

# Turbulence Prediction for Adaptive Optics

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## SUMMARY

We have developed linear zonal predictors of turbulence for adaptive optics. Prediction of turbulence has relevance to improving servo lag error in real time adaptive optics correction of the blurring of images due to:

- Atmospheric turbulence and
- Dynamic processes in the eye.

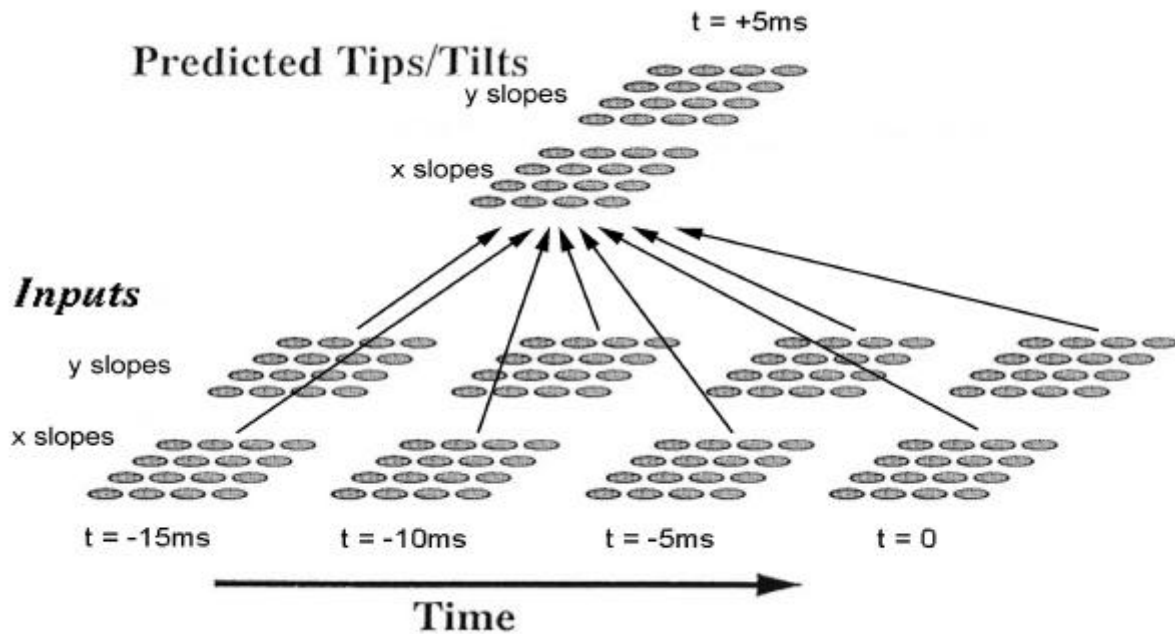
Zonal prediction has the possible advantage of being able to interpret and utilize wind-velocity information from the wavefront sensor better than modal prediction. For simulated open-loop atmospheric data for a 2-meter 16-subaperture AO telescope with 5 millisecond prediction and a lookback of 4 slope-vectors, we find that Widrow-Hoff (WH) Delta-Rule training of linear nets and Back-Propagation training of non-linear multilayer neural networks is quite slow, getting stuck on plateaus or in local minima. Recursive Least Squares (RLS) training of linear predictors is two orders of magnitude faster and it also converges to the solution with global minimum error, as also found with the Adaptive Natural Gradient Learning (ANGL) and Matrix Inversion Least Squares (MILS) zonal predictors.

In the case of bright guidestars, the ANGL, RLS, and MILS algorithms all converge to the same global minimum linear total phase error ( $\sim 0.18 \text{ rad}^2$ ), which is only  $\sim 5\%$  higher than the spatial phase error ( $\sim 0.17 \text{ rad}^2$ ), and is  $\sim 33\%$  lower than the total 'naïve' phase error without prediction ( $\sim 0.27 \text{ rad}^2$ ). The noise performance in the case of dim guidestars is equally impressive. Nonetheless, if each of the dominant turbulence layers in the atmosphere can be independently sensed, then prediction of turbulence for adaptive optics becomes trivial. We have:

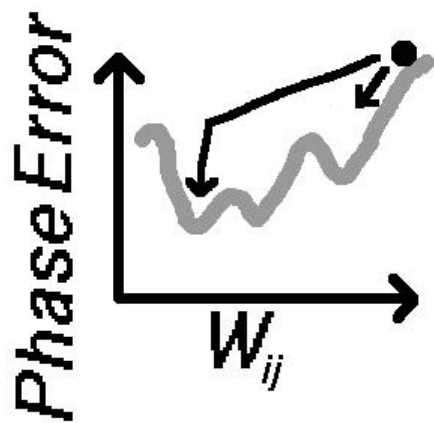
- Scaled our linear work to the  $\sim 108$ -subaperture 6.5 meter MMT AO system (with simulations), in which we discovered that prediction is **much** easier for the high order 6.5 meter AO system than with the low order 2.0 meter AO system. This improvement is caused by the lesser effect of unmeasured turbulence beyond the edge of the pupil since fewer of the subapertures border the edge of the pupil in the case of a high order system.
- Successfully applied our linear work to data from the high order 1.5 meter Starfire Optical Range AO system,

Soon we will:

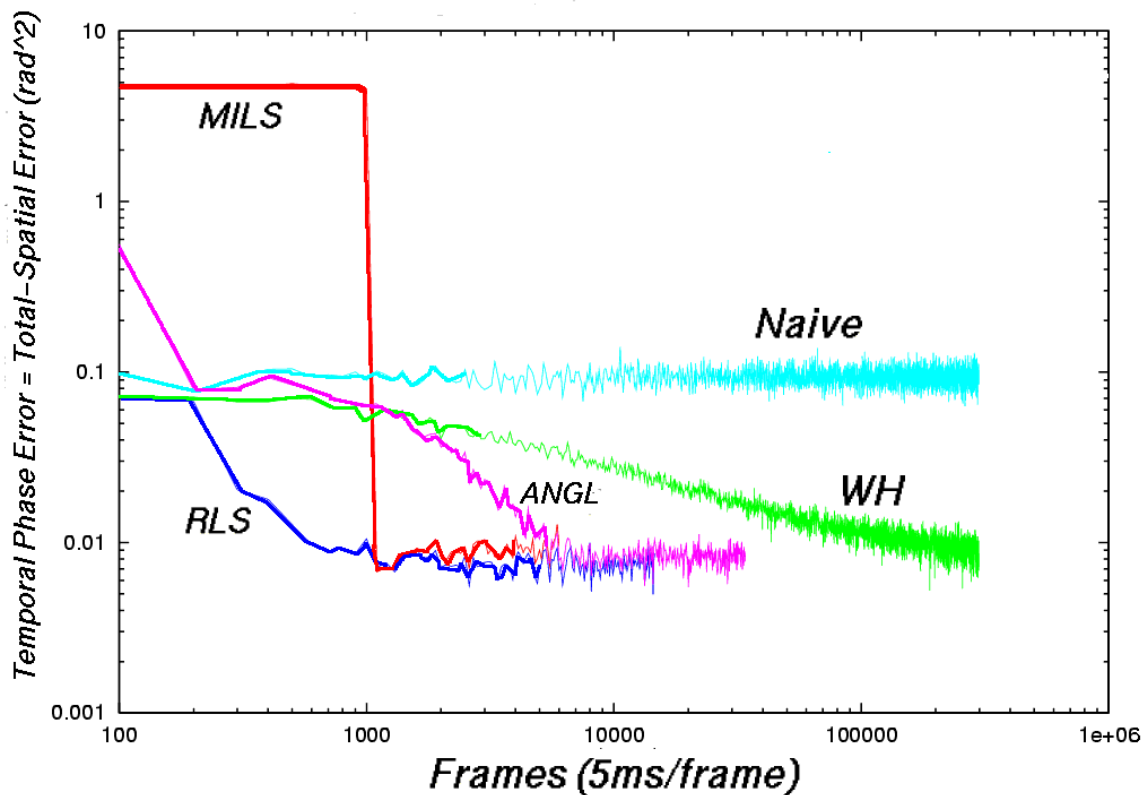
- Apply these linear predictors to real wavefront sensor data from MMT F/9 Cassegrain focus,
- Extend this work to the non-linear regime by employing Support Vector Machines for Regression.



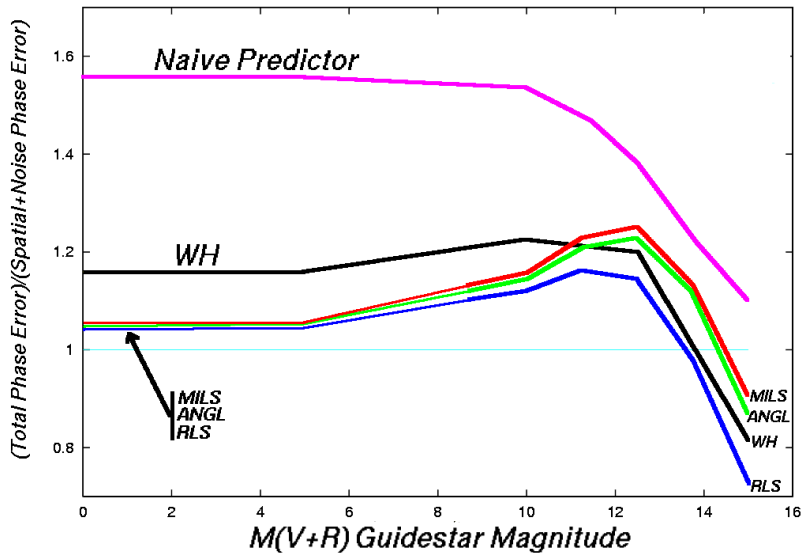
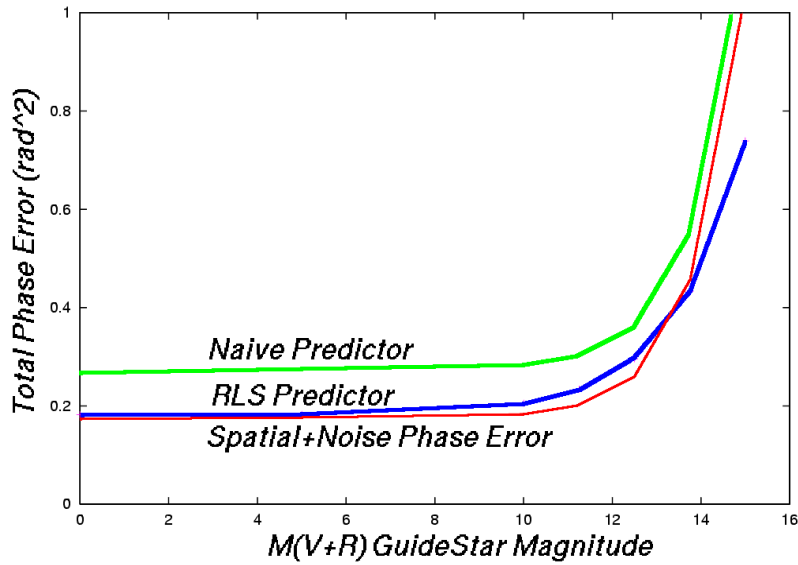
**Figure 1:** A drawing showing how all the slopes from a 4x4 Shack-Hartmann array for the past four frames are used to predict the future slopes (adapted from Refs. 1 & 4).



**Figure 2:** A sketch showing the phase error of a predictor as a function of of a single connection weight,  $W_{ij}$  (between past input  $j$  and future output  $i$ ), with all the other weights fixed. We show how unaided gradient descent algorithms (denoted by the ball following the local slope of the short arrow) potentially can get trapped in local minima of the phase error surface, whereas more sophisticated algorithms can actually find the global minimum.



**Figure 3:** We plot the temporal phase error as a function of training time for 4 different training algorithms. We also show the 'naïve' predictor temporal phase error, in which it is assumed that the atmosphere is random walking, so that the best prediction is that the next slope will be identical to the last. Clearly, the Widrow-Hoff (WH) gradient descent linear network takes over two orders of magnitude longer training time than the other three algorithms. The recursive least squares (RLS) algorithm allows continual updating of the predictor matrix, so it can do better more quickly than the matrix inversion least squares (MILS) solution, and RLS can continue to update and improve slightly even before enough additional data can be acquired for the next MILS matrix inversion. The adaptive natural gradient descent method (ANGL) also converges to the global minimum rather quickly (relative to WH gradient descent), which suggests that the WH gradient descent algorithm is getting stuck in a local minimum.



**Figure 5:** We show how the predictor error for the four different algorithms changes with guidestar magnitude. Figure (a) shows the total phase error (for only the RLS algorithm), and figure (b) shows the ratio of the total phase error to the spatial phase error (for all four algorithms). The spatial+noise phase error is a combination of the fitting and reconstructor errors and the photon noise error. The total phase error encroaches below the spatial phase error for dim guidestars ( $M(V+R) > 13.5$ ) due to the allowance the predictor provides for temporal averaging of several frames together. Clearly, the Widrow-Hoff (WH) algorithm performs worse than the other three algorithms for bright guidestars, though WH performs admirably for the high noise case of dim guidestars ( $M(V+R) > 11$ ). The other three algorithms converge to the same total phase error ( $\sim 0.18 \text{ rad}^2$ ) for bright guidestars ( $M(V+R) \leq 5$ ), which is only  $\sim 5\%$  higher than the spatial phase error ( $\sim 0.17 \text{ rad}^2$ ), and  $\sim 33\%$  lower than the naive predictor total phase error ( $\sim 0.27 \text{ rad}^2$ ).

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