Classical Computation of NeutraCHAMP Energy Loss due to Polarization by ^{14}N

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15 March 1991

Abstract

Recently, several theoretical and experimental studies have been completed concerning the existence of a charged massive particle (CHAMP) as the missing matter of the universe. In particular, if a negatively charged CHAMP, X^-, is neutralized by a proton, then the neutraCHAMP can remain unobserved, except to the most astute observer. In order to observe better, researchers need a better estimate of the neutraCHAMP energy loss . In the classical approximation, we calculate the polarization energy loss of the neutraCHAMP, X^-p, in ^{14}N, as a function of velocity. We can approximate quite well the attractive $(^{14}N, X^-p)$ potential, by neglecting electronic and protonic screening of the X^- from the ^{14}N nucleus, and retaining only the r^{-4} term. Therefore, due to the power-law form of the potential, the scattering angle equation can be integrated exactly, giving us analytical results for the variation of scattering angle, $\theta(b)$, and energy transfer, $T(b)$, with impact parameter, b. We show that the energy loss in ^{14}N is proportional to velocity: $dE/\rho dx = 0.84\ v/v_0\ MeVcm^2/g$, where $v_0 = 300km/s$ is the velocity dispersion of the hypothetical galactic X^-p ensemble. We also include protonic screening, by introducing a cutoff to the r^{-4} potential, as $(r^2 + a^2)^{-2}$, where the cutoff parameter is given by $a = \sqrt{1.5}a_p$, where $a_p = 28.8fm$ is the Bohr radius of the neutraCHAMP. For the velocities of interest, $v \leq 1000km/s$, the energy loss is still proportional to velocity: $dE/\rho dx = 0.14\ v/v_0\ MeVcm^2/g$. We discuss the impact parameter limitations on our classical calculation of energy loss. This calculation of the neutraCHAMP energy loss resides within the rough lower and upper limits set by previous authors, and gives a more definitive estimate of the detectability of X^-p in cosmic ray experiments.

De Rújula, Glashow and Sarid [?] speculate that charged massive particles, euphemistically called CHAMPs, close the universe, and comprise dark galactic haloes. CHAMPs come in two forms, $X^{\pm}$, with a possible mass range of $M_x \in [10^4 GeV, 10^6 GeV]$. If we assume that CHAMPs are the only dark matter component, then we can calculate a local number density for X's: $n_x \approx 9 m_p n_p / m_x$, where $n_p \approx 0.24\, cm^{-3}$. With this number density and assuming a Maxwellian

velocity distribution, with velocity dispersion of the order of the stellar velocities in the solar neighborhood of $v_0 \approx 300 km/s$, we can calculate the mean flux through a stationary surface [?], as

$$\Phi = \frac{n_x v_0}{2\sqrt{\pi}} \approx 2 \frac{10^6 M_p}{M_x} cm^{-2} s^{-1}.$$

CHAMPs can also be neutralized by binding to oppositely charged particles, e.g. $X^+ e^-$ and $X^- p$. $X^- p$ has a tightly-bound ground state, $E_0 = 25 keV$, $r_{xp} = 28.8 fm$, and hence interacts relatively weakly with other particles and photons. Therefore, since $X^- p$ systems interact relatively weakly and move relatively slowly (as compared to neutrinoes), they can constitute cold dark matter, and are dubbed "neutraCHAMPs."

Since the sun-earth system moves at the same speed as the velocity dispersion of the galactic neutraCHAMP ensemble, we might expect, in a satellite-based detector, to see neutraCHAMPs with speeds within the approximate interval $v \in (0, 300 km/s]$, with strong sidereal modulation. Ground- and balloon-based neutraCHAMP detection experiments have been completed or are currently in progress [?]. Experiments in the atmosphere require a good estimate of the rate of energy loss and range of neutraCHAMPs, in order to give experimentalists a good idea of what $X^- p$ flux and velocity distribution they can expect, given their atmospheric depth. Even in satellite experiments [?], the rate of energy loss needs to be known well, in order to optimize detector response.

Previous authors [?, ?, ?] only compute *estimates* of the upper and lower bounds of the energy loss of $X^- p$ in the atmosphere. Estimates of upper and lower bounds of the energy loss are of some use in speculating whether or not neutraCHAMPs can be detected as cosmic rays, but experimentalists desire a more refined calculation of the energy loss, if they hope to plan and carry out experiments in the atmosphere, giving definitive results.

1 Basic Equations

The energy loss of a neutraCHAMP ,$X^- p$, traversing pure ^{14}N, as a function of velocity will be calculated, and the result will be scaled for other light elements. Since the velocity of the collision between the $X^- p$ and a nitrogen atom is much less than the orbital velocities of the electrons in the nitrogen electron cloud ($v_e \approx Z\alpha c$), and also much less than the velocity of the neutraCHAMP proton ($v_p \approx \alpha c$), the colliding $X^- p$ only slightly disturbs the electron cloud of nitrogen atom. Since in this adiabatic approximation, no quantum transitions can occur, we can take the *undisturbed* charge distribution of the nitrogen atom as the electric field source. The electric field due to the nitrogen atom then polarizes the $X^- p$ when it is within the nitrogen atom [?, ?, ?]. Since the Bohr radius of the $X^- p$ is about $r_{xp} = 28.8 fm$, which is much smaller than the

typical radius of a nitrogen electron, $r_n \approx 4000 fm$, we can effectively ignore the polarization of the nitrogen atom by the $X^- p$.

The standard quantum theory of the linear Stark effect [?] gives the polarizability of the of the ground state of the hydrogenic neutraCHAMP as:

$$\chi = -2e^2 \sum_{n>1} \frac{|< n,0,0 \mid r \mid 1,0,0 >|^2}{E_0 - E_n}.$$

Following the elegant summation method of Dalgarno and Lewis [?, ?, ?, ?], the polarizability of the $X^- p$ ground state is:

$$\chi = 4.5 r_{xp}^3 = 4.5 \left(\frac{m_e}{m_p}\right)^3 r_{pe}^3 = 1.08 \times 10^5 \ fm^3,$$

where r_{pe} is the hydrogen Bohr radius. Notice that the neutraCHAMP polarizability is about 10^8 times the neutron polarizability, $\chi_n \approx 1.1 \times 10^{-3} \ fm^3$[?], and about 10^{-9} times the hydrogen atom polarizability, $\chi_h = 6.67 \times 10^{14} \ fm^3$. Therefore, we cannot ignore the neutraCHAMP structure.

Herman & Skillman [?, ?, ?] calculate the radial wavefunctions of the nitrogen atom by the Thomas-Fermi approximation. Using their data, we fit the electronic charge density of the nitrogen atom to a sum of two exponentials (Fig.1):

$$\begin{aligned} \rho(r) \ d^3r &= \left(Ze\delta^3(\vec{r}) - \frac{e}{4\pi} \frac{2\psi_{10}^2 + 2\psi_{20}^2 + 3\psi_{21}^2}{r^2} \right) d^3r, \\ &\approx \left(Ze\delta^3(\vec{r}) - \frac{e}{4\pi} \sum_{i=1}^{2} A_i e^{-\frac{r}{r_i}} \right) d^3r \end{aligned}$$

where $A_1 = 1.767 \times 10^{-11} \ fm^{-3}$, $r_1 = 3900 \ fm$, $A_2 = 1.289 \times 10^{-13} \ fm^{-3}$, $r_2 = 26700 \ fm$, and ψ_{nl} is the (n,l)th radial wavefunction of the nitrogen atom, multiplied by r. Notice that: $Z = 7 = \sum_{i=1}^{2} 2A_i r_i^3$. From Gauss' law, the electric field, $\vec{E}$, due to this charge distribution is:

$$\vec{E} = \frac{e}{r^2} \sum_{i=1}^{2} 2A_i r_i^3 e^{-\frac{r}{r_i}} \left[1 + \frac{r}{r_i} + \frac{1}{2} \left(\frac{r}{r_i} \right)^2 \right].$$

Using Jackson's result [?] for the interaction energy of an external-field induced dipole with the external field, the interaction potential is :

$$V(r) = -\frac{1}{2} \vec{p} \cdot \vec{E} = -\frac{1}{2} \chi \mid \vec{E} \mid^2 .$$

With the above result for the electric field, the potential is:

$$V(r) = -\frac{1}{2} \frac{e^2 \chi}{r^2} \left[\sum_{i=1}^{2} 2A_i r_i^3 e^{-\frac{r}{r_i}} \left(1 + \frac{r}{r_i} + \frac{1}{2} \left(\frac{r}{r_i} \right)^2 \right) \right]^2 .$$

However, for $r > r_1$, the interaction energy is very small compared to the kinetic energy of the collision, so that only the region where $r \ll r_1$ contributes to significant energy loss due to elastic scattering. We can approximate quite well the interaction potential by ignoring electronic screening of the nucleus and protonic screening of the X^- (Fig. 2):

$$V(r) = \frac{-Z^2 e^2 \chi}{2r^4}.$$

Given the interaction potential, in the center of mass frame, and using classical collision theory [?], we can calculate the scattering angle as a function of impact parameter, b, and the center of mass kinetic energy, $E_c = \frac{M_n E}{M_n + M_x}$, where E is the initial kinetic energy of the neutraCHAMP, in the lab frame. The center of mass kinetic energy, E_c, for nitrogen is:

$$E_c = 7keV \left(\frac{v}{v_0}\right)^2.$$

The equation for the scattering angle is [?]:

$$\theta = \pi - 2\int_{r_0}^{\infty} \frac{b dr}{r^2 \left(1 - \frac{V(r)}{E_c} - \frac{b^2}{r^2}\right)^{\frac{1}{2}}},$$

where r_0 is the largest positive root of the denominator of the above integrand.

The scattering cross-section as a function of angle is:

$$\frac{d\sigma}{d\Omega} = \frac{-b}{sin\theta}\frac{db}{d\theta}.$$

From this result, the energy transfer cross-section is:

$$\frac{d\sigma}{dT} = \frac{4\pi}{T_m}\frac{d\sigma}{d\Omega},$$

where T is the energy transferred to the nitrogen atom, $T = T_m sin^2(\theta/2)$, and $T_m = 4M_x M_n E/(M_x + M_n)^2 = 28keV(v/v_0)^2$ is the maximum energy transferred. Then the energy loss is:

$$\frac{1}{\rho}\frac{dE}{dx} = -\frac{N_a}{A}\int_0^{T_m} T\frac{d\sigma}{dT}dT,$$

N_a is Avogadro's number, and A is the atomic weight of nitrogen [?, ?].

If the impact parameter, b, is taken as the independent variable for the integration, $d\sigma = 2\pi b db$ and $T(b) = T_m sin^2(\theta(b)/2)$, so that the energy loss is:

$$\frac{1}{\rho}\frac{dE}{dx} = -\frac{2\pi N_a T_m}{A}\int_{bmin}^{bmax} sin^2\left(\frac{\theta(b)}{2}\right) b db.$$

2 Scattering in a r^{-4} potential

From the above attractive form of the interaction potential, we find that the centripetal potential [?],

$$V'(r) = V(r) + \frac{E_c b^2}{r^2},$$

will then go to $-\infty$ as r goes to zero, and to zero from the positive side as r goes to ∞. Therefore, somewhere in between 0 and ∞, the effective potential will have a maximum (Fig. 1). If we only include the r^{-4} term of the potential, with no screening, the maximum of the effective potential occurs at $r_m = \sqrt{\chi Z^2 e^2 / E_c b^2}$, and has a value $V'_m = E_c^2 b^4 / 2\chi Z^2 e^2$. If $E_c = V'_m$, an unstable $(X^- p, {}^{14}N)$ orbit arises. Near $E_c = V'_m$, the colliding ${}^{14}N$ will loop several times around the $X^- p$ before scattering at a finite angle. If E_c is fixed, and the impact parameter, b is varied, we can observe this critical phenomenom from a different aspect. For $b \gg b_c$, the scattering angle, $\theta(b)$ (in center of mass coordinates) is negligible. If

$$b = b_c = \sqrt[4]{2\chi Z^2 e^2 / E_c}$$

$$= \frac{b_{c0}}{(v/v_0)^{0.5}},$$

then $\theta(b)$ goes to $-\infty$, due to the unstable orbit, and for $b \ll b_c$, $\theta(b) \approx \pi$ (Fig. 2). For nitrogen, $b_{c0} = 216$ fm.

The $\theta(b)$ curve can be calculated analytically (Fig.3), if we ignore the screening of the potential, and take the term that dominates near $r = 0$: $V(r) = -0.5\chi Z^2 e^2 r^{-4}$. For $b < b_c$, by Gradshteyn and Ryzhik, equations 3.165.2, 8.121.3, and 8.112.1:

$$\theta(b) = \pi - \frac{2b}{b_c} K\left(\frac{1}{\sqrt{2}}\sqrt{1+\sqrt{1+\frac{b^2}{b_c^2}}}\right),$$

where $K(r)$ is the complete elliptic integral of the first kind. For $b \geq b_c$, by Gradshteyn and Ryzhik, equation 3.157.2:

$$\theta(b) = \pi - 2\frac{\sqrt{2}}{\sqrt{1+\sqrt{1-\frac{b_c^4}{b^4}}}} K\left(\sqrt{\frac{1-\sqrt{1-\frac{b_c^4}{b^4}}}{1+\sqrt{1-\frac{b_c^4}{b^4}}}}\right).$$

With the $\theta(b)$ curve, we can easily calculate the energy loss:

$$\frac{1}{\rho}\frac{dE}{dx} = -2\pi \frac{N_a}{A} b_c^2 T_m \; I,$$

where $I(s_{min}, s_{max})$ is the dimensionless integral

$$I(s_{min}, s_{max}) = \int_{s_{min}}^{s_{max}} sin^2(\theta(s)/2))sds,$$

$s = b/b_c$,$T_m \equiv 4M_nM_xE/(M_n + M_x)^2$, E is the kinetic energy of the X^-p system, and N_a is Avogadro's number.

3 Impact Parameter Limits

We may classically calculate a differential cross-section for scattering when the de Broglie wavelength of the colliding nucleus is small compared to any significant dimension of the collision, and when the collision is well-defined in terms of the uncertainty principle [**?**].

The de Broglie wavelength of a particle of reduced mass $\mu = 14M_p$, moving at a velocity, $v = \beta c$, is $\lambda = \hbar c/\mu\beta c$. At $v_0 = 10^{-3}c = 300km/s$, $\lambda = 14fm$. This wavelength may be compared with the smallest screening distance, $r_1 = 3900fm$, the X^-p radius, $r_{xp} = 28.8fm$, and the nitrogen nuclear radius, $r_n = 3.4fm$.

Another limit, r_e, is set by the condition that the electric field, $\vec{E}$, due to the nucleus, must be much smaller than the internal electric field of the X^-p system, $\vec{E}_{xp}$, so that the induced X^-p dipole moment, $\vec{p}$, is linear in $\vec{E}$, and so that the X^-p is not torn apart:

$$\mid \vec{E} \mid \cong \frac{Ze}{r_e^2} \ll \mid \vec{E}_{xp} \mid \cong \frac{e}{r_{xp}^2},$$

or:

$$r_e \gg Z^{\frac{1}{2}} r_{xp} = 76fm.$$

Since we have an attractive potential, proportional to r^{-4}, we can calculate the relationship between the turning point of the collision (distance of closest approach), r_0, and the impact parameter, b, for $b > b_c$:

$$r_0 = \frac{b}{\sqrt{2}} \sqrt{1 + \sqrt{1 - \left(\frac{b_c}{b^4}\right)^4}},$$

which can be inverted to get:

$$b = r_0 \sqrt{1 + \frac{1}{4}\left(\frac{b_c}{r_0}\right)^4}.$$

The minimum impact parameter, for which $r_0 \geq r_e$, is:

$$b_{min} = r_e \sqrt{1 + \frac{1}{4}\left(\frac{b_{c0}}{r_e}\right)^4 \left(\frac{v_0}{v}\right)^2}$$

$$= 76fm\sqrt{1 + 16.3\left(\frac{v_0}{v}\right)^2}.$$

For $v = v_0$, $b_{min} = 1.46b_c = 316fm$. When we require that $b_{min} > \alpha b_c$, where $\alpha \geq 1.0$, we find that the range of velocity for which our calculation applies is:

$$v \in [4.04v_0(\alpha^2 - \sqrt{\alpha^4 - 1}), 4.04v_0(\alpha^2 + \sqrt{\alpha^4 - 1})]$$

.

In classical theory, a critical impact parameter, b_c, appears for an attractive r^{-4} scattering potential. For impact parameters, $b < b_c = \sqrt[4]{2\chi Z^2 e^2/E_c} = (216/\sqrt{v/v_0})fm$, the turning point of the collision is $r_0 = 0fm$. Therefore, for such sub-critical impact parameters, the distance of closest approach, r_0, is less than both λ and r_e, and the classical calculation of a differential scattering cross-section does not apply, and a quantum mechanical calculation including inelastic nuclear effects would be more appropriate [?, ?, ?]. Since such a calculation is outside the scope of the present paper, we require that $b_{min} > 2.0b_c$, so that $v \in [154, 9540]km/s$, is the range over which the present calculation holds.

For well-defined classical collisions, the uncertainty principle warrants a lower limit on the scattering angle, $\theta > \theta_{min} \approx \lambda/2\pi r_1$. For a particle of reduced mass $\mu = 14M_p$, moving at a velocity, $v = \beta c$, $\theta_{min} = \hbar c/2\pi\mu\beta c r_1$. With $v_0 = 10^{-3}c = 300km/s$, and $r_1 = 3900fm$, $\theta_{min} = 6 \times 10^{-4} rads$. We find that this corresponds to a maximium impact parameter, $b_{max} \approx 5b_c$. By using the approximate b^{-4} scaling dependence of $\mid \theta(b) \mid$, for $b > b_c$, we find:

$$b_{max} \approx 5b_c \left(\frac{v}{v_0}\right)^{\frac{1}{4}}.$$

Setting $b_{max} = 10b_c$, to estimate the minimum velocity for which this calculation applies, we find $v_{min} \sim 6.3 \times 10^{-2} v_0 = 19km/s$.

Therefore, in the present classical calculation, the uncertainty principle does not limit us as much as the requirement that the collision is elastic. The interval of integration should be $b \in [2b_c, 10b_c]$, which limits us to the velocity interval $v \in [154, 9540]km/s$.

4 More realistic Interaction Potentials

5 Results

The energy loss is proportional to velocity:

$$\frac{1}{\rho}\frac{dE}{dx} = -S_0\left(\frac{v}{v_0}\right),$$

where $v_0 = 300km/s$. If we neglect the above discussion on the limitations of the classical approximation, and integrate over all impact parameters, then $S_0 = 0.84MeVcm^2/g$, for all $v \ll Z\alpha c$. However, when we only integrate over the interval discussed above, $b_{min} = 2b_c$, and $b_{max} = 10b_c$, then $S_0 = 8.4 \times 10^{-4} MeVcm^2/g$. There is negligible dependence of the energy loss on the mass of the neutrachamp, in the mass and velocity ranges of interest, since $M_x >> M_n$. Since the critical impact parameter, b_c, scales with $\sqrt{Z/M_n}$, the $X^- p$ energy loss scales with $Z/\sqrt{M_n}$ (Table 1).

6 Range-Energy Relations

From our calculation of the stopping power, we may calculate a range-energy relation for the neutraCHAMP in air (assuming $S_0 = 0.84MeVcm^2/g$):

$$R = \frac{2\sqrt{\frac{1}{2}M_p v_0^2}}{S_0}\sqrt{\frac{M_x}{M_p}}\sqrt{E_i} = 0.053\sqrt{\frac{M_x}{M_p}}\sqrt{E_i}\ \frac{g}{cm^2},$$

where E_i is the initial Xp kinetic energy, M_p is the proton mass, and M_x is the neutraCHAMP mass, all given in MeV. The reciprocal energy-range relation for Xp is:

$$E_i = 356\frac{M_p}{M_x}R^2 MeV,$$

where R is given in g/cm^2.

Therefore, if we hope to see Xp of mass $M_x = 10^6 M_p$, as cosmic rays at sea level, where the atmospheric depth is $T = 1000g/cm^2$, we require a minimum energy Xp: $E > E_{min} \approx 356MeV$, or $v_{min} \approx 253km/s$. Likewise, if we expect to see Xp of the same mass as cosmic rays on a mountain-top, where the atmospheric depth is $T = 700g/cm^2$, we require $E > E_{min} = 174MeV$,or $v_{min} \approx 177km/s$. If we assume a Maxwellian velocity distribution, with $v_0 = 300km/s$, this is a significant improvement. However, in a balloon-borne experiment, where $T = 10g/cm^2$, we only require $E > E_{min} \approx 35keV$, or $v_{min} \approx 2.5km/s$, so that we can essentially sample the entire velocity range.

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