$

$$200i^2 + 290i - 31 = 0 \tag{3}$$

From here, using the formula for the solution of a general quadratic equation,

$$ax^2 + bx + c = 0 \tag{4}$$

one can obtain

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{5}$$

and thus

$$i_{1,2} = \frac{-290 \pm \sqrt{290^2 - 4 \times 200 \times (-31)}}{2 \times 200}$$

The interest rate is bounded to be *positive*, by definition, so we would consider only the positive of the two roots,

$$i = \frac{-290 + \sqrt{290^2 - 4 \times 200 \times (-31)}}{2 \times 200}$$

$$i = \frac{-290 + \sqrt{84100 + 24800}}{400}$$

$$i = \frac{-290 + \sqrt{108900}}{400}$$

$$i = \frac{-290 + 330}{400}$$

$$i = \frac{40}{400} = \frac{1}{10} = 0.1, \text{ i.e., } 10\%.$$

Alternatively, we can transform (3) or (4) – by dividing them through by 200 and a, respectively – into a *normalised* version with the first coefficient becoming 1 (instead of 200 and a). For (3), one could thus write

$$i^2 + \frac{290}{200}i - \frac{31}{200} = 0$$

$$i^2 + 1.45i - 0.155 = 0 \tag{6}$$

The solutions for the *normalised* general quadratic equation

$$x^2 + px + q = 0 \tag{7}$$

are then given by

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \tag{8}$$

that is, by

$$i_{1,2} = -\frac{1.45}{2} \pm \sqrt{\left(\frac{1.45}{2}\right)^2 - (-0.155)}$$

in our special case. The formula (8) is computationally prefereable – in absence of a calculator or computer – to formula (5) when b is a whole, pair number. Again, we take the *positive* root only

$$i = -\frac{1.45}{2} + \sqrt{\left(\frac{1.45}{2}\right)^2 - (-0.155)}$$

$$i = -0.725 + \sqrt{(0.725)^2 + 0.155}$$

$$i = -0.725 + \sqrt{0.52563 + 0.155}$$

$$i = -0.725 + \sqrt{0.68063}$$

$$i = -0.725 + 0.825 = 0.1, \text{ i.e., } 10\%.$$

6) If a two-year simple loan of \$1000 at 10 percent interest, the amount payable in two years is (Mishkin, Chapter 4)
(a) \$1010.
(b) \$1100.
(c) \$1121.
(d) \$1200.
(e) \$1210.
Answer: (e).

$$1000 = \frac{CF_2}{(1+0.1)^2}$$

$$1000 \times 1.1^2 = CF_2$$

$$1000 \times 1.21 = CF_2$$

$$CF_2 = 1210 = \text{ principal } + \text{ interest}$$

7) If \$1102.50 is the amount payable in two years for a \$1000 simple loan made today, the interest rate is (Mishkin, Chapter 4)
(a) 2.5 percent.
(b) 5 percent.
(c) 10 percent.
(d) 12.5 percent.
(e) 20 percent.
Answer: (b).

$$1000 = \frac{1102.5}{(1+i)^2} = \text{... discount tables / calculator / computer ...} = 0.05, \text{ i.e., } 5\%.$$

8) If the amount payable in two years is \$2420 for a simple loan at 10 percent interest, the loan amount is (Mishkin, Chapter 4)
(a) \$1000.
(b) \$1210.
(c) \$2000.
(d) \$2200.
(e) \$2400.
Answer: (c).

$$PV = \frac{2420}{(1+0.1)^2} = \frac{2420}{1.1^2} = \frac{2420}{1.21} = \frac{242000}{121} = 2000 = \text{ principal.}$$